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# BANDWIDTH ENHANCEMENT OF AN ANCTENNA BASED METAMATERIAL USING CHARACTERISTIC MODE ANALYSIS FOR MICROWAVE APPLICATIONS

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**Abstract**

In this paper, a Metamaterial (MTM) bowtie antenna, operating at 28 GHz, is designed based on the Characteristic Mode Analysis (CMA). The theory of characteristic modes is a modal analysis technique for antennas of arbitrary shape. It is applied to examine the behaviour of the bowtie antenna. Thanks to CMA we can find the adequate placement of the MTM unit cells allowing us to increase the antenna bandwidth. The radiating behaviour of this antenna is analysed using the first four characteristic modes. The simulation results show that the bandwidth enhances from 32% to 55%. This structure is suitable for millimetre-wave 5G applications.

Keywords: Bowtie antenna, Characteristic mode analysis (CMA), Metamaterial, 5G applications.

## 1. Introduction

The fifth-generation (5G) mobile communication technologies have gotten a lot of interest in providing mobile services and applications of various kinds for 2020 and later. 5G technology that embraces the millimetre wave is required to overcome existing constraints such as slow data transfer rate and spectrum scarcity [1]. To allow communication between wireless devices operating at higher frequencies, we require conformal, small, and compact antennas that are inexpensive and simple to fabricate. Bowtie antennas have been the preferred choice over other types of antennas in this regard. There are planar antennas with lightweight, which are easy to fabricate and give a better symmetry to radiation. The main limit of bowtie antennas is their bandwidth. This aspect is being improved significantly through a variety of approaches such as the use of a large number of antennas in array configuration [2], increased substrate thickness [3], the use of high permittivity substrates [4], and the use of MTM [5].

The Metamaterial is a promising tool for optimizing antenna performances, such as the bandwidth. They are frequently associated with antennas and other microwave devices due to their exceptional capacity to control electromagnetic waves. The specificity of MTM is the possibility of having a negative refractive index due to the negative permittivity and permeability.

In 1968, Veselago [6] presented the possibility of the propagation of an electromagnetic wave in a linear, homogeneous and isotropic medium and simultaneously possessing a negative permeability and permittivity. He also described the properties of structures possessing negative refractive index properties.

In 2001 the Experimental Validation of Negative Refraction was carried out by Shelby et al. [7], while in 2006, they started the use of MTMs with system devices [8]. The most intriguing feature of MTM is the ability to control or modify the permittivity and permeability of the material to achieve a behavior adapted to a specific application. MTMs can be used with antennas for various applications ; including antenna miniaturization [9, 10], bandwidth increase [11] and gain enhancement [12]

Systematic methodologies, such as Characteristic Mode Analysis (CMA), are now frequently utilized for antenna design and analysis of MTMs since they provide a complete physical understanding of the radiation phenomenon. Indeed, the CMA is an appealing tool for antenna design and may allow for the proper association of MTMs to a specific antenna. It is an efficient method for antenna design because of its capacity to provide direct insights into antenna propagation, and it allows for a more systematic antenna design rather than a brute-force approach. These insights will help us decide where to locate the MTM unit cell [13].

In this regard, Characteristic Modes Analysis is proposed to design an efficient MTM Bowtie antenna. Based on the Theory of Characteristic Modes, we used MTM unit cells to adequately place it with the antenna to obtain a wide bandwidth. CMA investigates how each part of the antenna structure influences the whole antenna performance. Through CMA, the physical insight of the radiation mechanism of the antenna can be easily observed. So that the radiation antenna’s geometry and the placement of MTM unit cells may be optimized. It can give a clear understanding of the antenna radiation patterns and the resonating modes. It is a form set of functions that, under specific boundary conditions, diagonalizes relating operator fields and induced sources.

CMA was proposed by Garbacz and Turpin [14[ in 1971. In the same year, the method was improved with simple technique derivation by Harrington and Mautz [15]. Many applications use CMA, including antenna size reduction [16], feeding network investigations [17], and radiation pattern modification [18]. We can find it also in bandwidth improvement, which will be the case in our study; Futter and Jakobus [19] analysed the antenna positioning to optimize the bandwidth. Perli and Rao [20] investigated the excitation position to enhance the bandwidth. While Tang et al. [21]suggested changing the antenna geometry to have a wide bandwidth.

In this paper, we proposed the design and analysis of a Bowtie antenna based on MTM. The novelty of this work is the use of a novel MTM unit cell and the CMA to examine the antenna behavior to enhance the bandwidth and significantly improve their performances. Simulation and analysis of the proposed antenna are carried out using, respectively, the ANSYS-HFSS Simulator and the computational electromagnetic software FEKO. This article will be divided into the following sections; Section 2 will illustrate the theoretical concept of the CMA method, and Section 3 and Section 4 will show, respectively, the antenna design and analysis. Section 5 will be devoted to results and discussion, and then the work will be achieved by a conclusion.

## 2. CMA Concept

Characteristic mode theory is an arbitrary form modal analysis technique. It also explains the resonating frequency of specific modes, radiation patterns, and mode currents in detail. The characteristic mode analysis CMA creates a set of orthogonal currents on a conducting body of any geometry. It expresses existing current patterns in a body using its natural resonance mode, similar to Fourier Series Expansion, which describes arbitrary waveforms by a sum of orthogonal harmonic functions. The resonance modes are determined by the body geometry and are not affected by excitation. When determining which modes will be present, the excitation, the geometry, and the location become essential. The Characteristic Modes can be calculated using a generalized Eigenvalue equation as shown in Eq. (1).

$$XJ_n = \lambda_n R J_n \tag{1}$$

where $X$ and $R$ represent the geometry's reactance and resistance, respectively, and $\lambda_n$ is an eigenvalue corresponding to the eigenvector $J_n$, which indicates the current density of the $n^{th}$ mode on the structure surface.

