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ROBUST BASEDBAND REED SOLOMON DETECTION OVER POWER LINE CHANNEL

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Abstract

The research studied the transmission of based-band data for conventional Reed Solomon Codes (RS) over hostile impulsive channel such as power-line channels (PLC). The research works identified the breakdown of well-designed conventional RS codes over PLC impulsive channel and proposed a mitigation technique to improve the performance of such RS codes over impulsive channel. Traditionally, Reed Solomon Codes is designed effectively to combat and correct burst errors arising from Addictive White Gaussian Noise (AWGN). However, it works ineffectively against non-Gaussian noise interference, particularly on PLC channel. Thus such decoding method failed severely in impulsive noise interference due to the deficiency in its design on part of the decoder which is optimized for AWGN channel. Traditionally, median value can statistically eliminate the influence of outliers. Hence, the excellent trait of median value was exploited in our study to eliminate impulses arising from PLC. As a consequence, median filtering framework is incorporated to enhance the detection of signals from PLC prior to hard decision RS decoding. The simulation results showed that performance of RS codes is substantially improved after median filtering. The incorporation of median filtering framework into RS codes has significantly strengthened the operation of RS codes over hostile PLC channel.

Keywords: Reed Solomon (RS) Codes, Symmetric α-stable ($S\alpha S$) distribution, Median filtering, Power Line Communication (PLC), Channel coding, Bit Error Rate (BER).

1. Introduction

Reed Solomon codes are nonbinary cyclic codes that represent a subclass of Bose-Chaudhuri-Hocquenghem (BCH) codes. It was proposed by Irving Reed [1] in his paper in the Journal of the Society for Industrial and Applied Mathematics in 1959.

Nomenclatures

$c(x)$	Code symbol
C_g	Exponential Euler constant
d_{min}	Minimum hamming distance
E_b/N_o	Energy per bit to noise power spectral density ratio
$e(x)$	Error sequence
$g(x)$	Generator polynomial
K	Number of data symbols
m	Number of bits of data symbols
$m(x)$	Message symbol
n	Number of code symbols
r	Code rate
$r(x)$	Received signal/codeword
S	Syndrome
S_o	Geometric noise power, W
t	Symbol error correcting capability
W	Weight of median filter

Greek Symbols

α	Characteristic exponent
β	Error position
γ	Degree of dispersion
σ (x)	Locator polynomial
$\Phi(\omega)$	Characteristics function
ω	Angular velocity, rad/s
$\Diamond$	Replication operator

Abbreviations

AWGN	Additive White Gaussian Noise
BCH	Bose-Chaudhuri-Hocquenghem
BER	Bit Error Rate
CD	Compact Disc
DVD	Digital Video Disc
FEC	Forward Error Correction
GF	Galois Field
GSNR	Geometric signal-to-noise ratio
PLC	Power Line Channel
RS	Reed Solomon Codes
$S\alpha S$	Symmetric alpha-stable distribution

Reed Solomon codes found many applications in modern digital communication systems, such as compact disc (CD) or Digital Video Disc (DVD) players, spacecraft for deep space probe, PLC, WiMax and etc. The development of Reed Solomon codes (RS) over PLC has been introduced to HomePlug 1.0 standard [2] for HomePlug payload encoding. The purpose of the paper is to study and propose a robust basedband decoding technique for RS codes over hostile baseband PLC. The recent researches and interests on transmitting data over power line are driven by the superior features of power line cable for data communication. The attractiveness of using power line is mainly attributed to economical setup cost and widely deployment of power line cables in urban and suburban areas throughout the world. However, the exciting development can only be realized if the problems faced can be overcome by researchers. Besides, AWGN noise, signals that are transmitted over power cable experience various degree of interferences. The main culprit that slows down the development of PLC for communication is impulsive noise. Impulsive nature of such noise is found as the major source of impairments that significantly paralyses the normal optimum operations of most the Forward Error Control (FEC) codes in AWGN. Impulse is defined as a source of noise arising from hostile transmission medium such as PLC which appears in the channel in the form of short duration and ideally has infinite amplitude [3]. Impulsive noise is the main factor to overcome for our study in order to enable the use of RS codes for reliable data transmission.

