

PERFORMANCE IMPROVEMENT OF A PARALLEL REDUNDANT SYSTEM WITH COVERAGE FACTOR

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Abstract

In this paper we propose a new coverage model which shows improvement in the availability of the system and the reduction in the cost which are important measures for reliability. The present study discusses the effect of coverage factor on the reliability characteristics of a parallel redundant complex system. The system is analysed under Preemptive resume-repair policy. Using the concept of coverage factor, the availability and cost analysis of the system have been computed and analysed by graphical illustrations.

Keywords: Coverage factor, Optimal maintenance policy, Performability, Availability, Cost analysis.

1. Introduction

A system is composed of subsystems. It is assumed that every subsystem must survive in order to survive the system. This assumption leads to the fact that the system reliability is the product of the divided reliability of each sub system. It is well known that each component and the system will be in one of the two states, either successful or failed. The components are independent and identically distributed (i.i.d.) and the reliability of each component are active or passive. The reliability characteristics are obtained by assuming that the fault detection and switch over mechanism are perfect.

As the coverage directly influences the system reliability, it is important to characterize the recovered fault proportion in order to accurately determine the coverage factor of redundant equipment and to calculate the impact of the parameter on the overall system reliability.

The coverage factor is defined as the conditional probability of recovery, given that a fault has occurred. It was first introduced by Bouricius et al. [1]. Coverage quantitatively characterizes the adequacy of fault-detection and recovery methods.

Coverage factor α = probability (fault detected system recovers/ fault occurs)

The coverage factor is associated to the supervising mechanism and the reliability of the system to promptly recover from the occurrence of fault. The coverage analysis brings the coverage factor influence on the availability of redundant equipment and the method used for its evaluation. The estimation of coverage is in practice a difficult problem because of its complex parameter dependency on both fault mechanisms and the recovery capability the system. Application of coverage is a useful method for enhancing the reliability of a system. It is an effective method for quantitative and qualitative analysis for failure method of complex systems.

The previous models work out system reliability using any standard solution method that ignores the concept of coverage. In many fault-tolerant systems, the main cause of system failure is not exhaustion of redundancy but imperfect coverage due to the inability of the system to successfully reconfigure from a component failure even when spare components are available as discussed by Arnold [2]. Fault coverage is a measure of a system's ability to perform fault detection, fault location, fault containment, and/or fault recovery. Fault recovery coverage is a system's ability to successfully recover from faults and maintain an operational status, after the occurrence of faults. Thus a covered fault is one from which the system can automatically recover. An uncovered fault is one of the reasons that lead to immediate system failure, apart from of the current state of the system. Fault coverage is normally assumed to be constants for simplicity, but they can be a function of time. Another application of coverage factor is that it is used to quantify the efficiency of fault-tolerant systems, since the validation of fault-tolerant systems is based on the efficiency of their fault tolerance mechanisms as shown by Powell et al. [3]. A modelling approach for evaluating the effectiveness of a reconfiguration scheme in a fault-tolerant network is discussed by Reibman and Zaretsky [4], in which the high-level model represents the occurrence of failures in the network and includes a coverage factor and a lower-level model represents the network reconfiguration system, and is used to estimate the coverage. Also Cai et al. [5], Prabhudeva and Verma [6], and Kumar et al. [7] used coverage factor in various reliability purposes. Ram and Singh [8] developed a mathematical model of a parallel redundant complex system with two types of failure under a Preemptive resume-repair discipline using Gumbel-Hougaard family copula in repair and obtained Laplace transforms of the transition state probabilities and other reliability characteristics of the system. They considered a complex system which consists of two independent repairable subsystems A and B in (1-out-of-2: F) arrangement. Subsystem A has two identical units arranged in parallel redundancy (1-out-of-2: G), subsystem B has n units in series (1-out-of- n : F) with two types of failure, that is to say partial and catastrophic. The subsystem A is a priority unit while subsystem B is non-priority, i.e., the subsystem A is the preferred unit for operation on line and subsystem B is allowed to operate on line only when the subsystem A is under failure. Whenever subsystem B is under repair and at the same time subsystem A fails, the repairing of subsystem B is stopped and subsystem A is taken for the repair. The repair of subsystem B is started from the point where it was left earlier

as soon as repair of A is completed, i.e., the system is analysed under Preemptive resume-repair policy. The study, however, does not incorporate an important aspect of the problem: the system may be analysed with a coverage factor. In the present study the above mentioned system is considered for availability and cost analysis incorporating the coverage factor.

