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EXERGY-BASED ECOLOGICAL ANALYSIS OF GENERALIZED IRREVERSIBLE HEAT PUMP SYSTEM

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Abstract

A reverse Carnot cycle forms the basis of all heat-pump cycles in providing heating and cooling loads. The optimal exergy-based Ecological analysis of an irreversible Heat-pump system with the losses of heat resistance, heat leak and internal irreversibility has been carried out by taking into account Exergy based ecological function (E) as an objective in the viewpoint of Finite-Time-Thermodynamics (FTT) or Entropy Generation Minimization (EGM). Exergy is defined here as the power required minus the lost power. The effects of irreversibilities along with internal heat leakage on coefficient on the performance of the system are investigated. The exergy based Ecological function decreases steadily with irreversibilites and heat leakages in the system. COP in such a system increases with the cycle temperature ratio. If a heat pump cycle is optimized with above mentioned criterion, there is a trade-off between its coefficient of Performance and the heating load it provides.

Keywords: Finite time thermodynamics, Exergy, Coefficient of Performance, Heating load, Irreversible.

1. Introduction

In the past era of abundant fuel supply, determining the efficiency of a system by the first law of thermodynamics was frequently adequate for technical and management decisions. The thermal Coefficient of Performance (COP), a first law efficiency measure, of a reversible Carnot cycle provides a bound on the optimal operation of heat pumps operating between the heat reservoirs at temperatures T_H

Nomenclatures

A	Total heat-transfer area, m^2
A_H	Hot working fluid heat-transfer area, m^2
A_L	Cold working fluid heat-transfer area, m^2
COP	Coefficient of performance of heat pump
f	ratio of the heat-transfer areas on the hot and cold side
E	Ecological objective function, kW
Ex	Exergy, kJ
Q_H	Rate of heat flow from the heat pump to the space, kW
Q_{HC}	rate of the heat transfer released to the heat sink, kW
Q_L	Rate of heat flow from low temperature source to the heat pump, kW
Q_{LC}	The rate of the heat transfer supplied by the heat source, kW
Q_{LK}	The rate of the heat leakage, kW
R	Irreversibility Parameter
S	Entropy generation rate, kW/K
T_o	Ambient Temperature, K
T_H	High-temperature sink temperature, K
T_L	Low temperature source temperature, K
T_X	Hot working fluid temperature, K
T_Y	Cold working fluid temperature, K
t	Time period of the cycle, s
U	Total overall heat-transfer coefficient, W/K
U_H	Overall heat-transfer coefficient on the high-temperature side, W/K
U_L	Overall heat-transfer coefficient on the low-temperature side, W/K

Greek Symbols

α	Conductance at High temperature side, W/m^{2}K
β	Conductance at Low temperature side, W/m^{2}K
ΔS	Entropy generation, kJ/K
χ	Heat leakage coefficient

and T_L. However, the process is in slow motion and the heat transfer takes place across an infinitesimal temperature difference. In recent years, however, as fuel supplies have become scarcer, therefore, such a performance parameter is of limited usefulness, since it does not provide any heating load. Thus, it has been necessary to resort to the exergy as a resource measure. However, attempts to determine the value of exergy have been from various points of views. The goal of any exergetic optimization procedure is to achieve the best compromise between the work-energy rate (e.g., power of an engine, cooling rate/Coefficient of Performance (COP) of a refrigerator, or heat supply rate of a heat engine) and its dissipation.