Another essential measure is the modal significance (MS), which is used to determine the resonant frequency and the bandwidth provided by a mode. It is represented by Eq. (2).

$$MS_n = \left|\frac{1}{(1+j\lambda_n)}\right| \tag{2}$$

The same information can be obtained from the characteristic angle curves provided by Eq. (3).

$$\beta_n = 180^{\circ} - tan^{-1}(\lambda_n) \tag{3}$$

When $\lambda_n = 0$ , $MS_n$=1 and $\beta_n = 180^{\circ}$, the modes resonate at a specific frequency. When $\lambda_n > 0$ and $\beta_n > 180^{\circ}$, the modes are storing magnetic energy, this indicates that we have an inductive mode. In otherwise, when $\lambda_n < 0$ and $\beta_n < 180^{\circ}$, the modes are storing electric energy, and this signifies that we have a capacitive mode.

## 3. Antenna Design

The proposed bowtie antenna is designed using ANSYS-HFSS. The antenna resonates at 28 GHz, which makes it suitable for 5G applications. The proposed MTM unit cell is structured in such a way that it operates at 28 GHz, as the antenna case.

### 3.1. Antenna without metamaterial

The suggested bowtie antenna is represented in Fig. 1. The type of substrate is Rogers Ultralam 1250 (TM) with a thickness of 2,25 mm. The elements, shown in orange, are modelled as a PEC and the yellow element placed at the center is the feed. The substrate's permittivity is equal to 2.5, and 0.0019 is the tangential loss. As mentioned previously the bowtie antenna was chosen for the application for many advantages such as the lightweight, the facility of fabrication, and the ability to give better symmetry in radiation. The employed excitation for the antenna is the lumped port. It is chosen to apply a uniform electric field between two metallic boundaries. Table. 1 lists the bowtie antenna parameter values.

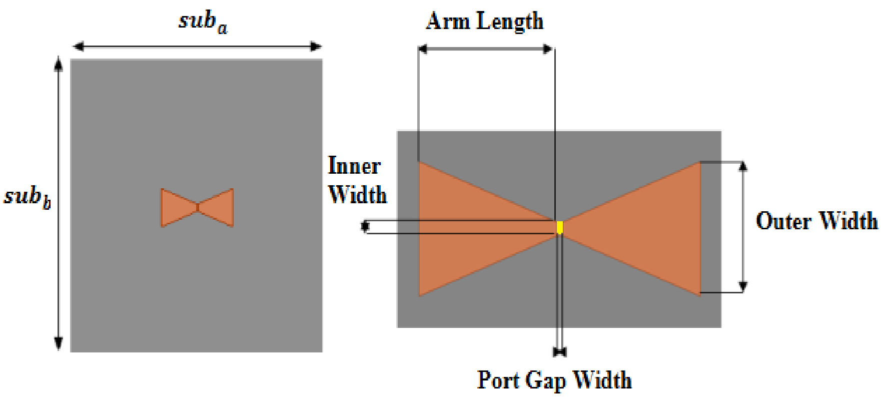



**Fig. 1. Bowtie antenna geometry (Front View).**

**Table 1. Antenna parameters.**

| Parameters | Dimensions (mm) |
|---|---|
| $sub_a$ | 10 |
| $sub_b$ | 10 |
| **Arm Length** | 1.38 |
| **Port Gap Width** | 0.05 |
| **Inner Width** | 0.25 |
| **Outer Width** | 1.28 |

The reflection coefficient S(1,1) of the bowtie antenna simulated with HFSS is shown in Fig. 2. We observe that the antenna resonates at 28 GHz and 39.5 GHz (≈ 40 GHz), with a bandwidth of 32% ( for the desired frequency of 28 GHz).

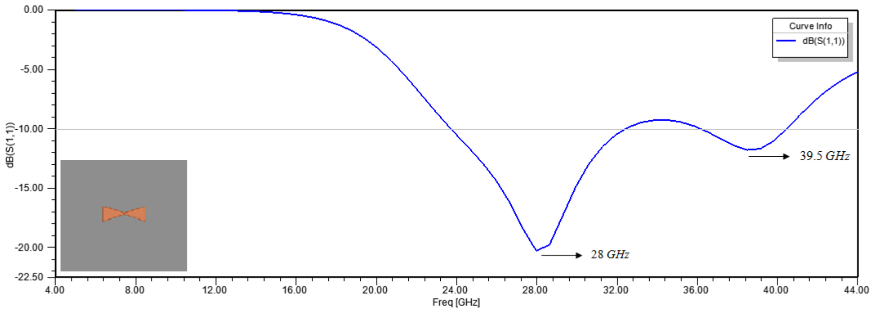



**Fig. 2. The reflection coefficient S(1,1) of the bowtie antenna.**

## 3.2. MTM design and extraction parameters

The proposed MTM, shown in Fig. 3, is a novel MTM structure. It is an evolved version of the Split Ring Resonator . The MTM mainly consists of an outer ring containing four slots behaving as capacitive loads and two inner elements acting as inductive loads. The same substrate as the antenna is chosen for the simulation .All of the parameters of the MTM unit cell are shown in Table 2.