RS codes are categorized into cyclic codes and it possesses the properties of cyclic codes [4]. Therefore, it packs m bits of information into non-binary symbols where we referred to as finite field. Typically, a RS code with m-bit symbols can be described by Eq. (1)

$$0 < k < n < 2^m + 2 \tag{1}$$

where k is the number of data symbols being encoded, and n is the number of code symbols from the RS encoder output. The (n, k) RS codes are given by Eq. (2)

$$(n , k) = (2m - 1 , 2m - 1 - 2t) \tag{2}$$

where t is the symbol-error correcting capability and $(n - k) = 2t$ is the number of parity symbols added. RS codes with t error correction capability are able to correct any combination of t or fewer errors.

The distance between two non-binary RS codes can be defined as the number of symbols in which the codewords differ. Hence the minimum distance of RS codes can be obtained by Eq. (3)

$$d_{min} = n - k + 1 \tag{3}$$

With reference to Eq. (3), t can be expressed in Eq. (4)

$$t = \frac{d_{\min} - 1}{2} = \frac{n - k}{2} \tag{4}$$

Burst error usually comes from various sources such as the scratching CD/DVD or Rayleigh, Rician fading phenomena in wireless channels. It causes contiguous number of bits of transmitted sequence in errors. The non-binary data structure of RS codes promise the effective burst-error correction [5] over such errors. This is because the use of non-binary data structure allows the detection and correction of error symbols in bytes level instead of bit level.

Galois fields (GF) are used in non-binary cyclic codes such as RS codes to describe the encoding and decoding principles. For a Galois Field of p^m elements, p is the prime number. For each element, we can refer it as one symbol produced from a source in non-binary form. A binary field GF(2) is a subfield of the extension field GF(2^m); hence we can construct an infinite set of elements starting with the elements $\{0,1,\alpha\}$ and successive elements can be obtained by progressively multiplying the last entry by α and yields [6].

$$\begin{aligned} F &= \{0,1,\alpha,\alpha^2,\alpha^3,......\alpha^{2m-2},\alpha^{2m-1},\alpha^{2m},....\} \\ &= \{0,1,\alpha,\alpha^2,\alpha^3,......\alpha^{2m-2},\alpha^0,\alpha^1,....\} \end{aligned} \quad (5)$$

where $\alpha^{2m-1} = 1 = \alpha^0$

The advantages of using GF to represent the binary signal are the compact notation that enables bulk processing and manipulation of symbols in non-binary form. Therefore, the GF representation of the symbols effectively strengthens the capability of RS codes to detect and correct transmitted symbols in the presence of burst noise corruption.

2. Reed Solomon Encoding and Decoding

A code $c = (c_0, c_1, c_2, c_3, ..., c_{n-2}, c_{n-1})$ is said to be cyclic if the shifted version of the code word $c' = (c_1, c_2, c_3, ..., c_{n-2}, c_{n-1}, c_0)$ is also a codeword. To encode message symbol $m(x)$, the generator polynomial has to be determined in advance. Typically, the generator polynomial $g(x)$ can be written into polynomial form with order of $2t$, where $2t$ is the number of parity symbols that are used as redundancy. Hence, it can be expressed as Eq. (6):

$$g(x) = (x-\alpha)(x-\alpha^2)(x-\alpha^3)........(x-\alpha^{2t}) = \prod_{j=1}^{2t}(x-\alpha^j) = \sum_{j=0}^{2t} g_j x^j \quad (6)$$

Non-systematic encoding or systematic encoding can be performed on the message symbols $m(x)$ by RS encoder. For non-systematic encoding, the output codeword $c(x)$ of order $n = k + 2t$ can be generated by multiplying information polynomial $m(x)$ with the generator polynomial $g(x)$ using modulo polynomial algebra as Eq. (7):

$$c(x) = m(x) \,.\, g(x) \quad (7)$$

where the message symbols which can be represented as ascending polynomials of order k as shown in Eq. (8)

$$m(x) = \sum_{j=1}^{k} m_j x^j = m_1x^1 + m_2x^2 + m_3x^3 + m4x^4 + ... + m_k x^k + \quad (8)$$