Many performance analyses for endoreversible and irreversible heat pumps have been performed by considering finite-time and finite-size constraints [1-7]. Angulo-Brown [8] proposed an ecological criterion $E= P- T_L\sigma$ for finite-time Carnot heat engines, where T_L is the temperature of cold heat reservoir, P is the power output and σ is the entropy generation rate. They derived a general property of non-endoreversible thermal cycles with this ecological criterion. Yan [9] showed that it might be more reasonable to use $E=P-T_o\sigma$ if the cold reservoir temperature T_L is not equal to the environmental temperature T_o. This criterion function is more reasonable than that presented by Angulo-Brown [8]. The optimization of the ecological function represents a compromise between the power output P and the lost power $T_o\sigma$ due to the entropy generation in the system. Recently, Bolaji [10] studied the exergetic performance of a domestic refrigerator using R12 and its alternative refrigerants. Researchers [11,12] found ecological figures of merit for thermodynamic cycles and carried out ecological optimization of heat engines cycles while [13,14] provided newer performance criteria of optimization. Based on the view point of the exergy analysis Chen et al. [11] provided a unified ecological optimization objective for all thermodynamic cycles as stated in Eq. (1)

$$E = \frac{Ex}{t} - T_o.\frac{\Delta S}{t} = \frac{Ex}{t} - T_o.S_g \tag{1}$$

where Ex is the exergy output of the cycle T_o is the environment temperature, ΔS is the entropy generation of the cycle, and t is the cycle period and whence s_g is the entropy generation rate of the cycle. For heat pump cycles the exergy output rate of the cycle is

$$\frac{Ex}{t} = Q_{HT}\left(1 - \frac{T_o}{T_H}\right) - Q_{LT}\left(1 - \frac{T_o}{T_L}\right) \tag{2}$$

where Q_{LT} is the rate of the heat transfer supplied by the heat source, Q_{HT} is the rate of the heat transfer released to the heat sink, and T_H and T_L are the temperatures of the heat sink and the source respectively. Thus, for heat pump cycles the exergy based ecological function is defined as under

$$E = Q_{HT}\left[\left(1 - \frac{T_o}{T_H}\right) - \left(\frac{T_L}{T_H - T_L}\right)\left(1 - \frac{T_o}{T_L}\right)\right] - T_o.S_g \tag{3}$$

2. Generalized Irreversible Heat-Pump Model

A model of Generalized Irreversible heat-pump system and its corresponding *T-s* diagram are shown in Figs. 1 and 2 respectively. The following assumptions are made for this model.

- The Carnot heat pump operates between a low temperature heat source at T_L, and a high temperature heat sink at T_H.
- The cycle consists of two isothermal (2-3, 4-1) and two adiabatics (1-2, 3-4). All processes being irreversible.
- The working fluid flows through the closed system in steady state conditions.
- The working fluid temperatures at higher and lower side are T_X and T_Y respectively. Hence, $T_x > T_H > T_L > T_Y$
- All processes in each cycle are assumed to be irreversible.

There are external irreversibilities due to heat transfer in the high and low temperature heat exchangers between the heat-pump and its surrounding heat-reservoirs. Because of this heat-transfer, the working fluid temperatures (T_X, T_Y) are different from the reservoirs' temperatures.

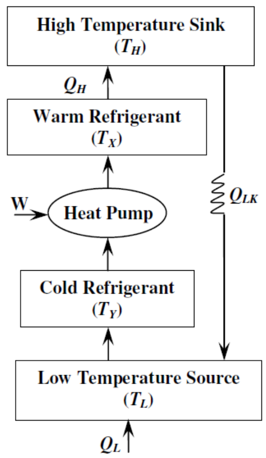

Fig. 1. Schematic of a Generalized Irreversible Heat Pump System.

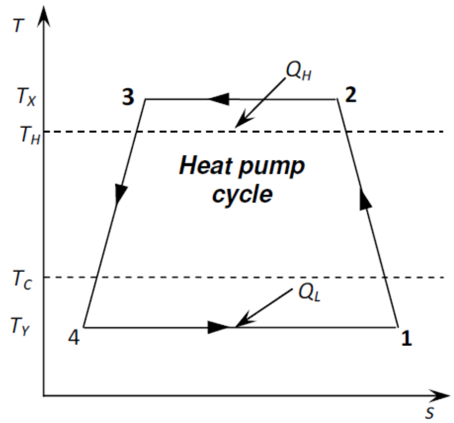

Fig. 2. *T-S* Diagram of a Generalized Irreversible Carnot Type Heat-Pump Cycle.

