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A DIRECT MATHEMATICAL METHOD TO CALCULATE THE EFFICIENCY OF BORE HOLE CYLINDRICAL DETECTORS

SALAM F. NOUREDDEEN[1,]*, MAHMOUD I. ABBAS[2]

[1]Basic Science Department, Faculty of Science, Arts, Science and Technology,
University of Lebanon, Beirut, Lebanon
[2]Physics Department, Faculty of Science, Alexandria University, 21511 Alexandria, Egypt
*Corresponding Author: schoolnetmlsud@mehe.gov.lb

Abstract

Several techniques can be used to determine the total efficiency including Monte Carlo simulations, semi-empirical methods and experimental measurements. The first technique requires a good definition of the geometry and materials, including window thickness together with an accurate set of cross-sections. The second technique requires two different types of experimental input, the first being from use of sources emitting cascade γ-rays and the second from use of sources emitting isolated γ-rays in order to cover the wide energy range and provide coincidence-summing corrections, respectively. In the present method we introduce a theoretical approach based on the Direct Statistical method proposed by Selim and Abbas to calculate the total efficiencies for point, line and thin circular disk sources for scintillation detectors. The present method combines calculation of the path length covered by the photon inside the detector active volume and the geometrical solid angle Ω, to obtain a straightforward mathematical expression for the efficiency calculation. By comparison, the total efficiency values are in good agreement with the theoretical calculations done by Monte Carlo simulation.

Keywords: Borehole scintillation detector, Total Efficiency, Direct mathematical method, Monte Carlo simulation.

1. Introduction

Gamma radiations can be emitted from different sources with different energies. To detect these radiations several detectors were used. The efficiency of these detectors can be determined experimentally and theoretically [1-12]. In our present work, the photon path length inside the detector is taken in order to determine

Nomenclatures

d_i	The distance covered inside the detector, m
L	Length of the detector, m
H	Height of the source, m
R_i	Radius of the inner cylinder, m
R_o	Radius of the outer cylinder, m

Greek Symbols

μ	Total attenuation coefficient of the detector, m^{-1}
μ_1	Total attenuation coefficient of the housing(Al), m^{-1}
ρ	Lateral distance, m
ξ_1	Efficiency of an axial point source
ξ_2	Efficiency of a non-axial point source
ξ_c	Efficiency of a cylindrical source
ξ_D	Efficiency of a disc source

the total efficiency of the bore hole cylindrical detector. The work described below involves the use of a straight forward analytical formula for the computation of bore hole geometrical and total efficiency. Section 2 presents direct mathematical formulae for the geometrical and total efficiencies in the case of isotropic radiating axial point, non-axial point, line and cylindrical sources. Section 3 contains comparisons between the calculated efficiency using the formulae derived in this work with the published Monte Carlo method for a disk source placed at the canter of the detector. The conclusion is presented in Section 4.

2. Mathematical Viewpoint

In the following, direct mathematical expressions for total efficiencies of a bore hole detector are derived using an isotropic radiating point source that is extended to line, disk and cylindrical sources. The quantities (ρ,h) specify the location of an arbitrarily positioned point source and the polar (θ) and the azimuthal (φ) angles at the point of entrance of the considered surface define the direction of the incidence of a gamma-ray photon. The effective rays passing through the detector active volume traverse a distance d until it emerges from the crystal.

2.1. Axial point source placed inside the detector void part

The incident photon may enter the detector's inner side and emerge. From Fig. 1:

i) Lower base one (LB1)

$$d_1 = \frac{R_0}{\sin(\theta)} - \frac{L}{\cos(\theta)} \tag{1}$$

ii) Detector side one

$$d_2 = \frac{R_0}{\sin(\theta)} - \frac{R_i}{\cos(\theta)} \tag{2}$$

The polar angle θ takes the steps

$$\theta_1 = \tan^{-1}\frac{R_i}{L} \tag{3}$$

$$\theta_2 = \tan^{-1}\frac{R_0}{L} \tag{4}$$

The total efficiency can be calculated by

$$\xi_1 = \int_0^{2\pi}\int_{\theta_1}^{\theta_2} f_1 d\theta\, d\phi + \int_0^{2\pi}\int_{\theta_2}^{\frac{\pi}{2}} f_2 d\theta\, d\phi \tag{5}$$

$$f_i = (1 - e^{-\mu d_i})\sin(\theta)\,.\,e^{-\frac{\mu_1 . t}{\sin(\theta)}} \quad , \text{i=1,2} \tag{6}$$