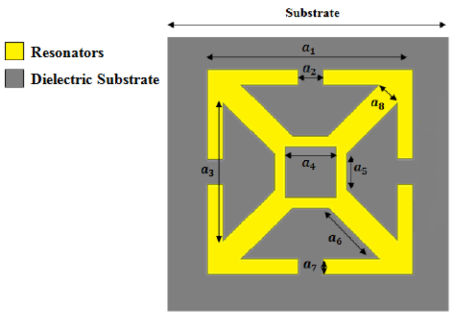



**Fig. 3. The geometry of the MTM Unit Cell (Front View).**

**Table 2. MTM Parameters.**

| Parameters | Dimensions (mm) |
|---|---|
| Substrate | 3 |
| $a_1$ | 2.2 |
| $a_2$ | 0.25 |
| $a_3$ | 1.34 |
| $a_4$ | 0.5 |
| $a_5$ | 0.34 |
| $a_6$ | 0.5 |
| $a_7$ | 0.15 |
| $a_8$ | 0.18 |

The S parameter for the magnitude of the modelled MTM structure is shown in Fig. 4. We observe that the MTM resonates at 28 GHz with a return loss reaching -37.5 dB, and a bandwidth of 12 %.

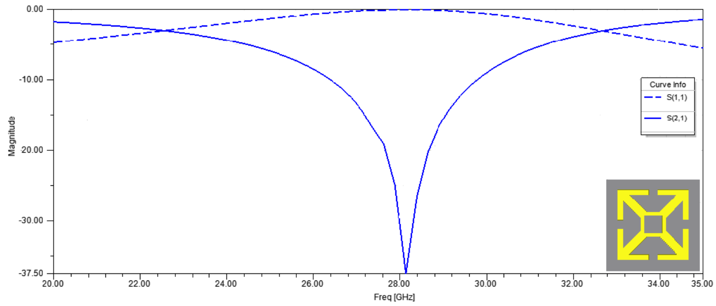


**Fig. 4. The reflection coefficient S(1,1) of the MTM unit cell.**

To demonstrate the validity of the MTM unit cell, we extract permeability and permittivity values. They are obtained with extraction software using MATLAB [22]. The results of the extracted parameters are shown, respectively in Figs. 5 and 6. We find that they are negative between 27 and 29 GHz, which corresponds to the transmission zone determined by the S parameter magnitude, and which verifies the properties of the MTM.

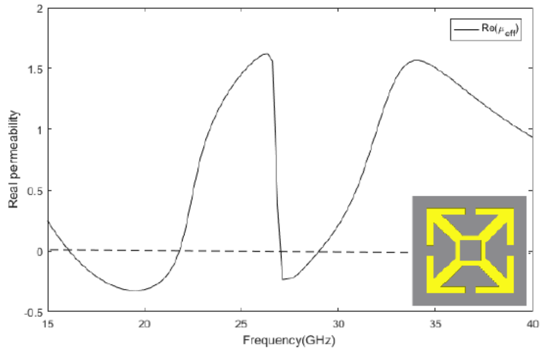


**Fig. 5. The real values of MTM permeability.**

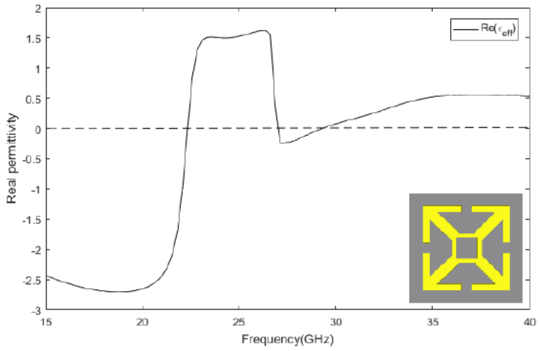


**Fig. 6. The real values of MTM permittivity.**

Figure 7 shows the equivalent circuit of the MTM unit cell. The equivalent circuit is extracted using the AWR simulator. The component values are chosen in such a way that the MTM works with the resonant frequency that they are as follows: inductors of 0.559 nH, capacitances of 0.0298 pF, and resistances of 6.76 ohm.

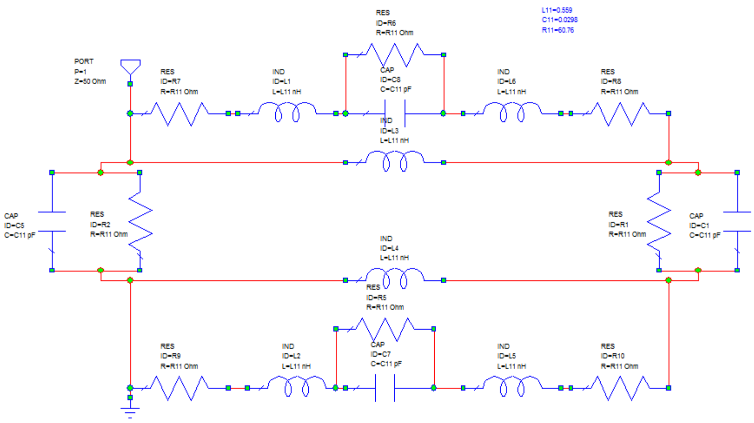
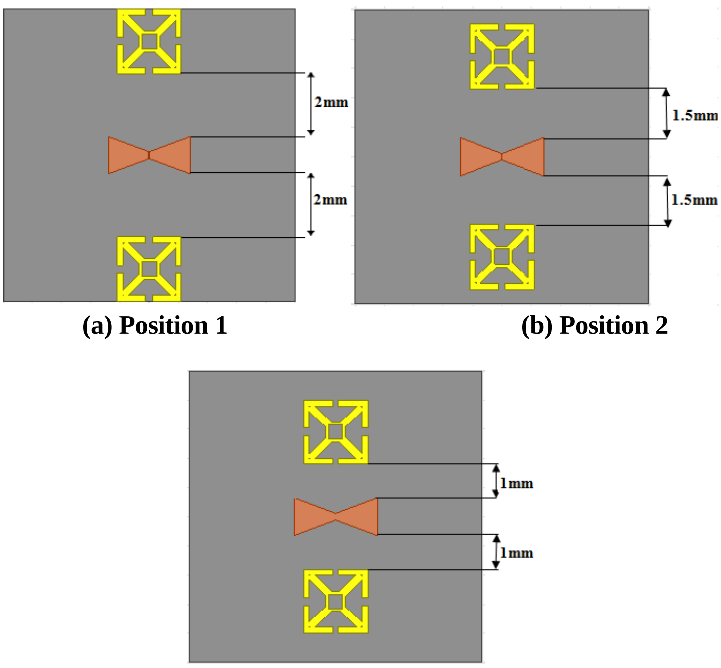