For systematic encoding, message symbols $m(x)$ are sent together with parity symbols $p(x)$ in their polynomial representations $c(x)$ as shown in Eq. (9)

$$c(x) = p(x) + x^{n-k}m(x) \quad (9)$$

The introduction of message symbols into codeword symbols for systematic encoding makes systematic encoding more attractive in its choices of implementation. For systematic encoding, the message symbols in its polynomial representation can be decoded directly from the received codeword. Hence plethora transmission error for non-systematic encoding does not appear in systematic RS codes. Therefore, systematic encoder is used in our encoding process for simulation modelling.

The corrupted received signal can be expressed as Eq. (10)

$$r(x) = c(x) + e(x) \tag{10}$$

where $e(x)$ are the error sequences that are induced by the impulsive noise from PLC. Bit-wise addition is performed to obtain the corrupted transmitted symbols over PLC channel.

To decode non-binary RS codes, a number of decoding algorithms for RS codes have been proposed by Peterson [7, 8], Clark and Cain [9], Blahut [10] and Berlekamp [11,12], Lin and Costello. Typically, syndrome is used to determine the error magnitude and location in the corrupted transmitted sequence. In block codes and cyclic codes, syndrome is the multiplication of the transpose parity check matrix H with the received sequence $r(x)$. The purpose is to validate if the received codeword is one of the member of the transmitted codeword set. Hence, the correctness of the received sequence can be verified. Typically, the syndrome computation is performed on RS codes to detect the erroneous location and magnitude on the received sequence $r(x)$. The syndrome of RS codes are calculated with the Eq. (11)

$$\begin{aligned} S_i &= r(x) = r\left(\alpha^i\right) \\ S_i &= \left(c(x)+e(x)\right)_{x=\alpha^i} = e\left(\alpha^i\right) \\ S_i &= e\left(\alpha^i\right) \end{aligned} \tag{11}$$

RS codes is capable in correction t errors with $2t$ parity symbols as given in Eq. (4) Therefore, there are $2t$ simultaneous equations can be formulated according to Eq. (11).

$$\begin{aligned} S_1 &= r(\alpha) = \left(e_{n-1}\cdot x^{n-1} + e_{n-2}\cdot x^{n-2} + e_{n-3}\cdot x^{n-3} + \ldots + e_1\cdot x + e_0\right)\big|_{x=\alpha} \\ S_2 &= r\left(\alpha^2\right) = \left(e_{n-1}\cdot x^{n-1} + e_{n-2}\cdot x^{n-2} + e_{n-3}\cdot x^{n-3} + \ldots + e_1\cdot x + e_0\right)\big|_{x=\alpha^2} \\ S_3 &= r\left(\alpha^3\right) = \left(e_{n-1}\cdot x^{n-1} + e_{n-2}\cdot x^{n-2} + e_{n-3}\cdot x^{n-3} + \ldots + e_1\cdot x + e_0\right)\big|_{x=\alpha^3} \\ &\vdots \\ S_{2t} &= r\left(\alpha^{2t}\right) = \left(e_{n-1}\cdot x^{n-1} + e_{n-2}\cdot x^{n-2} + e_{n-3}\cdot x^{n-3} + \ldots + e_1\cdot x + e_0\right)\big|_{x=\alpha^{2t}} \end{aligned} \tag{12}$$

The procedure of solving such $2t$ non-linear simultaneous equation is known as RS decoding algorithm. Berlekamp-Massey algorithm is used in the simulation to decode the RS codes in impulsive noise [11-13].

If there are non-zero syndromes in the Eq. (12), it signifies the errors in the received sequence $r(x)$. The equation can be linearized by introducing error locator polynomial $\sigma(x)$. To find out the error symbol's magnitude and its location, we use error locator polynomial Eq. (13)

$$\sigma(x) = (1+\beta_0 x)\cdot(1+\beta_2 x)\cdots\cdots(1+\beta_{n-1}x)$$
$$\sigma(x) = 1+\sigma_1 x+\sigma_2 x^2+\cdots\cdots+\sigma_{n-1}x^{n-1} \tag{13}$$

where $\sigma(x)$ is the reciprocal of β which indicates the error position.