2.1. Thermodynamic analysis

Using the First Law of Thermodynamics, the heat-transfer process is equated to the energy change of the working fluid in the system. The rate of heat-flow from the low temperature source

$$Q_L = U_L.A_L.(T_L - T_Y) \tag{4}$$

where, U_L and A_L, are the overall heat-transfer coefficient and heat-transfer area of the heat-exchanger between the heat-pump and low temperature heat-source respectively. T_Y is the cold working fluid temperature.

The rate of heat-flow to high temperature heat-sink is

$$Q_H = U_H.A_H.(T_X - T_H) \tag{5}$$

where, U_H and A_H, are the overall heat-transfer coefficient and heat-transfer area of the heat-exchanger between the heat-pump and high-temperature heat-sink respectively. T_X is the warm working fluid temperature.

The ratio of the heat-transfer areas on the hot and cold side *(f)* is

$$f = \frac{A_H}{A_L} \tag{6}$$

The finite-size heat-exchanger constraint requires the total area to remain constant and x is the thermal allocation ratio for the high temperature side to total heat exchange area

$$A_H + A_L = A \tag{7}$$

$$x.A_H = A \tag{8}$$

The heat-leakage from the high-temperature sink to low-temperature source

$$Q_{LK} = \chi(T_H - T_L) \tag{9}$$

where, χ is the heat-leakage coefficient.

The net heat transferred from the low-temperature source (Q_{LC})

$$Q_{LT} = (Q_L - Q_{LK})$$

$$Q_{LT} = U_L.A_L.(T_L - T_Y) - \chi(T_H - T_L) \tag{10}$$

The net heat transferred to the high temperature heat sink (Q_{HC})

$$Q_{HT} = (Q_H - Q_{LK})$$

$$Q_{HT} = U_H.A_H.(T_X - T_H) - \chi(T_H - T_L) \tag{11}$$

For Heat pump, the power required using the first law of thermodynamics is

$$W_i = Q_H - Q_L \tag{12}$$

The second law of thermodynamics for an irreversible cycle requires that

$$\frac{Q_H}{T_X} - R.\frac{Q_L}{T_Y} = 0, \qquad R \succ 0 \tag{13}$$

The entropy generation rate in the cycle is (s_g) and is given by

$$S_g = \frac{Q_L}{T_Y} - \frac{Q_H}{T_X} \tag{14}$$

Using Eqs. (4) and (5) in (13), one gets the sink side high temperature of the working fluid

$$T_X = \frac{T_H.T_Y.U_H.f}{\left(T_Y.U_H.f + R.T_Y.U_L - R.T_H.\tau.U_L\right)} \tag{15}$$

The entropy generation rate in the cycle using Eqs. (4), (5), (8) and (9) in (14) one gets

$$s_g = \left(\frac{1}{T_H} - \frac{x}{R.\tau.T_H}\right).U_H.A_H\frac{(R.\tau.T_H - R.x.T_H)}{(R.x + U_H.x.\frac{f}{U_L})} - \chi(T_H - T_L).\left(\frac{1}{\tau.T_H} - \frac{1}{T_H}\right) \tag{16}$$

2.2. Maximum ecological function

Cheng and Chen [12] defined an exergy based ecological function for Carnot heat pump as

$$E = Q_{HT}\left[\left(1 - \frac{T_o}{T_H}\right) - \left(\frac{T_L}{T_H - T_L}\right)\left(1 - \frac{T_o}{T_L}\right)\right] - T_o.S_g \tag{17}$$