**Fig. 7. Equivalent Circuit of MTM unit cell.**

## 3.3. Antenna with MTM design

To place adequately the MTM unit cells with the antenna, we did a parametric study of the dimensions between the antenna and the MTMs. Figure 8 shows three tested positions of the MTM unit cells, and Fig. 9 illustrates the reflection coefficient S(1,1) of different positions. With Position 1, the antenna resonates at the band of interest and the bandwidth enhances in a good way. While with Position 2, the antenna resonates with a very narrow bandwidth. And with Position 3, the antenna operates with a wide bandwidth but not on the wanted resonating frequency. The adequate position then is position 1.




**(a) Position 1** **(b) Position 2**

**(c) Position 3**

**Fig. 8. Geometry of MTM antenna: (a) Position 1, (b) Position 2, (b) Position 3.**

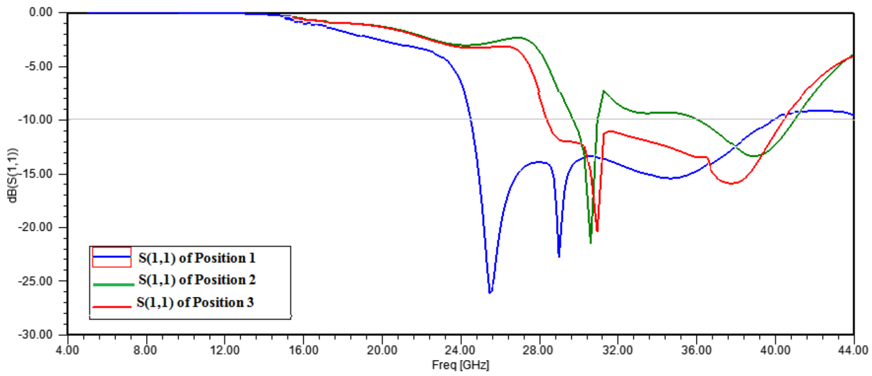



**Fig. 9. Parametric Study of S (1,1).**

Figure 10 depicts the bowtie antenna with MTM unit cells with the chosen position.

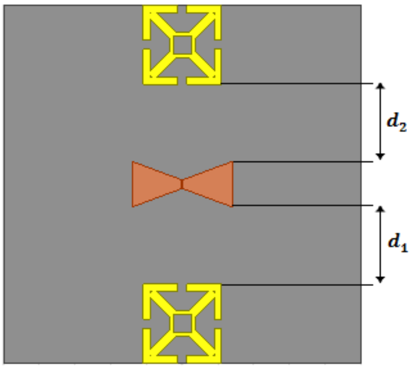



**Fig. 10. Geometry of MTM antenna (Front View).**

The MTM unit cells are mounted on the same substrate on the sides of the antenna with a distance of d1=d2=2 mm. Figure 11 presents the reflection coefficient S(1,1). We can see an improvement in the level of bandwidth , besides the two resonant frequencies of 26 GHz (≈27 GHz) and 29 GHz (≈28 GHz), with a bandwidth of 55% ( for 28 GHz).

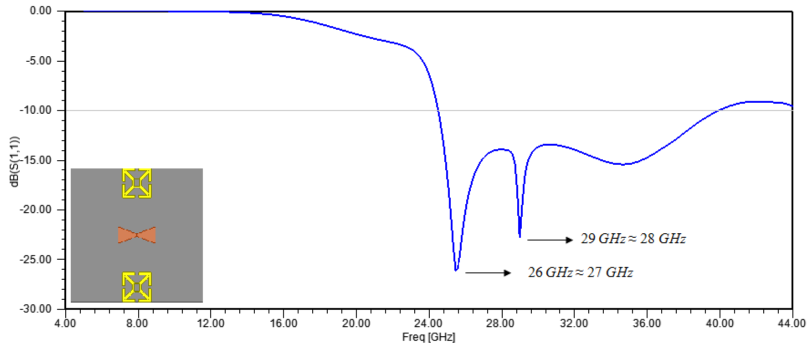



**Fig. 11. The reflection coefficient S(1,1) of the MTM bowtie antenna.**

## 4. Antenna Analysis

The proposed antenna's characteristic mode analysis (CMA) is performed using FEKO. The antenna resonates at 28 GHz, which makes it suitable for 5G applications. The theory of Characteristic Mode Analysis is applied here in a series of steps to investigate how each component of the antenna structure influences the overall performance of the antenna. First, we start by examining the ground plane only by the Theory of Characteristic Modes. The antenna including the ground plane is then examined. And afterward, The bowtie is investigated using the suggested MTM unit cells. The idea here is to know how to control the bandwidth of the bowtie using MTM unit cells. We first identify the frequency band modes, and then the MTMs will be integrated to enhance the bandwidth..