Therefore, matrix can be formed in the form of Vandermonde matrix as shown in Eq. (13):

$$\begin{bmatrix} S_1 & S_2 & S_3 & \cdots\cdots\cdots S_t \\ S_1 & S_2 & S_3 & \cdots\cdots\cdots S_{t+1} \\ . & & & \\ . & & & \\ S_t & S_2 & S_3 & \cdots\cdots\cdots S_{2t-1} \end{bmatrix} \cdot \begin{bmatrix} \sigma_t \\ \sigma_{t-1} \\ . \\ . \\ \sigma_1 \end{bmatrix} = \begin{bmatrix} -S_{t+1} \\ -S_{t+2} \\ . \\ . \\ -S_{2t} \end{bmatrix} \tag{14}$$

Solving the Vandermonde matrix in Eq. (14) enables us to find the location of the errors because $\sigma(x)$ is defined as the reciprocal of the location of erroneous symbol. Hence, if there are errors detected in the received sequence, the determinant of Vandermonde matrix is non-singular and can be inverted. If the root of the locator polynomial has been identified via trial and error by solving Eq. (13), we are ready to find the magnitude of error by substituting all non-zero elements of β into Eq. (12).

3. Impulsive Noise Model

Gaussian distribution has been widely used in digital communication systems as a typical noise source. However, impulsive noise is the major issue to deal with in PLC communication systems [14]. In PLC channel, impulsive noise is more detrimental than AWGN noise. Generally, the impulsive noise in PLC has tail heavier than AWGN noise in its distribution. In order to facilitate the development of our simulation model, symmetric alpha-stable ($S\alpha S$) distribution is used to empirically model the impulsive nature of the noise source over PLC channel [15]. $S\alpha S$ distribution has the advantages of smooth, unimodal, symmetric and bell-shaped closely resembles to conventional used AWGN. The characteristic function of $S\alpha S$ distribution is given in Eq. (15)

$$\phi(\omega) = e^{-\gamma^{\alpha}|\omega|^{\alpha}}, \quad -\infty \prec \omega \prec \infty \tag{15}$$

where parameter γ describes the degree of dispersion and measures the spread of distribution and parameter α is usually known as characteristic exponent which is restricted to the interval $0 \leq \alpha \leq 2$. α determines the tail heaviness or impulsiveness of the $S\alpha S$ distribution. Typically, lower the value of α goes the heavier tail is generated. α is known as tail constant of the distribution. However, for a constant α, larger the values of γ correspond to larger dispersion.

$S\alpha S$ distribution does not have closed form expression [16] with few exceptional case with $\alpha = 2$ for zero-mean Gaussian distribution, $\alpha = 1$ Cauchy distribution, and $\alpha = 0.5$ Levy distribution. If $\alpha < 2$ tail constant, it is implying infinite variance appearing in the process. For other values of α, there is no closed form expression in existence. Hence, alternatively, geometric signal-to-noise ratio (GSNR) is used in our evaluation:

$$GSNR = \frac{1}{2G_g}\left(\frac{A}{S_0}\right)^2 \quad (16)$$

where A is the signal amplitude and S_0 denotes the geometric noise power of a symmetric α-stable variable and given by Eq. (17).

$$S_0 = \frac{\left(C_g \gamma\right)^{1/\alpha}}{C_g} \quad (17)$$

where $C_g \approx 1.78$ is the exponential of the Euler constant.

To evaluate the performance of RS channel coding scheme over PLC channel, BER (Bit Error Rate) is used to determine the performance of RS codes over PLC channel. In our simulation, we characterised BER in term of E_b/N_o (Bit Energy to Noise Spectral density Ratio). The parity symbols used in RS codes are not part of the transmitted information and their bit energy can be averaged over information bits, and E_b/N_0 can be defined for $S\alpha S$ distribution as

$$\frac{E_b}{N_0} = \frac{GSNR}{2r} \quad (18)$$

where r is the code rate of RS encoder.