Using the Eqs. (5), (6) and (16) in (17) yields

$$E = \frac{(T_H - T_o).\left[\chi.T_H(\tau-1) - \frac{ART_H U_L U_H f(T_Y - \tau T_H)}{(f+1)(T_Y U_H f + T_Y U_L R - T_H U_L \tau R)}\right]}{T_Y}$$
$$- T_o\left[\frac{\chi.T_H(\tau-1)^2}{\tau} + \frac{AU_L(T_Y - \tau T_H)\begin{pmatrix} T_Y U_H f + T_Y U_L R - \\ T_H U_L \tau R - f T_H U_H \tau R \end{pmatrix}}{T_H \tau (f+1)(T_Y U_H f + T_Y U_L R - T_H U_L \tau R)}\right] \tag{18}$$
$$+ (T_o - \tau T_H)\left[\chi.T_H(\tau-1) - \frac{AU_L f^2 (T_Y - \tau T_H)}{f+1}\right]$$

If the exergy based ecological function is maximized with respect to the low working fluid temperature, the optimum value of the temperature comes out as

$$T_Y = \frac{T_H.\tau.\alpha.f.\sqrt{\begin{matrix} 2.R.T_o^2 - T_H.R.T_o + 2.R.T_o^2.f^2 + 2.\tau.T_H^2.f^2 - 2.T_H.T_o.f^2 - 2.\tau.T_H.T_o.f^2 \\ - R.T_H^2.\tau^2.\beta.f^2 + R.T_H.T_o.\tau.\beta + R.T_H.T_o.\tau.\beta.f^2 \end{matrix}}}{(R.\beta + \alpha.f).(T_o + T_o.f^2 - T_H.\tau.f^2)}$$

Putting the value of the cold working fluid temperature obtained above in Eq. (15) hot working fluid temperature is obtained. Whence the optimum value of the ecological function be obtained from Eq. (18).

The ratio of the exergy based ecological function to the Coefficient of Performance, Y, is defined here,

$$Y = \frac{E}{COP} \tag{19}$$

The maximum heating load delivered by the heat pump (Q_H) can be found out by using Eq. (5) above, rewritten here,

$$Q_H = U_H.A_H.(T_X - T_H) \tag{5}$$

While the Coefficient of Performance, COP, is defined as

$$COP = \frac{Q_H}{W} \tag{20}$$

3. Results and Discussion

In the simulation of the heat pump model considered using Mathcad 14, while varying any one parameter all other parameters are assumed to be constant as given : T_H=350 K, T_H=300 K, U_A=1 W/K, α=β =1, f=2.

Variations of the COP and Exergy based Ecological Function with cycle temperature-ratio τ for different values of cycle irreversibility parameter R are shown in Fig. 3. It can be seen that the COP of the heat-pump systems increases with the cycle temperature-ratio. It is more so for higher values of the ratio. When R=1.02, at τ=0.1 the values of COP and E are 0.981 and 48.02 kW respectively, while at τ=0.7, the values of COP and E become 1.805 and 8.769 kW respectively. Similarly, when R=1.5, at τ=0.1 the values of COP and E are 0.964

and 10.015 kW respectively while at τ=0.7, the values become 1.461 and 2.139 kW respectively. The corresponding values Exergy based Ecological function, however, increases to a maximum before decreasing down. The function reaches its maximum value at τ=0.2 when the R=1.05, while it shifts to τ=0.225 when R=1.2. However, both, COP and the Exergy based Ecological function increase as the values of the irreversible parameter R are lowered. The same can also be obtained from the Eqs. (18) and (20).