### 4.1. Ground plane analysis

From a physical point of view, the phase angle between the characteristic current $J_n$ and the associated characteristic field $E_n$ is represented by a characteristic angles. Figure 12 presents the variation characteristic angles vs frequency, and Fig. 13 presents the Modal Significance for the first four modes of the ground plane. It is observed in Fig. 12 that only Modes 1 and 2 are contributing to the band of interest at 28 GHz (since βn = 180°at 28 GHz ), while Mode 4 starts contributing after 31.36 GHz (since βn = 180° at 31.36 GHz ), and Mode 3 don't resonate in the band of interest. According to Fig. 13, the first two modes have a high modal significance around 1 at 28 GHz and the Modal Significance of Mode 4 reaches 1 at 31.22 GHz, while the Modal Significance of Mode 3 does not reach 1, and by that, Modes 1 and 2 radiate efficiently around their resonant frequencies. Mode 3 and mode 4 will not be considered in further work. Figure 14 shows the current distributions for Modes 1 and 2 at 28 GHz. It is observed that the vector direction of the current of Mode 1 is along the length of the ground plane according to the x-axis and runs across the ground plane according to the y-axis for Mode 2.

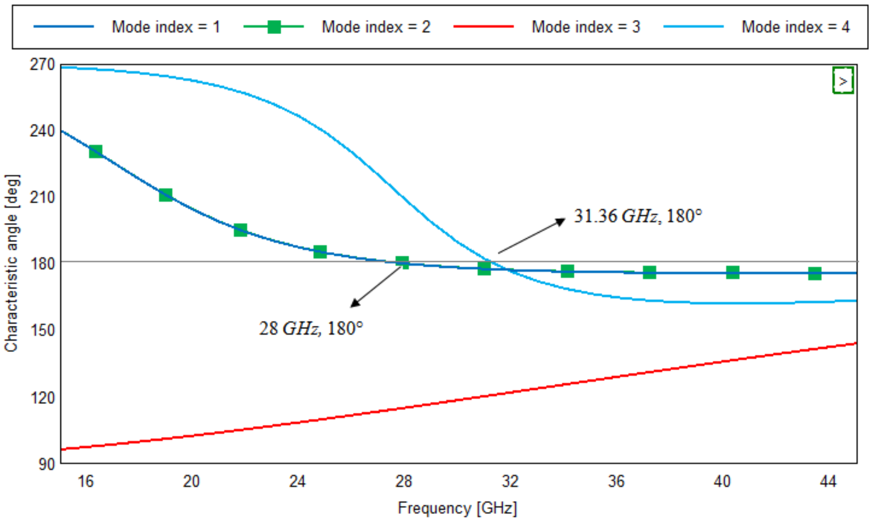



**Fig. 12. Characteristic angles of modes 1-4 of the ground plane.**

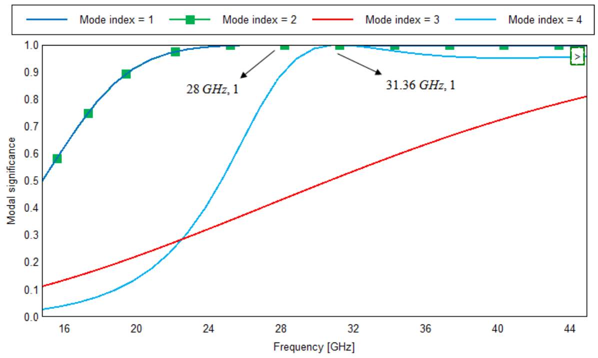
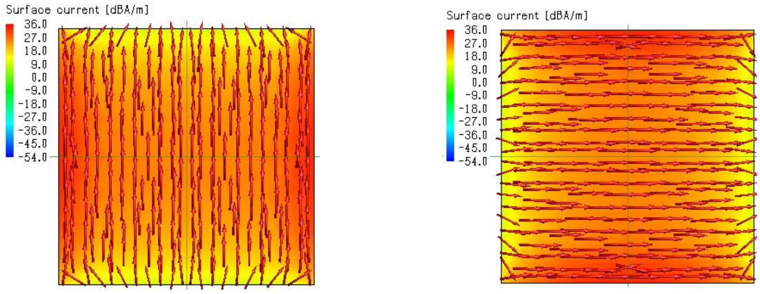



**Fig. 13. Modal Significance of modes 1-4 of the ground plane.**




**(a) Mode 1**

**(b) Mode 2**

**Fig. 14. Current distribution of the first two modes on the ground plane at 28 GHz**

## 4.2. Bowtie antenna analysis

CMA is applied here to the bowtie antenna design, presented recently in Fig. 1, including its substrate. The characteristic angles and Modal Significance for the first two modes of the bowtie antenna are shown, respectively, in Figs. 15 and 16, while the current distributions for the same modes are shown in Fig. 17.