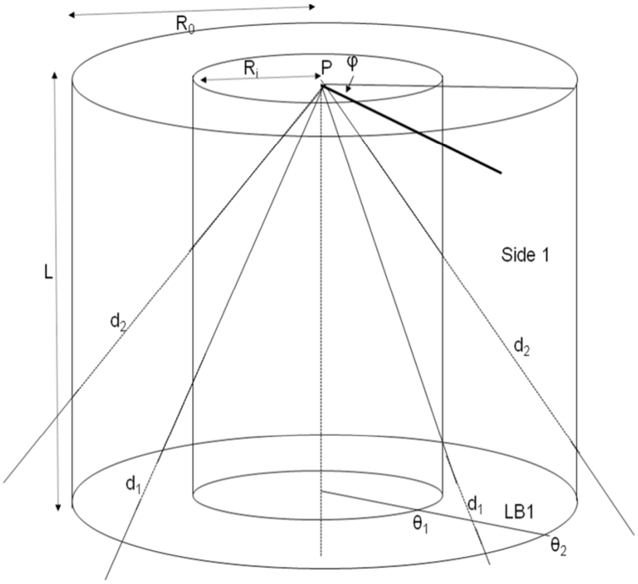

Fig. 1. Schematic View of all the Possible Path Lengths through the Active Medium of a Borehole Cylindrical Detector for a Photon Emitted from an Isotropic Axial Radiating Point Source.

2.2. Non-axial point source placed inside the detector void part

The incidence photon may enter the detector's inner side and emerges. From Fig. 2:

i) lower base 1 (LB1)

$$d_1 = \frac{L}{\cos(\theta)} - \frac{R_i - 2\rho\cos(\phi)}{2\sin(\theta)\cos(\phi)} \tag{7}$$

ii) detector side (1)

$$d_2 = \left|\frac{R_0 - R_i}{2\sin(\theta)\cos(\phi)}\right| \tag{8}$$

iii) lower base 2 (LB2)

$$d_3 = \frac{L}{\cos(\theta)} - \frac{R_i}{2\sin(\theta)\sin(\phi)} + \frac{\rho}{\sin(\theta)} \tag{9}$$

iv) detector side 2

$$d_4 = \left|\frac{R_0 - R_i}{2\sin(\theta)\sin(\phi)}\right| \tag{10}$$

v) lower base 3 (LB3)

$$d_5 = \frac{L}{\cos(\theta)} - \frac{R_i + 2\rho\cos(\phi)}{2\sin(\theta)\cos(\phi)} \tag{11}$$

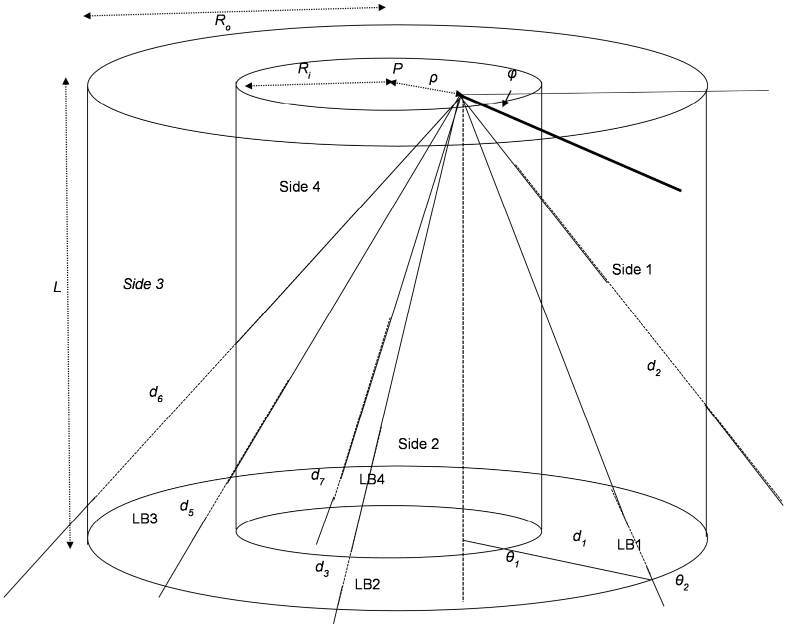

Fig. 2. Schematic View of all the Possible Path Lengths through the Active Medium of Borehole Cylindrical Detector for a Photon Emitted from an Isotropic Non-Axial Radiating Point Source.