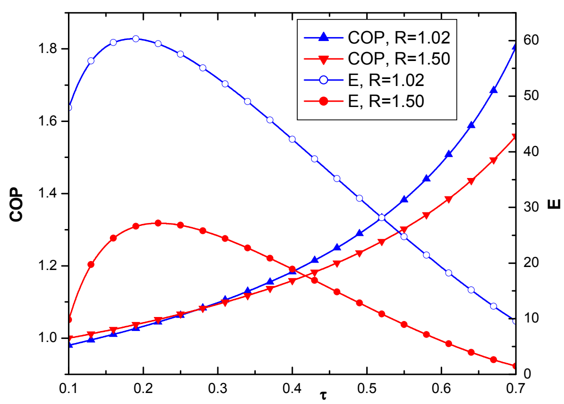

Fig. 3. Variation of COP and the Ecological Function E with Extreme Temperature-Ratio τ for Different Values of Irreversibility Factor R.

Variations of the Heat-load provided by the heat-pump and Exergy based Ecological Function with cycle temperature-ratio for different values of R are shown in Fig. 4. The heat-load provided by the heat-pump goes on decreasing as the cycle temperature-ratio increases. When R=1.02, at τ=0.1 the values of Q_H and E are 96.891 kW and 48.02 kW respectively while at τ=0.7, the values become 31.631 kW and 8.769 kW respectively. Similarly, when R=1.5, at τ=0.1 the values of Q_H and E are 84.138 kW and 10.015 kW respectively while at τ=0.7, the values become 15.675 kW and 2.139 kW respectively. The corresponding values Exergy based Ecological function, however, increases to a maximum before decreasing down. Both, heat-load and the Exergy based Ecological function increase as the values of the irreversible parameter R are lowered. The same can also be obtained from the Eqs. (5) and (18).

Variations of the Exergy based Ecological to COP ratio, Y, with cycle temperature-ratio for different values of the leakage coefficients are shown in Fig. 5. Y decreases with an increase in leakage for a given cycle temperature ratio. When χ=0.01, at τ=0.1 the value Y is 47.445 kW and at τ=0.7, the value becomes 0.495 kW. However, When χ=0.02, at τ=0.1 the value of Y is approaches to zero, while at τ=0.7, the values become 38.875 kW.

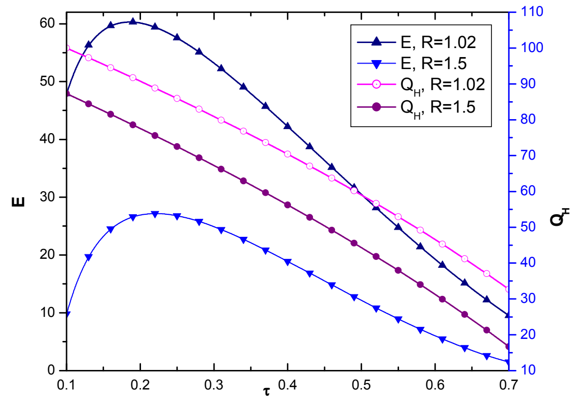

Fig. 4. Variation of the Ecological Function *E* with Extreme Temperature-Ratio *τ* for Different Leakage Coefficients and Irreversibilty Factor *R*.

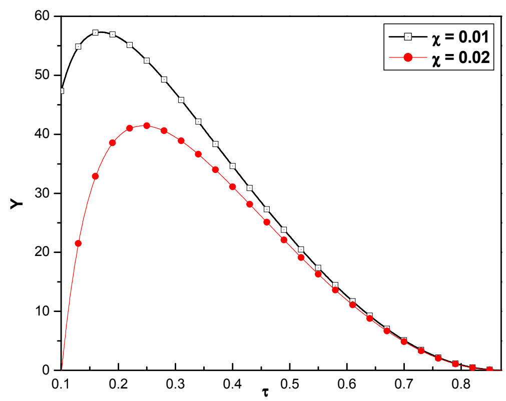

Fig. 5. Variation of the Ecological Function *E* to COP ratio *Y* with Cycle Temperature-Ratio *τ* for Different Leakage Coefficients.