2. Materials and Methods

After a failed module is repaired, it is assumed to be in a new condition. It is then returned immediately to an active state if the subsystem is operating in degraded mode. An uncoverable failure in an active module causes system failure immediately.

Upon a unit failure in subsystem A , the system transits to a more failure representative state (with probability $\lambda_a \Delta t \alpha$, if the fault is detected, or $\lambda_a \Delta t (1-\alpha)$, if the fault is not detected). It is noticeable that with coverage factor when faults are detected in subsystem A then there is no detection in subsystem B . In this paper authors consider that fault is detected only in the subsystem A and no fault in the subsystem B . This fact has been incorporated in [6] where the coverage factor is applied in equation (38). The reliability of a system may be defined as the conditional probability that the system will perform fully throughout the interval $[t_0, t]$, given that the system was performing fully at time t_0 under the stipulated environmental condition. When the coverage factor (the capacity of failure occurrence detection in a module of a system) is considered, the Markov model is a more suitable modelling. Markov method can be used to evaluate the availability of a system in which the failure rate and the repair rate (λ, μ) are assumed to be a constant in time continuation. The Markov model incorporates two main concepts: the system state that describes the system at any given instant of time, and the state transition that represents the state transition probability. Electronic components may be considered with constant failure rate (λ) during their useful life period and, during a small period of time Δt , the probability that a module will fail within this time period is approximately $\lambda \Delta t$.

The reconfiguration operation will detect and remove the failed subsystem from the system, however; all the other operating subsystems will continue to operate as it is, that is the fault detection and the switch over mechanism are perfect. The probability of successful reconfiguration operation is defined as coverage factor. We denote this reconfiguration parameter or system coverage factor by α . The major motivation of this study is to bring reliability theory into a real-world applicable availability problem.

3. Numerical Computation

3.1. Availability analysis

(a) Let the failure rates of subsystems A and B for partial and catastrophic failures be $\lambda_A=0.1, \lambda_P=0.2, \lambda_C=0.3$ and α varies within 0.1 to 0.9.

(b) Again taking the failure rates of subsystems A and B for partial and catastrophic failures to be $\lambda_A=0.3, \lambda_P=0.2, \lambda_C=0.1$ and α varies within 0.1 to 0.9.

Using cases (a) and (b) in Eq. (38) of [6] and $t=0, 1, 2, 3, 4, 5, 6, 7, 8, 9$, and 10 units of time and taking the other parameters same as in a previous research, we

get the availability of the design system as shown in Figs. 1 and 2. (Eq. (38) of Ref. [6] is given in Appendix 1.)

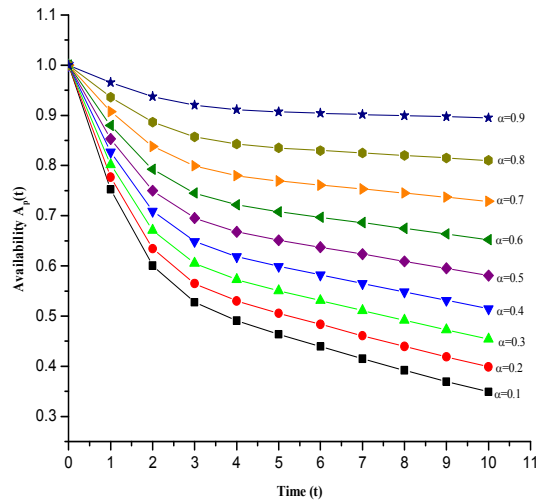


Fig. 1. Availability as Function of Time.

