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FRACTIONAL ORDER SYSTEM IDENTIFICATION BASED ON GENETIC ALGORITHMS

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Abstract

System identification deals with estimating the plant parameters under control using input-output measuring data. Most of practical plants have fractional order dynamic properties which are based on integration and differentiation of non-integer order. In this work the structure and the parameters of fractional order unknown transfer function are estimated using input-output data. Integer order Least Squares identification is used first to confirm the structure (order) of the unknown transfer function. Then, Genetic Algorithms (GAs) is followed to find the most accurate fractional order estimate that represents the system. Illustrative examples are presented in which fractional order transfer functions are identified in a way that faithfully estimates the dynamics of the unknown plants.

Keywords: System Identification, Least Squares, Fractional order, Genetic Algorithms.

1. Introduction

Prior knowledge of the parameters of system under control is often the first step in designing controllers. The concepts of system identification focused on proper estimation of the parameters of real plants. Many statistical and geometric methods such as least square, maximum likelihood and regression models are extensively used for real-time parameter estimation [1, 2]. The real world plants or processes are generally of fractional order [3].The problem of parameter identification becomes more difficult for a fractional order system compared to an integral order one. However, it is used to approximate a fractional order process as an integer order one. In general, this approximation can cause significant differences between a real system and its mathematical model. The idea of using fractional-order calculus in modelling fractional order control systems is well-addressed in [4, 5]. Advantages of using fractional-order system identification have been introduced in a number of publications [6-8].

Nomenclatures

D^r	Fractional differentiation operator
na	Positive integer number represents the number of coefficients in the denominator
nb	Positive integer number represents the number of coefficients in the nominator
u	Input signal
w_A	Low frequency of the bandwidth
w_B	High frequency of the bandwidth.
y_m	Model output
y_p	Plant output

However, the Evolutionary Algorithms (EAs) is used by many researchers to find the "optimal" solution relative to the defined problem. Different approaches were proposed in this context to choose an objective function that really meets the desired specifications and obeys the practical constraints (see for example [9] in which Particle Swarm Optimization (PSO) algorithm is used).

In this work, the Least Squares (LS) method is adopted in order to facilitate the problem of finding the unknown transfer function structure (i.e., the nearest integer numerator and denominator order). Then the (LS) results are utilized as an initial guess in the GAs to obtain the coefficients and the fractional order of the transfer function.

2. Fractional Order System Modelling

Fractional order systems are described by fractional order differential equations. Fractional calculus allows the derivatives and integrals to be arbitrary number. There are several definitions of fractional derivatives [10]. Grunwald-Letnikov definition is perhaps the best known one due to its most suitable for the realization of discrete control algorithms. The rth order fractional derivative of continuous function $f(t)$ is given by:

$$D^r f(t) = \frac{d^r f(t)}{dt^r} = \lim_{h \to 0} h^{-r} \sum_{j=0}^{[x]} (-1)^{-j} \binom{r}{j} f(t - jh) \tag{1}$$

where $[x]$ is a truncation and $x = \frac{t-r}{h} \binom{r}{j} = \frac{r(r-1)......(r-1+j)}{j!}$. The general calculus operator, including fractional order and integer order is defined as:

$$aD_t^r = \begin{cases} d^r / dt^r \cdots\cdots \Re(r) \rangle 0 \\ 1 \cdots\cdots\cdots\cdots \Re(r) = 0 \\ \int_a^t (d\tau)^{-r} \cdots\cdots \Re(r) \langle 0 \end{cases} \tag{2}$$

where a and t are the limits related to operation of fractional differentiation and r is the calculus order ($0 < r < 1$). Accordingly, Fractional order systems can be represented by the generalized fractional differential equation given by [11]:

$$y(t) + \sum_{i=1}^{na} a_i D^{\alpha_i} y(t) = \sum_{j=0}^{nb} b_j D^{\beta_j} u(t) \tag{3}$$

where $\alpha_1 < \alpha_2 < \cdots < \alpha_{na}$, $\beta_0 < \beta_1 < \cdots < \beta_{nb}$ are non-integer positive numbers and i=1, 2, …, na, j=0, 1, 2,…, nb. The Laplace transform of Eq. (3) can be given by:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{j=0}^{nb} b_j s^{\beta_j}}{1 + \sum_{i=1}^{na} a_i s^{\alpha_i}} \tag{4}$$

Equation (4) can be approximated using indirect discretization method which consists of two steps. In the first step a rational analog transfer function that approximates the irrational transfer function of the fractional system is obtained. In the second step an analog to digital transformation of the rational analog approximation is done.