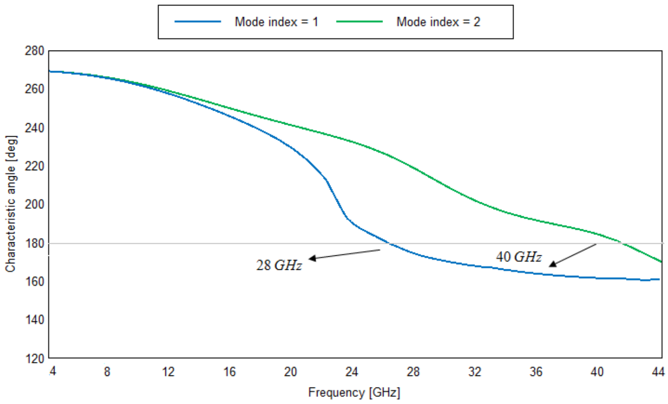



**Fig. 15. Characteristic angles of modes 1-2 of the bowtie antenna.**

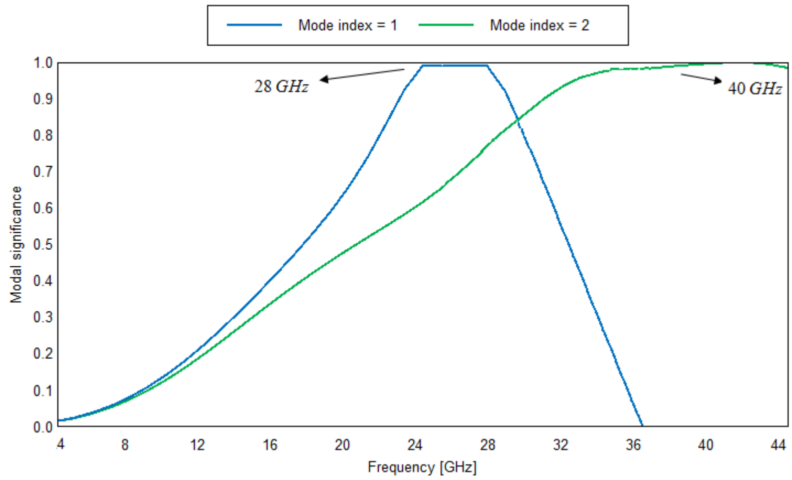
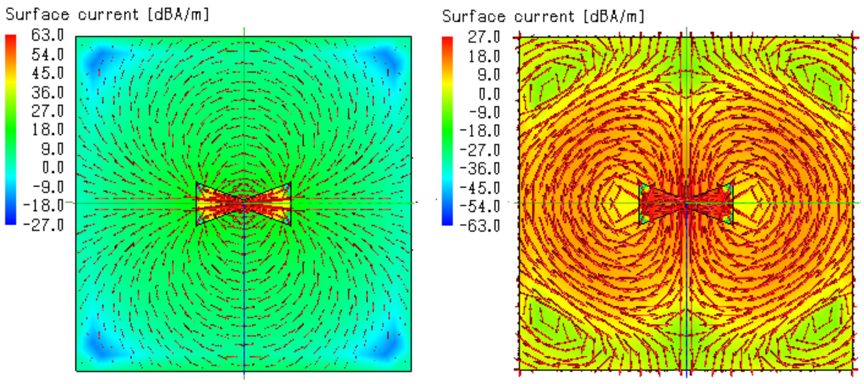



**Fig. 16. Modal Significance of modes 1-2 of the bowtie antenna.**

It is observed in Fig. 15 that Mode 1 contributes to the band of interest at 28 GHz, and Mode 2 resonates at 40 GHz. We can verify the previous results by the Modal Significance curve, as shown in Fig. 16. According to Fig. 17, Mode 1 has a double rotational mode, whereas Mode 2 has a double rotational mode as well, but with a cross mode located in the corners of the substrate. The goal here is to bring Mode 2 closer to Mode 1 to have large bandwidth. We will put the MTM unit cells to make the vector direction of Mode 2 behave like the vector direction of Mode 1. We aim so to have double rotational modes for both Modes.




**(a) Mode 1 at 28 GHZ** **(b) Mode 2 at 40 GHz**

**Fig. 17. Current distribution on the bowtie antenna.**

## 4.3. MTM Bowtie antenna analysis

The bowtie antenna incorporating the MTM unit cells, shown recently in Fig. 10, is analysed here using CMA. Figures 18 and 19 show, respectively, the Characteristic Angle and the Modal Significance of the antenna-based MTM.

The MTMs are placed to bring Mode 1 and Mode 2 closer together to get wideband behavior. Now Mode 1 and Mode 2 closely resonate at 28 GHz and 27 GHz, as shown in Figs. 18 and 19, and this subsequently enhances the bandwidth antenna almost twice compared with the antenna without MTM, from 32% to 55%. Figure 20 shows the current distribution of Mode 1 at 28 GHz and Mode 2 at 27 GHz of the bowtie antenna with MTM unit cells, we can notice that we have a double rotational mode for both Modes.