vi) detector side 3

$$d_6 = \left|\frac{R_0 - R_i}{2\sin(\theta)\sin(\phi)}\right| \tag{12}$$

vii)lower base 4 (LB4)

$$d_7 = \frac{L}{\cos(\theta)} - \frac{R_i + 2\rho\sin(\phi)}{2\sin(\theta)\sin(\phi)} \tag{13}$$

viii) detector side 4

$$d_8 = \left| \frac{R_0 - R_i}{2\sin(\theta)\sin(\phi)} \right| \tag{14}$$

The final expression of the total efficiency of a non-axial point source at different positions is given by:

$$\begin{aligned}\xi_2 = &\int_0^{\frac{\pi}{2}} \int_{\theta_1}^{\theta_2} f_1 \, d\theta \, d\phi + \int_0^{\frac{\pi}{2}} \int_{\theta_2}^{\frac{\pi}{2}} f_2 \, d\theta \, d\phi + \int_0^{\phi_5} \int_{\theta_1}^{\theta_2} f_3 \, d\theta \, d\phi + \int_{\frac{\pi}{2}}^{\pi} \int_{\theta_1}^{\theta_2} f_4 \, d\theta \, d\phi \\ &+ \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_{\theta_5}^{\theta_6} f_5 \, d\theta \, d\phi + \int_{\pi}^{\frac{3\pi}{2}} \int_{\theta_6}^{\frac{\pi}{2}} f_6 \, d\theta \, d\phi + \int_{\frac{3\pi}{2}}^{2\pi} \int_{\theta_7}^{\theta_8} f_7 \, d\theta \, d\phi + \int_{\frac{3\pi}{2}}^{2\pi} \int_{\theta_8}^{\frac{\pi}{2}} f_8 \, d\theta \, d\phi\end{aligned} \tag{15}$$

The polar angle θ takes the steps

$$\theta_1 = \tan^{-1}\left(\frac{R_i - 2\rho\cos(\phi)}{2L\cos(\phi)}\right) \tag{16}$$

$$\theta_2 = \tan^{-1}\left(\frac{R_0 - 2\rho\cos(\phi)}{2L\cos(\phi)}\right) \tag{17}$$

$$\theta_3 = \tan^{-1}\left(\frac{R_i - 2\rho\sin(\phi)}{2L\sin(\phi)}\right) \tag{18}$$

$$\theta_4 = \tan^{-1}\left(\frac{R_0 - 2\rho\sin(\phi)}{2L\sin(\phi)}\right) \tag{19}$$

$$\theta_5 = \tan^{-1}\left(\frac{R_i + 2\rho\cos(\phi)}{2L\cos(\phi)}\right) \tag{20}$$

$$\theta_6 = \tan^{-1}\left(\frac{R_0 + 2\rho\cos(\phi)}{2L\cos(\phi)}\right) \tag{21}$$

$$\theta_7 = \tan^{-1}\left(\frac{R_i + 2\rho\sin(\phi)}{2L\sin(\phi)}\right) \tag{22}$$

$$\theta_8 = \tan^{-1}\left(\frac{R_0 + 2\rho\sin(\phi)}{2L\sin(\phi)}\right) \tag{23}$$

2.3. Line source placed inside the detector void part of the bore hole cylindrical detector

The efficiency of a 4πNaI (Tl) bore hole cylindrical detector arising from a line source of length ℓ is derived as [3]

$$\xi_L = \frac{1}{h_2 - \mathrm{h}_1} \int_{h_1}^{h_2} \xi_1 dh \tag{24}$$

where, ξ_1 is the total efficiency of an axial point source as identified before in Eq. (5). The total efficiency of the line source is determined at different positions along the axial axis of the detector.