- **Analog rational approximation**

The approximation is carried out in the following two steps:

- Determine a frequency range $[\omega_A, \omega_B]$ where it is needed to approximate the fractional order transfer function.
- Perform an analog integer-order approximation of the fractional-order transfer function:

$$s^{-r}_{[\omega_A - \omega_B]} \cong \frac{\omega_B(\omega_A + s)}{(\omega_A)^n (rs^2 + \omega_B s + (1-r)\omega_A \omega_B)} \left(\frac{1 + \frac{s}{\omega_B}}{1 + \frac{s}{\omega_A}} \right)^r \tag{5}$$

Equation (5) is expanded using continued fraction expansion CFE, Taylor series expansion, or the method of recursive poles and zeros to obtain an analog approximation. The analog approximation with recursive poles and zeros is summarized below [12]:

$$\left(\frac{1 + \frac{s}{\frac{d}{b}\omega_A}}{1 + \frac{s}{\frac{b}{d}\omega_B}} \right)^r = \lim_{N \to \infty} \prod_{k=-N}^{k=N} \frac{1 + \frac{s}{\omega'_k}}{1 + \frac{s}{\omega_k}} \tag{6}$$

$$\omega_k' = \left(\frac{d}{b}\omega_A\right)^{\frac{r-2k}{2N+1}} \tag{7}$$

$$\omega_k = \left(\frac{b}{d}\omega_B\right)^{\frac{r+2k}{2N+1}} \tag{8}$$

$$s^r \approx K\left(\frac{ds^2+bs\omega_B}{d(1-r)s^2+bs\omega_B+dr}\right)\prod_{k=-N}^{k=N}\frac{s+\omega_k'}{s+\omega_k} \tag{9}$$

$$K = \left(\frac{d}{b}\omega_A\right)^r \prod_{k=-N}^{k=N}\frac{\omega_k}{\omega_k'} \tag{10}$$

In this work b = 10, d = 9 , ω_A=0.01 rad/s, ω_B=10000 rad/s, and N=7 as in [12].These parameters are selected to ensure the dynamic range of most industrial plants. The procedure for the approximation can be briefly summarized in the following steps:

a) The frequency range [ω_A, ω_B] and N are given.

b) Based on the fractional order r, calculate ω_k' and ω_k according to Eqs. (7) and (8).

c) Compute K from Eq. (10).

d) Obtain the approximate rational transfer function from Eq. (9) to replace s^r.

- **Discretizing the analog approximation**

To discretize the analog rational approximation of Eq. (9) select suitable s-to-z transforms. The bilinear transformation is employed by replacing s in Eq. (9) by the right hand side of Eq. (11)

$$s \approx \frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})} \tag{11}$$

3. The Genetic Algorithm [13, 14]

The Genetic Algorithms (GAs) is a direct random search technique, which can find the optimal solution in complex multi-dimensional search space. It is a model of natural evolution operators which manipulate individuals in a population over several generations to improve their fitness gradually. These individuals that, represent a candidate solution to the problem under consideration, often encoded as a bit string.

There are two basic issues in the GA, the first one is how to code the problem this is called *coding*. The second issue is how to qualify each individual (string) which is called the *fitness evaluation.* This step depends on the problem nature. It is either mathematic equation or a rule-based procedure or in some cases a combination of both. These two issues are problem dependent.

The remaining part of the GA is problem independent which consists mainly of the GA operators. Three operators are basically important, that are *selection (reproduction), crossover, and mutation*. The selection procedure is to reproduce more strings whose fitness functions are higher than those whose fitness functions are low. This is important to drive the search towards better and better solution. Reproducing can be implemented by many methods (for example *Roulette Wheel, Tournament*…etc.).The crossover is important to create new individuals from already exist ones. The mutation is an operator that avoids the search process to be trapped at a local area in the hyper search space by exchanging bits of a certain string. The following algorithm clarifies the main steps in GA:

Step-1: Create an initial population of strings.