Variations of the Exergy based Ecological to COP ratio, *Y*, with irreversibility parameter *R* for different leakage coefficients are shown in Fig. 6. The ratio steadily decreases with *R* for any value of leakage coefficient. However, for a given irreversibility parameter it decreases as the heat leakage increases. When χ=0.01, at *R*=1.0 the value *Y* is 53.985 kW and at *R*=1.10, the value becomes 46.785 kW. However, When χ=0.02, at *R*=1.0 the value of *Y* is 43.059, while at *R*=1.10, the values become 35.895 kW.

Variation of the Exergy based Ecological function with irreversibility parameter *R* for different Leakage coefficients is shown in Fig. 7. The exergy based Ecological function decreases steadily with *R* for any value of leakage coefficient. However, for a given irreversibility parameter the Exergy based Ecological function decreases as the heat leakage increases. When χ=0.01, at *R*=1.0 the value *E* is 59.285 kW and at *R*=1.09, the value becomes 51.959 kW. However, When χ=0.02, at *R*=1.0 the value of *E* is 45.875, while at *R*=1.09, the values become 38.445 kW.

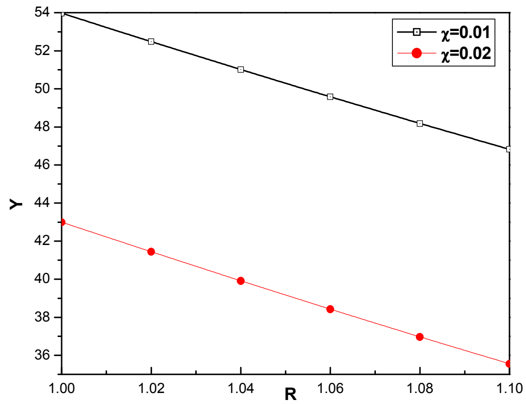

Fig. 6. Variation of the Ecological Function *E* to COP ratio *Y* with Cycle Irreversibility Parameter *R* for Different Leakage Coefficients.

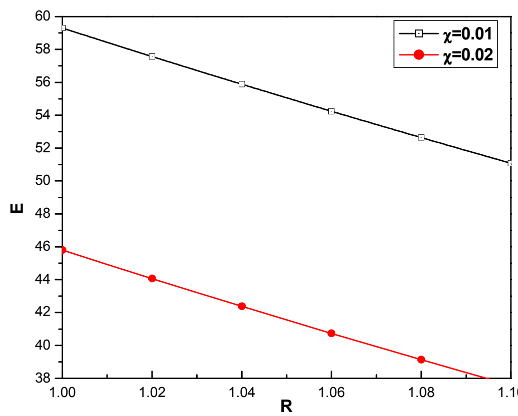

Fig. 7. Variation of the Ecological Function *E* with Cycle Irreversibility Parameter *R* for Different Leakage.

4. Conclusions

From theoretical analyses carried out, it can be inferred that though the COP of the reversible cycle provides an upper bound on the COP of any real cycle but it does not provide any heating load.

For a more realistic optimization, a new thermo-ecological performance analysis for a generalized irreversible Carnot heat pump has been presented. The ecological performance criterion is defined as the power output per unit loss rate of availability is chosen as the objective for optimisation.

The optimal performance conditions which maximize the objective function are investigated. The effects of internal irreversibility, heat leakage and source temperature ratio on the global and optimal performances were discussed. It is concluded the COP of the heat-pump systems increases with the cycle temperature-ratio.

Moreover, the COP and the Exergy based Ecological function increase as the values of the irreversible parameter *R* are lowered. If a measure of the ecological function per unit COP is analysed by taking a ratio, it decreases with an increase in leakage for a given cycle temperature ratio.

The ratio steadily decreases with *R* for any value of leakage coefficient. However, for a given irreversibility parameter it decreases as the heat leakage increases. The exergy based Ecological function decreases steadily with *R* for any value of leakage coefficient.

If a heat pump cycle is optimized with above mentioned criterion, there is a trade-off between its coefficient of Performance and the heating load it provides.

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