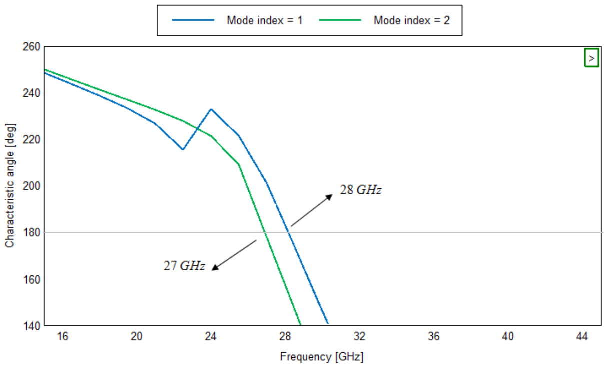



**Fig. 18. Characteristic angles of modes 1-2 of the MTM bowtie antenna.**

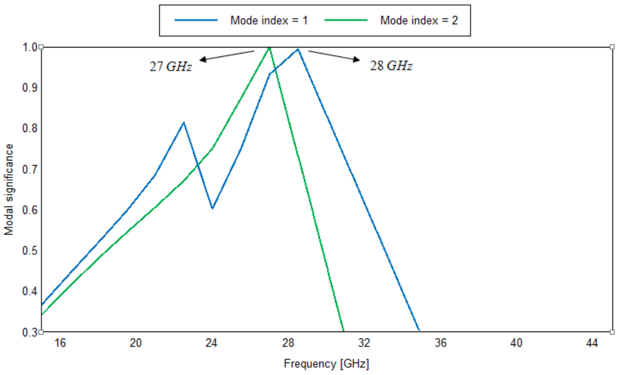
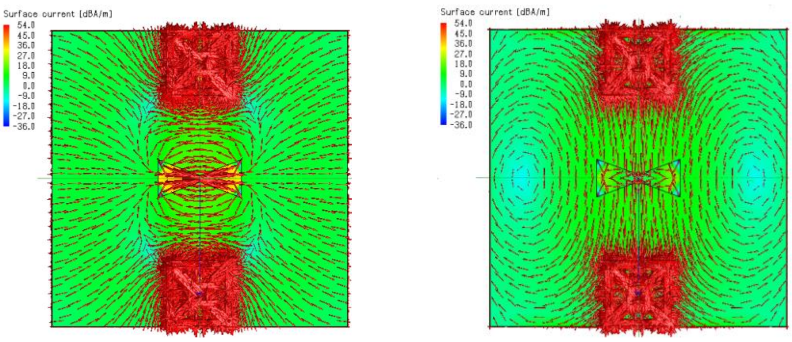



**Fig. 19. Modal Significance of modes 1-2 of the MTM Bowtie antenna.**




**(a) Mode 1 at 28 GHz** **(b) Mode 2 at 27 GHz**

**Fig. 20. Current distribution of the MTM bowtie antenna**

## 5.Results and Discussion

The reflection coefficient S(1,1) and the gain of the bowtie antenna without and with MTMs are presented in Figs. 21 and 22. We notice an improvement in the level of the bandwidth, which varies from 32 % (9.05 GHz) for the antenna without MTM to 55% (15.42 GHz) for the antenna with MTM. And an enhancement at the level of the gain that varies from 6.25 dB to 8.56 dB. The characteristic mode analysis helps us know the ideal location for the MTM unit cells to improve the bandwidth without increasing the antenna size and without influencing its radiation. It gives us a fundamental physical insight, unlike the driven simulation. Contrary to the Full Wave Solutions Methods, CMA can help in antenna design by determining where to place excitations and loads. The Full Wave Solutions solve Maxwell equations for the electric and magnetic fields in the time domain or the frequency domain. This type of rigorous analysis, when performed for the general 3D case, makes no simplifying assumptions about the nature of the EM problem.

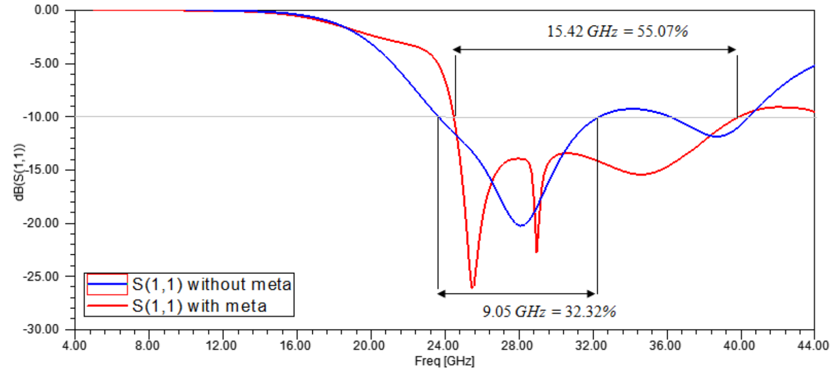



**Fig. 21. The reflection coefficient S(1,1) of the Bowtie antenna with and without MTM unit cells.**

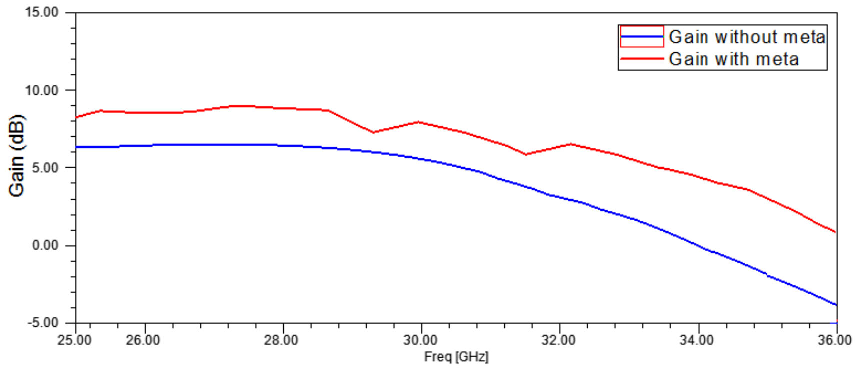
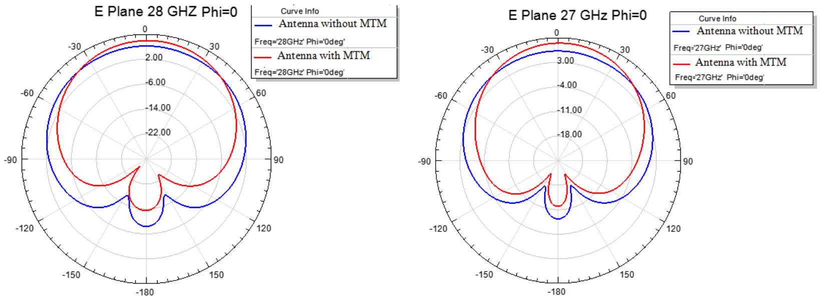



