Journal of Engineering Science and Technology
Vol. 18, No. 1 (2023) 521 - 533

ESTIMATION OF THE WAVE-INDUCED ROLL–PITCH ANGLE OF SURFACE VESSELS

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Abstract

Surface vessels such as frigates, destroyers, carriers, battle cruisers, coastal guard ships, and fisheries surveillance are capable of operating in severe weather conditions. One of the most challenges in such conditions is the capsizing phenomenon. In this paper, an engineering model occupied with only five primary design parameters of a vessel including length, breadth, draught, weight, draft, and displacement was suggested to estimate the Roll and Pitch angle of a surface vessel and estimated the capsizing capability in heavy sea states. A dataset of six coastal guard ships was used to process the simulation. The simulation results showed that the typical resonance frequency of the Roll angle is between 0.05 to 0.2Hz while that of the Pitch angle is between 0.1 to 0.18 Hz. The Roll and Pitch angle profiles with respect to frequency were plotted to determine the safe operating condition of these ships. To verify the theoretical model in real sea conditions, an Inertia Moment Unit (IMU) – based device was built and implemented. The experiment was implemented in a fisheries boat in the South China sea to test the performance of the device and the correlation with the suggested profile. The measurement results provide relatively coherent matching with the prediction from the simplified model. For the first time, the results suggest a simple and low-cost method to predict the motion of surface vessels and the marine devices in such ships to operate functionally in heavy sea conditions by the use of five primary ship parameters and one sea state parameter.

Keywords: Engineering model, Hydrostatic simulation, Pitch motion, Roll motion, Ship motion, Surface vessel.

1. Introduction

Surface vessels such as frigates, destroyers, carriers, battlecruisers, coastal guard ships, and fisheries surveillance have been built larger, faster, and even unmanned all over the world [1]. They are capable of working in severe weather conditions such as typhoons, storms, and even tsunamis. In addition, many surface vessels today are equipped with comparatively delicate equipment and instrument whose performance and life span can be influenced by excessive vibrations and motion [2]. Therefore, the study of ship motion/vibration induced by external factors like waves and wind has been a critical issue for decades and played an important role in the aspect of both simulation/modeling and experimental measurement [3-13].

Huang et al. [3] presented numerical simulation results of large-scale vessels in the high sea state of level 7. Jia et al. [4] experimentally evaluated the quality of the sensor to perform measurements of an Inertially Stabilized Gimbal Platform which is the major part of a ship-borne Electro-Optical system in warfare. Vishnubhatila et al. [13] applied a closed-form analytic solution to predict vessel dynamics in a realistic seaway.

Thu et al. [14] established a mathematical model of a ship in waves with the consistency of coupled motions in Roll – Yaw, and Sway – Roll - Yaw angles. However, most of the publications do not include the “dead ship” situation in which the vessel is unable to manoeuvre itself in a severe sea state. In this case, the natural frequencies of the large surface vessel may fall into the range of waves. Hence, the resonance may happen and cause damage to the equipment working in the ship, or even catastrophic damage such as capsizing [15].

Capsizing is considered the sudden transition of a ship from a position of equilibrium to another position with the entrance of a large quantity of water. Such a phenomenon is normally very fast and devastating with human loss. Capsizing often happens when sudden large waves hit the ship and affect the Roll and Pitch angle immediately (Fig. 1). To avoid this phenomenon, modern ships have been equipped with anti-capsize solutions such as roll damping using Fuzzy-extension control, roll-stabilizers systems [7, 13, 16].

In the experimental aspect, the measurement of ship vibration in harsh sea conditions is usually not practical. Hence, researchers and marine engineers developed numerical models to predict the behavior of the ship in storms or large waves [3, 13, 17, 18]. However, implementing such models usually requires either lots of ship and sea parameters or computational resources which are not usually published for free. Therefore, a simplified approach to determine the limited ship motion angle of ships in high sea states is needed intensively.

In this paper, we suggested an engineering model built from basic ship design parameters to estimate the Roll and Pitch angle limitation of surface vessels and estimated the capsizing capability in heavy sea states of six coastal guard vessels.