Figure 1 shows the impulsive noise that is superimposed on the transmitted sequence and eventually corrupts the sequence with Poisson arrival of impulses. The impulses are shown as spikes with ideally zero width and infinite amplitude.

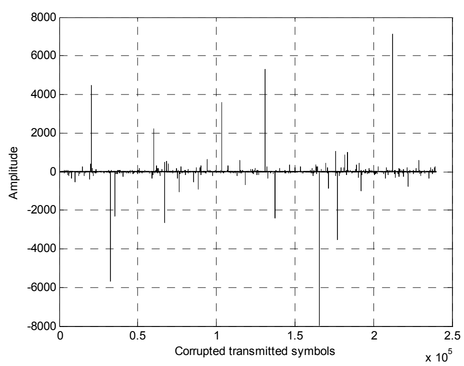

Fig. 1. Corrupted Basedband Received Samples in $S\alpha S$ Impulsive Noise (α =1.5, γ=1).

4. Median Filter

Robust detection can be achieved by introducing median filtering prior to symbol detection. Assuming the continuous received samples x are sampled regularly from noisy channel at time instant T, hence, it produces $x_n = x(nT)$ discrete noisy samples. Subsequently, the samples of the received signal in its discrete form are fed into median filter. The purpose of the median filtering is achieved by performing median operation statistically over all the noisy received samples to reduce the entries of outliers.

The median values $\tilde{x}$ of two subsequent samples are given by Eq. (19), if the observations of r.v. (random variable) are denoted by $x_1, \ldots\ldots, x_n$

$$\tilde{x} = \begin{cases} x_{\left(\frac{n+1}{2}\right)} & \text{if } n \text{ is odd number} \\ \frac{x_n + x_{n+1}}{2} & \text{if } n \text{ is even number} \end{cases} \tag{19}$$

Due to the effectiveness of median operation that is used statistically in removing the effects of outliers in statistics; hence, in our simulation, a powerful median filter with FIR structure was constructed and applied to safe-guard the receiver against the entries of outliers arising from hostile PLC channel with $S\alpha S$ distribution. Therefore, the sample mean $MEDIAN(x_1, x_2, \ldots\ldots, x_n)$ can be generalized to the class of linear FIR filters [16] as given in Eq. (20)

$$\tilde{x} = MEDIAN\left(|W_1| \Diamond \operatorname{sgn}(W_1) x_1, \ldots\ldots, |W_n| \Diamond \operatorname{sgn}(W_n) x_n\right) \tag{20}$$

with $\tilde{x} = W_i \in R$ are weights of filter for i = 1, 2, …… , n and ◊ is the replication operator.

W_n represents the weighting operation that is performed by the median filter with N numbers of weighted taps. Each weighed tap has coefficient which is given by W_n. The vital functionality of the median operator in Eq. (20) is to choose middle or median values from the samples containing outliers. In addition, modulo is used to restrict the weights W_n in the range of positive value hence it does not affect the sign of the samples.

To perform median filtering, a FIR transversal filter was constructed with varying weighted taps W_n. Median values of the samples were extracted and weighted accordingly through the median filter that was run on sliding window basis to produce outputs. The filter estimated the magnitudes of the received samples in median range among noisy samples. Thus, such operation produced samples that closely resembled to the received signal obtained in AWGN environment. Figure 2 shows the output from the median filter with weighted taps N = 20. The corrupted received signal was filtered and smoothened to reduce the entries of impulses over power line channel. The spike of received signal is significantly attenuated from the median filter output. The reduction of spiky amplitude from median filter output can be observed from Fig. 2 in comparison with Fig. 1.

5. Simulation and Results

According to the RS encoding and decoding theorem, median filtering and noise model described above, we are ready to construct a simulation model to study the behaviour of RS codes over PLC.