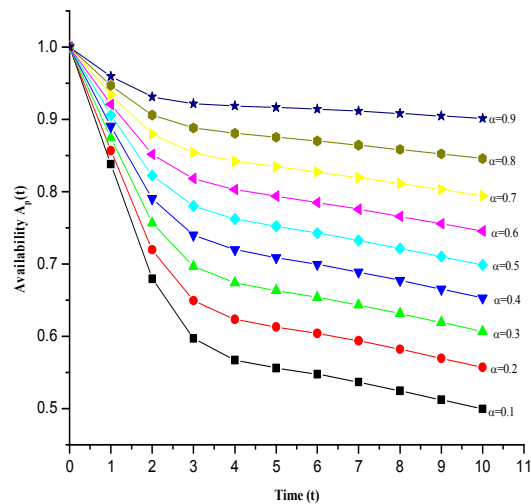


Fig. 2. Availability as Function of Time.

3.2. Cost analysis

Let the service facility be always available, then the expected profit during the interval (0, t] is

$$E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t \tag{1}$$

where K_1 and K_2 are revenue cost and service cost per unit time, respectively.

Using Eq. (1), for the same set of parameters as in 3.1(a) and (b) respectively and taking $K_1= 1$; $K_2= 0.01, 0.05, 0.10, 0.15, 0.25$ and setting $t = 0, 1, 2, 3, 4, 5, 6, 7$ units of time, one can get the computed values of $E_p(t)$ as shown in Figs. 3 and 4 respectively.

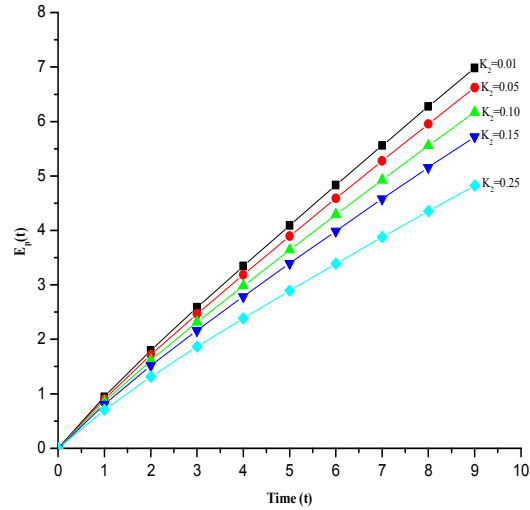


Fig. 3. Expected Profit as a Function of Time.

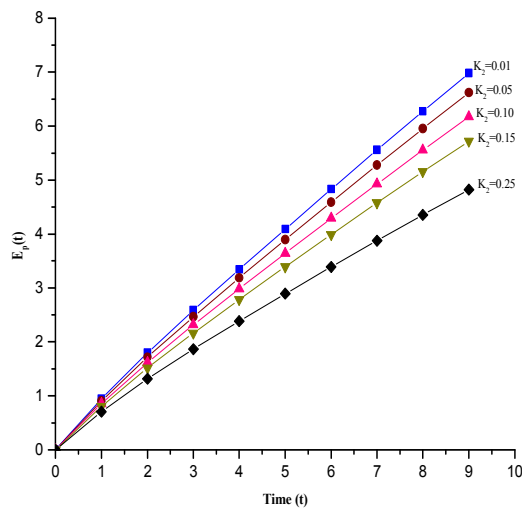


Fig. 4. Expected Profit as a Function of Time.

4. Result, Discussions and Conclusions

In this paper, we have shown that the results of paper [6] have been improved by using the coverage factor involving the Markov model. The Markov analysis

method has one major drawback. It is necessary to assume constant failure and repair rates. It is also essential to assume that the events are independent. These assumptions are not always valid in real life, so we shall have to overcome this drawback by using other efficient methods for further improvements.

The effect of coverage factor on the availability of the system has been shown in Figs. 1 and 2. One can easily see that in both cases, the availability of the system decreases as time increases. Also with the increment in coverage factor the availability of the system increases. In the first case when $\lambda_A = 0.1$, $\lambda_{pj} = 0.2$, $\lambda_c = 0.3$, varying α from 0.1 to 0.9 the values obtained are shown in Fig. 1. Also in the second case fixing $\lambda_A = 0.3$, $\lambda_{pj} = 0.2$, $\lambda_c = 0.1$, varying α from 0.1 to 0.9 for different values of time the values obtained are shown by Fig. 2.