Step-2: Calculate the fitness of each string.

Step-3: While (an acceptable solution is not found) OR (does not exceed maximum number of generation)

- Perform reproduction for next generation.
- Perform crossover between parents to create new offspring.
- Apply mutation with certain probability.
- Calculate the fitness of each offspring.

End while

4. Genetic-Based Tuning of FO System

The undertaken identification problem is to find the optimal tuning gains for the FO system given in Eq. (4) that ensures certain objective function. The objective function is suggested here to force FO system to behave similar to a predefined model (the plant). Therefore, each GA solution has to minimize the following objective function:

$$Fitness = \sum_{T_o} \left| y_p(t) - y_m(t) \right| \tag{12}$$

To is the number of collected data samples. Binary coding is used to represent the FO system tuning parameters as shown in Fig. 1. For each exponent (α_i or β_j), 9bits representation is used to ensure resolution of 2^{-9}=0.00195. While for each coefficient (a_i and b_j), 25 bits representation is used: 7 bits for the integer part (0 to 127) and the remainder 18 bits for the fractional part (i.e., with resolution of $2^{-18}=3.8\times10^{-6}$).

a_1	a_2	⋯	a_{na}	α_1	α_2	⋯	α_{na}	b_0	b_1	⋯	b_{nb}	β_0	β_1	⋯	β_{nb}

Fig. 1. FO System String Coding.

The whole block diagram representing the proposed design method is shown in Fig. 2.

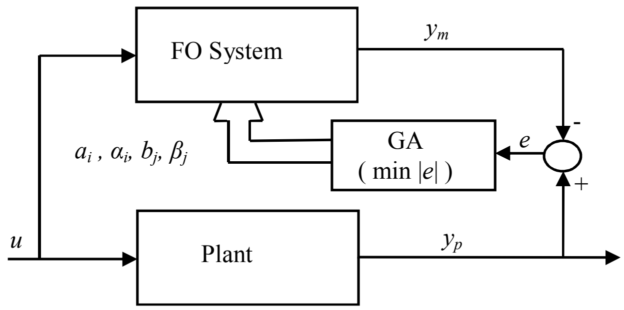

Fig. 2. The Block Diagram of the Proposed Design Method.

5. Proposed Procedure

The problem under consideration is to find the structure and the parameters of a fractional order system defined by [4] using input-output measured data. Therefore, statistical standard Least-square based linear estimators have been employed with ARMAX (Auto-Regressive Moving Average with eXogenous input). Such step gives an initial guess of the system integer order transfer function. In other words, it fits the input-output data into the following model:

$$\frac{Y(s)}{U(s)} = \frac{b_0 + b_1 s + ... + b_n s^n}{1 + a_1 s + ... + a_m s^m} \tag{13}$$

Practically, $m > n$ .Now, knowing the system integer order (n and m) and its parameters, the GAs will employed such information as an initial guess to search for a fractional version of [11].

6. Illustrative Examples

In order to illustrate the above procedure in identification of fractional order system, two examples are presented here.

6.1. Example-1: Fractional order hypothetical model

Consider the following fractional order transfer function,

$$\frac{Y(s)}{U(s)} = \frac{s^{0.5} + 1}{s^{1.75} + 3s^{0.6} + 1} \tag{14}$$

Following the procedure of Sec 2, the simulation of Eq. (14) is accomplished under MATLAB environment. The sampling time is chosen to be 0.001 s.

Several trials have been accomplished using LS identification method (*arx.m* in MATLAB identification Toolbox) to discover the 'best' integer order model from a set of input-output data as shown in Table 1. It is found that the following model has the least sum absolute errors between the actual and the model outputs.

$$\frac{Y(s)}{U(s)_{LS}} = \frac{0.008153s + 16.53}{s^2 + 14.11s + 30.33} \tag{15}$$

Table 1. LS Identification of Example-1.