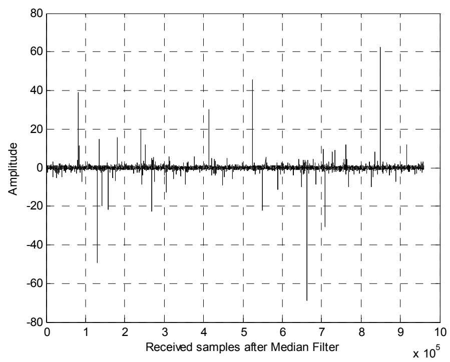

Fig. 2. Corrupted Basedband Received Signal after Median Filter, where Number of Weighted Taps *N* =20.

Simulation was performed based on the Matlab Simulink model as shown in Fig. 3. The performance of the median filter in impulsive noise is shown in Fig. 4. From the outputs, Fig. 4(a) shows the binary antipodal signal from the transmitter. Subsequently, the antipodal signal was filtered with square root raise cosine filter with roll-off factor r = 0.5 as shown in Fig 4(b). Impulsive noise (α=1.5) was added to the square root raise-cosine signal and corrupted the signal at E_b/N_o =5 dB in Fig 4(c). From Fig 4(d), we can observe that impulsive noise corrupted signal was filtered and the magnitudes of the impulses were attenuated significantly from the output of median filter.

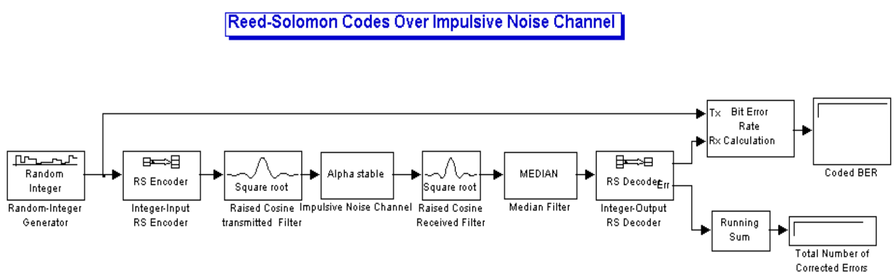

Fig. 3. Simulink Model for RS Codes over Impulsive Noise Channel with Median Filtering.

Corrupted received signal from impulsive channel was filtered and the outliers were removed from the received signal from the median filter output. Hence, median filter can protect the RS decoder against the influence of impulsive noise and converts the impulse corrupted received signal to AWGN-like corrupted signal. The filtering process effectively reduces the entries of impulses from received sequence hence it enables the optimal decoding performance of conventional RS codes over PLC channel. The following BER of RS codes were simulated with (15, 10) RS codes over GF(16) with t = 2, where 1000 codewords were used in simulation.

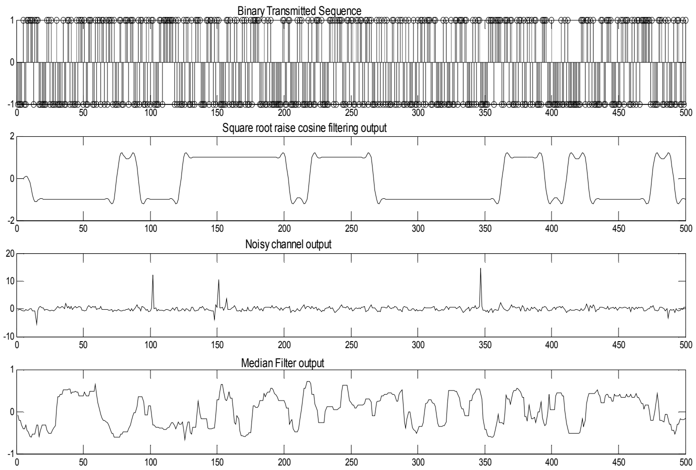

Fig. 4. Filtering Process of Median Filter prior to RS Decoding, (a) Binary Transmitted Sequence, (b) Output from Square Root Raise Cosine Filter, (c) Baseband Signal under Impulsive Noise (α=1.5) (d) Median Filter Output (N=10).

Figure 5 depicts the BER performance of (15, 10) RS codes over impulsive channel with α=1.5. It shows that decoding performance was improved substantially from the median filter output in comparison with direct RS decoding from impulsive channel without median filter. Hence, approximately 4 dB coding gain is observed from Fig. 5.