**Fig. 22. The gain vs frequency of the bowtie antenna with and without MTM unit cells.**

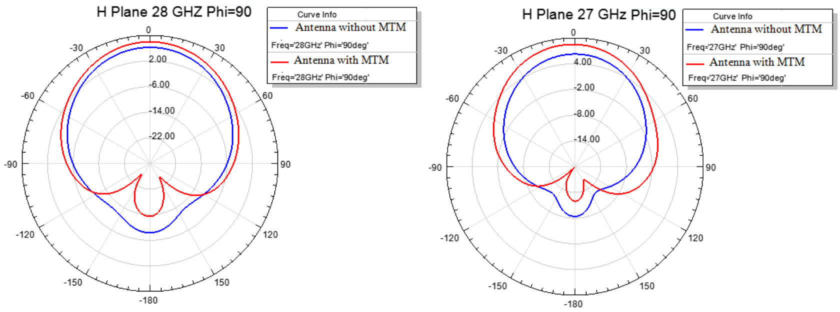
Figures 23 and 24, illustrate, respectively, the radiation pattern of the proposed antenna with and without MTM for the two resonant frequencies of Mode 1 and

Mode 2, on the E and H plane. We can notice the improvement of the antenna propagation on both the E and H Plane, thanks to MTMs.




**(a) Mode 1 at 28 GHz** **(b) Mode 2 at 27 GHz**

**Fig. 23. Radiation Pattern of Bowtie antenna with and without MTM- E Plane**




**(a) Mode 1 at 28 GHz** **(b) Mode 2 at 27 GHz**

**Fig. 24. Radiation pattern of Bowtie antenna with and without MTM- H plane.**

To ensure the validity and accuracy of the proposed method, the results are compared to other improvement techniques. The different results of these methods are shown in Table. 3.

Awan et al. [23] proposed the Defected Ground Surface (DGS) as a solution to optimize antenna performances. As we can see, this method does not achieve a wide bandwidth despite the small size of the structure. While Haraz et al. [24] used the EBG and a Superstrate above the array antenna. Yoon and Seo [25] suggested the use of a U Slot Array antenna to improve the propagation. The U-slot patch antenna structure consists of a patch, two U-shaped slots, and a ground plane. These two methods offer an acceptable bandwidth, but the structure is large. In [26], the Array Antenna method is also used, but at this time a 1x16 elements tapered antenna array feed with tapered microstrip lines and employing bending lines between the patches was proposed to enhance the antenna bandwidth. This method does not give a wide bandwidth, it has furthermore a big structure size. Bhavani et al. [27] proposed to use of an MTM-inspired non-uniform circular array superstrate for antenna applications to enhance the bandwidth and efficiency.

**Table 3. Improvement Techniques Comparison.**

| Reference | Frequency (GHz) | Improvement Technique | Dimensions | Bandwidth (GHz) | Bandwidth (%) |
|---|---|---|---|---|---|
| **[23]** | 28 | DGS | 0.46*0.46*0.07 $\lambda^3$ | 27.1-28.5 | 4.92 |
| **[24]** | 28 | EBG and Superstrate | 1.86*1.86*0.65 $\lambda^3$ | 27.1-30 | 10.70 |
| **[25]** | 28 | U Slot Array Antenna | 3.85*4.29*0.04 $\lambda^3$ | 26.9-31 | 14.27 |
| **[26]** | 28 | 1*16 Array Antenna | 3.27*2.61*0.07 $\lambda^3$ | 27.5-28.5 | 3.57 |
| **[27]** | 5.75 | CMA and Metasurface | 0.52(radius)*0.03$\lambda^3$ | 5.5-6.1 | 10.43 |
| **This work** | 28 | CMA and MTM | 0.93*0.93*0.21 $\lambda^3$ | 24.4-39.8 | 55 |

From the different results, we note that our proposed technique has the highest bandwidth value compared to other techniques' results by guaranteeing a small size. The use of MTM unit cells and the Theory of Characteristic Mode Analysis on the antenna structure may be an adequate solution for 5G microwave applications thanks to the good performance that offers.

## 6. Conclusions

A bowtie antenna based on MTM, resonating at 28 GHz, is proposed in this paper The suggested antenna has been modelled and designed using Characteristic Modes Analysis, which has been proven to be an efficient method. Compared with the antenna without MTM, the proposed structure has a wide bandwidth of 55%. and a gain of 8.56 dB. Compared also with recent works, the proposed antenna enhances bandwidth by guaranteeing a small size. The improvement of this paper is to use novel MTM unit cells to add to the antenna using the Characteristic Mode analysis method, to enhance the antenna bandwidth without increasing its size. The proposed antenna good be an ideal candidate for the 5G mobile applications.

### Nomenclatures

| | |
|---|---|
| $J_n$ | Eigenvector |
| *MS* | Modal Significance |
| *R* | Resistance of the geometry |
| *X* | Reactance of the geometry |

***Greek Symbols***

| | |
|---|---|
| $\beta$ | Characteristic Angles |
| $\lambda_n$ | Eigenvalue corresponding to the eigenvector $J_n$ |

**Abbreviations**

| | |
|---|---|
| CMA | Characteristic Mode Analysis |
| DGS | Defected Ground Structure |
| EBG | Electromagnetic Band Gap |
| MTM | Metamaterial |
| TCM | Theory of Characteristic Mode |

## References

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