Model Order	Sum of absolute errors	RMS of error
1	776.2830	0.0479
2	355.8586	0.0222
3	360.3229	0.0238
4	331.0720	0.0213

Knowing the system structure, the GAs is now used to find the unknown parameters which are shown in Eq. (16). The population size is 1200, the maximum number of generation is 150, the number of collected data sample is samples, and the crossover probability and the mutation probability are chosen by trial and error to be 0.2 and 0.35, respectively (the crossover probability and the mutation probability values depend on the complexity degree of the hyper surface that relates all the parameters to be optimized [14]). The fitness function used is the sum of absolute deviations from the actual set of output observations.

$$\frac{Y(s)}{U(s)} = \frac{s^{0.227539} + 0.23814}{s^{1.37891} + 0.670231s^{0.488281} + 1.70802} \tag{16}$$

The response of the considered system, the LS identified and the GAs identified models are shown in Fig. 3. The error of LS identified and the GAs FO identified models are shown in Fig. 4. The sum of absolute errors of the GAs FO identified model is (76.5425) (RMS=0.0053).

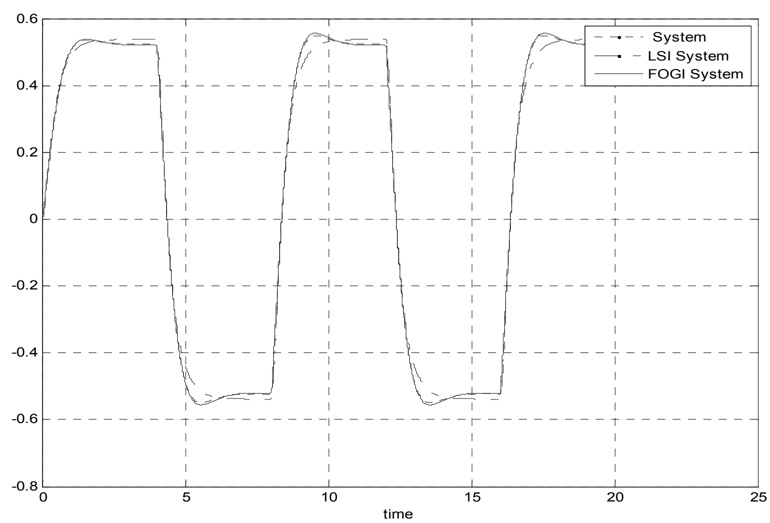

Fig. 3. The Step Response of the Considered System, LS and GA FO Identified Models.

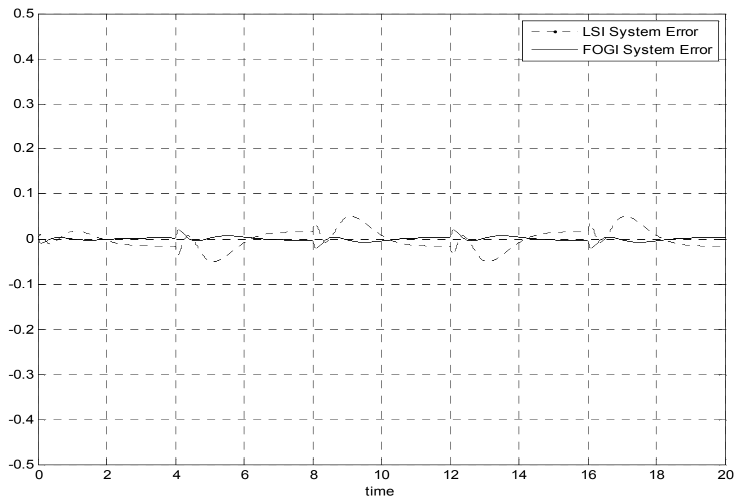

Fig. 4. The error of LS and GA FO Identified Models.