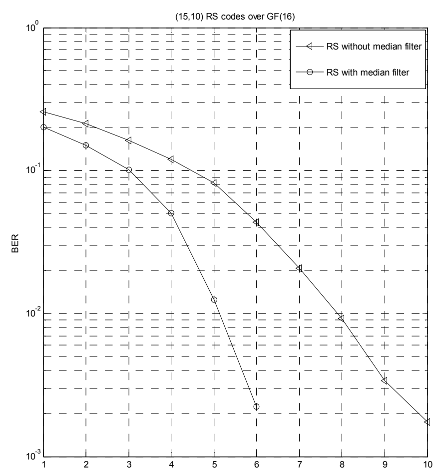

Fig. 5. BER Performance of (15, 10) RS Codes over Impulsive Channel with α=1.5.

Figure 6 compares the BER performance of (15, 10) RS codes in AWGN and impulsive noise (α=1) with median filtering. We can observe from Fig. 6 that the BER performance of RS codes has approximately 3 dB gain with $p_b = 10^{-3}$. Although the improvement is not as perfect as it is run on AWGN noise, however, better performance is obtained in comparison with direct detection from *SαS* noise channel.

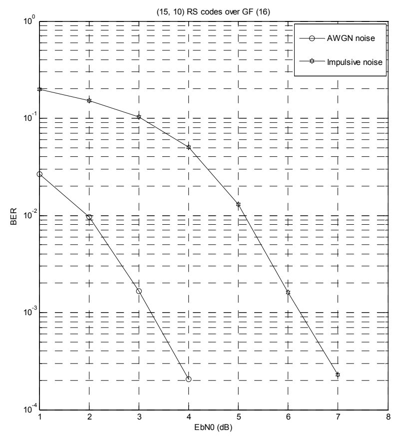

Fig. 6. BER Performance of (15, 10) RS Codes under AWGN Noise and Impulsive Noise (α=1.5) Interference.

The BER performance of median filter under various degree of α (between 0 to 2) was studied and the outcomes were presented in Fig. 7. Typically, the smaller the value of α, the heavier is the tail in distribution, therefore, while α = 0.5 has larger tail than α =1 and so on. With each increment of α indicates the decaying of tail of the *SαS* distribution. Finally, α=2 resembles to the Gaussian distribution.

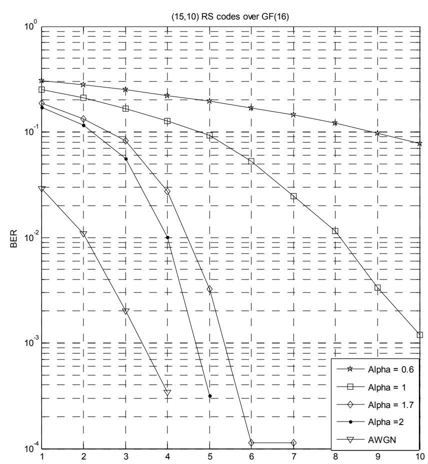

Fig. 7. BER Performance of (15, 10) RS Codes in Various Degree of Impulsiveness α.

6. Conclusions

Classical RS codes that are optimized for AWGN channel fail severely in new emerging PLC data communication. The study conducted here investigated the baseband transmission of data in impulsive noise. The abrupt change in magnitude of impulses from such channel has compromised the performance of RS codes that was mathematically used to correct burst errors in AWGN channel. To overcome the fundamentally deficiency of RS codes in impulsive noise, median filter was incorporated as part of detection mechanism of RS codes over basedband transmission system. Median filter can be easily implemented as FIR filter. The superior performance of median against outliers influence is fully demonstrated in our study.

The simulation results show the substantial BER improvement of RS codes over impulsive noise channel with $S\alpha S$ distribution. Therefore, the introduction of median filter into RS codes in baseband has enabled the implementation of RS codes as channel coding techniques for PLC communication. The use and realization of PLC communication would provide opportunity for internet services reaching suburban areas or those areas that are out of wireless internet coverage. In addition, PLC internet connection would provide more reliable and economical network to internet surfers.

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