2.4. Disc source placed inside the detector void part of the bore hole cylindrical detector

The efficiency of a 4πNaI (Tl) bore hole cylindrical detector arising from a disc source is derived as [3]:

$$\xi_D = \frac{2}{s^2} \int_0^s \xi_2 \rho d\rho \tag{25}$$

where ξ_2 is the total efficiency of a non-axial point source as identified before in Eq. (15). The total efficiency of the disc source is determined at different positions along the axis of the detector.

2.5. Cylindrical source placed inside the detector bore hole cylindrical detector

The efficiency of a 4πNaI (Tl) bore hole cylindrical detector arising from a cylindrical source is derived as [3 and 6-8]

$$\xi_C = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} \xi_D dh \tag{26}$$

where ξ_D is the total efficiency of a disc source as identified before in Eq. (25).

3. Results and Discussion

The integrals in the present efficiency equations are elliptic integrals and do not have a closed solution. Then a numerical solution is obtained using the MATHCAD 14 software. The total efficiency is determined in our present work by the direct mathematical method using isotropic point source which is extended to line and disc sources. The bore hole detector is made up of a cylinder in which the central part is left void so the radiating source can be moved easily as shown in Figs. 1 and 2. The radii of the central part R_i and outer surface R_0 are 0.1m and 0.3m respectively, while the length is 0.4m. The housing of the NaI(Tl) crystal is 5×10^{-4} m thick aluminum where the diameter of the hole inside the detector is 35×10^{-3}m. The solid angle of the detection geometry is high, so that more than 95% can reach the detector surface. Three different sources point, line (with length 0.02 m) and disc (with radius 1.6×10^{-2} m) sources have been used to calculate the total efficiency. Table 1 shows the separation distance between the source position and the detector centre increases the total efficiency decreases for the different sources which is consistent with the results obtained by Abbas for other detectors [1-10].

Table 2 shows the variation of the total efficiency with the energy of incident photons. These values are compared to the values obtained by Monte Carlo [12]. In our work the height is taken from zero which is from the centre to 0.15 meters which is at the surface of the detector. The same calculations are repeated again on the lower side of the detector. It is clear also that the total efficiency for disc source is greater than that of line and point sources for the same energy. The percentage difference between the calculated values and the measured ones [12] are shown in Table 2.

Table 1. The variation of the Efficiency with Height for Axial and Non-Axial Point Source and Line Source.

	Efficiency		
	Point Source		**Line source**
***h* (cm)**	**Axial point source**	**Non-axial point source**	
0	0.886	0.862	0.919
1	0.886	0.859	0.917
2	0.884	0.85	0.913
3	0.879	0.834	0.907
4	0.869	0.807	0.896
5	0.842	0.763	0.873
6	0.778	0.692	0.818
7	0.62	0.569	0.693

Table 2. Comparison between the Calculated Values of the Total Efficiency of a Disc Source Placed at the Center of the Bore Hole Scintillator Detector Data with Efficiencies Calculated by Monte Carlo Method.

Energy MeV	**Efficiency M.C. [12]**	**Efficiency Present Work**	$\Delta = \frac{\varepsilon_{cal} - \varepsilon_{meas}}{\varepsilon_{cal}} 100\%$
0.1	0.55	0.934	-69.81
0.2	0.889	0.944	-6.18
0.3	0.949	0.944	0.52
0.5	0.968	0.926	4.33
0.7	0.961	0.906	5.72
1	0.941	0.878	6.69
1.2	0.932	0.862	7.51
1.3	0.925	0.854	7.67
1.4	0.914	0.848	7.22
1.5	0.913	0.84	7.99
1.7	0.893	0.83	7.05
2	0.879	0.816	7.16
2.5	0.871	0.8	8.15
3	0.861	0.79	8.24
3.5	0.848	0.778	8.25
4	0.847	0.78	7.91
5	0.845	0.778	7.92
7	0.85	0.784	7.76
8	0.855	0.78	8.77
10	0.864	0.798	7.63
15	0.882	0.816	7.48

4. Conclusions

Direct mathematical expressions to calculate the total efficiency of 4πNaI(Tl) bore hole detector have been derived in the case of point, line and disk sources. The derived efficiency of the detector is studied as a function of the energy of the incident photon when the disk source is placed at the center of detector. The results shows good agreement between the present and the published values except for very low energy source because Monte Carlo method [12] does not give accurate values at these small energies, the high discrepancies being less than 9%.

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