6.2. Example-2: Fractional order Model of DC Motor

A practical gathered data from 3KW DC motor is considered. The data are sampled at 0.01 s; the excitation of the motor is DC voltages whereas the response is speed in rpm. Using LS identification method, it is found that the best integer order model is Eq. (17).

$$\frac{Y(s)}{U(s)_{LS}} = \frac{9.3438s + 82.706}{s^2 + 12.2307s + 12.5447} \tag{17}$$

Knowing the system structure, the GAs is now used to find the unknown parameters which are shown in (23). The population size is 1200, the maximum number of generation is 150, the number of collected data sample is samples.

$$\frac{Y(s)}{U(s)} = \frac{3.2625s^{0.9971} + 1.9643}{s^{1.6484} + 0.3063s^{0.0127} + 0.3497} \tag{18}$$

The response of practical system, the LS identified and the GAs FO identified models are shown in Fig. 5. The error of LS identified and the GAs FO identified models are shown in Fig. 6. The sum of absolute errors of LS identified and the GAs FO identified models are (11655) (RMS=23.1176) and (10104) (RMS=22.3176) respectively.

7. Conclusions

In this paper a procedure was suggested to find the 'best' structure and coefficients of a fractional order control system. This procedure utilizes firstly the LS method to find the suitable integer order structure (poles and zeros) that fits the input-output system data. Then, GA was exploited to find the fractional order version of that obtained using LS method.

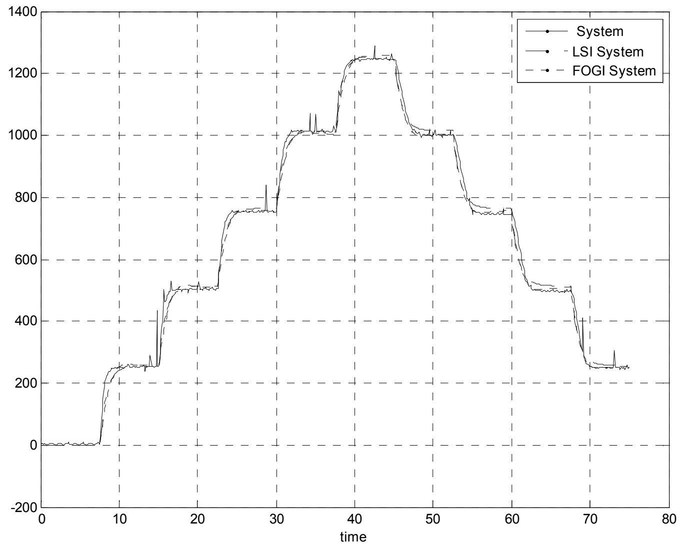

Fig. 5. The Response of the Practical System, LS and GA FO Identified Models.

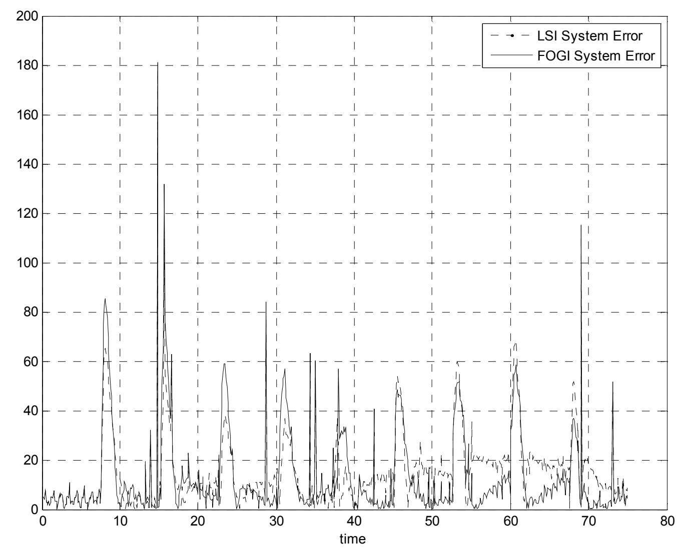

Fig. 6. The Error of LS and GA FO Identified Models.

This procedure is illustrated using an example of fractional order model and practical voltage-speed data of a 3KW DC motor. It is found that the sum of the absolute error between the 'best' LS step response and the GA fractional order response are (355.8586) and (76.5425) for the first example and (11655) and (10104) for the second example, respectively. Therefore, the identified GA fractional order models obtained by the proposed procedure are more suitable to be used in control algorithms that fully require the system dynamics.

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