Keeping the revenue cost per unit time K_1 at value 1, $\alpha = 0.5$ and varying service cost K_2 as 0.01, 0.05, 0.10, 0.15, 0.25, Figs. 3 and 4 respectively are obtained. By critical examination of these two figures one can observe that increasing service cost leads decrement in expected profit. The highest and lowest values of expected profit in the first case are 6.183 and 0.674 respectively while in the other case these values are 6.981 and 0.706 respectively. One can easily find that in the second case the profit is higher than the profit in the first case.

Consequently it is obvious that with the coverage factor, the results of the present model are in fairly good agreement as shown in the graphs with respect to [6]. The present study brought into focus the more realistic issue, i.e., coverage factor, incorporation which can address practical situations in reliability analysis of the complex systems. It is further observed that coverage factor has led to improvement in the availability of the system and the reduction in the cost which are important factors of any reliability analysis of a system. In future work one can try sensitivity analysis of the system involving coverage factor.

References

1. Bouricius, W.G.; Carter, W.C.; Jessep, D.C.; Schneider, P.R.; and Wadia, A. B. (1971). Reliability modeling for fault-tolerant computers. *IEEE Trans. Computers*, C-20(11), 1306-1311.
2. Arnold, T. (1973). The concept of coverage and its effect on the reliability model of a repairable system. *IEEE Transactions on Computers*, 22(3), 251-254.
3. Powell, D.; Martins, E.; Arlat, J.; and Crouzet Y. (1995). Estimators for fault tolerance coverage evaluation. *IEEE Transactions on Computer*, 44(2), 261-274.
4. Reibman, A.; and Zaretsky, H. (1990). Modelling Fault coverage and reliability in a fault-tolerant network. *Global Telecommunications Conference*, 2, 689-692.
5. Cai, K.-Y.; Wen, C.-Y.; and Zhang, M.-L. (1991). Fuzzy reliability modeling of gracefully degradable computing systems. *Reliability Engineering and System Safety*, 33(1), 141-157.
6. Prabhudeva, S.; and Verma, A.K. (2007). Coverage modeling and reliability analysis using multi-state function. *International Journal of Automation and Computing*, 4(4), 380-387.
7. Kumar, K.; Singh, J.; and Kumar, P. (2009). Fuzzy reliability and fuzzy availability of the serial process in butter-oil processing plant. *Journal of Mathematics and Statistics*, 5(1), 65-71.

8. Ram, M.; and Singh, S.B. (2008). Availability and cost Analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family copula in repair. *International Journal of Reliability, Quality & Safety Engineering*, 15(4), 341-365.

Appendix 1

Equation (38) of ref. [6] is given as:

$$\begin{aligned} \bar{P}_{up}(s) = & [1 + \lambda_{pj} \frac{r_{pj}(s + 2\lambda_A + \lambda_C)}{1 - 2\lambda_A \bar{S}_A(s)[r_{pj}(s + \lambda_A + \lambda_C) - r_{pj}(s + 2\lambda_A + \lambda_C)]}] \\ & + 2\lambda_A \lambda_{pj} \frac{r_{pj}(s + 2\lambda_A + \lambda_C)}{1 - 2\lambda_A \bar{S}_A(s)[r_{pj}(s + \lambda_A + \lambda_C) - r_{pj}(s + 2\lambda_A + \lambda_C)]} r_{pj}(s + \lambda_A + \lambda_C) \bar{P}_0(s) \\ & + [1 + \lambda_{pj} \frac{\bar{S}_A(s)[r_{pj}(s + \lambda_A + \lambda_C) - r_{pj}(s + 2\lambda_A + \lambda_C)]}{1 - 2\lambda_A \bar{S}_A(s)[r_{pj}(s + \lambda_A + \lambda_C) - r_{pj}(s + 2\lambda_A + \lambda_C)]}] \\ & + \lambda_{pj} r_{pj}(s + \lambda_A + \lambda_C) \{1 + 2\lambda_A \frac{\bar{S}_A(s)[r_{pj}(s + \lambda_A + \lambda_C) - r_{pj}(s + 2\lambda_A + \lambda_C)]}{1 - 2\lambda_A \bar{S}_A(s)[r_{pj}(s + \lambda_A + \lambda_C) - r_{pj}(s + 2\lambda_A + \lambda_C)]}\} \bar{P}_1(s) \end{aligned}$$

Where $r_i(s) = \frac{[1 - \bar{S}_i(s)]}{s}$