Of all the six motions of a ship, the Roll and Pitch angles are the most difficult ones to compensate for and control in practice. These two rotation motions are also the most serious factors affecting the capsizing phenomenon. Therefore, the model focuses on the Roll-Pitch profile angle with respect to major surface vessel engineering dimensions such as the length, the breath, the draught, etc.

To verify the model, an Inertia Moment Unit (IMU) – based device was developed which is capable of measuring the vibration in Roll – Pitch angle. An experiment in a fisheries boat was conducted in the South China sea to test the performance of the device and the correlation with the suggested profile. The measurement results provide relatively coherent matching with the prediction from the simplified model. The results suggest a simple method to predict the motion of surface vessels and the marine devices in such ships to operate functionally in heavy sea conditions.

The study is divided into five main sections. The Introduction part describing the needs of the study is presented in Section 1. Section 2 – Model and Simulation – introduces the model and the simulation approach on the basis of data of Rumanian coastal guard vessels. Section 3 presents the measurement basis and data logging implementation. Results and discussion are presented in Section 4 and the Conclusion is summarized in Section 5.

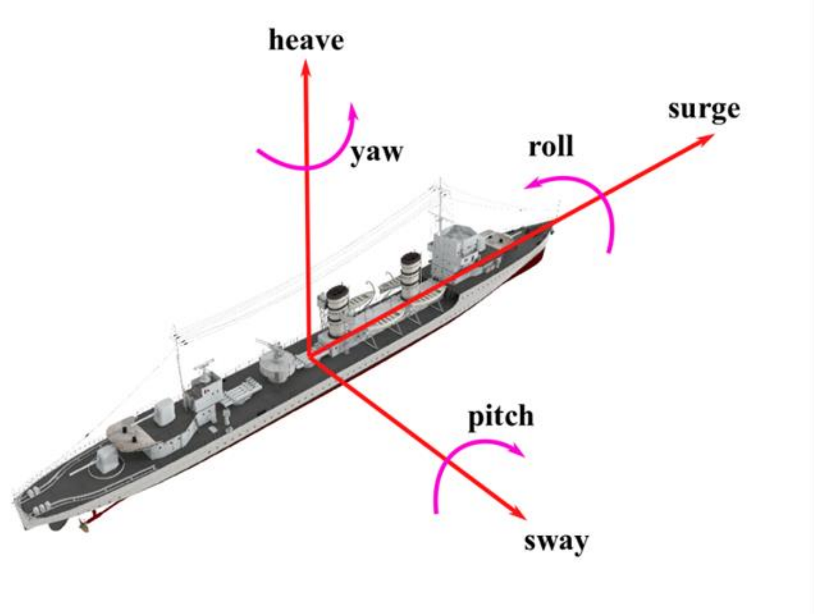

Fig. 1. Six motions of a destroyer.

2. Model and Simulation

2.1. Terminology and explanation

2.1.1. Sea state

Sea state is the condition of the wind waves and swells in the sea at a certain location and moment and can be validated by experienced observers rather than delicate equipment. To qualify the sea state, the weight heights are statistically collected and classified into 9 codes followed by The World Meteorological Organization (WMO) in Table 1. In marine engineering, two characteristics of sea states are often used: the significant wave height and the mean wave period. The significant wave height $H_{1/3}$ is the mean wave height of one-third of the highest waves over a period of time, while the mean wave period T is the mean of all wave periods. It is noted that the sea states of the target typhoons in the South China Sea are above sea state 7 and the wave period of 10 to 15 seconds [19].

Table 1. Sea state code followed by WMO.

Sea state code	Wave height (meter)	Characteristic
0	0	Calm
1	0 to 0.1	Calm
2	0.1 to 0.5	Smooth
3	0.5 to 1.25	Slight
4	1.25 to 2.5	Moderate
5	2.5 to 4	Rough
6	4 to 6	Very rough
7	6 to 9	High
8	9 to 14	Very high
9	Over 14	Phenomenal

2.1.2. Wave and wave frequency

Waves are oscillations or disturbances of the water surface and are caused by various effects such as wind, gravitational forces, and earthquakes. In the sea, the most common waves are formed by wind, therefore, National Weather Service (NWS) established the Beaufort scale to estimate the wind speed and sea state for weather forecasts. To estimate the wave frequency, researchers may use the real data measured or use the wave spectral densities formula by Pierson and Moskowitz (PM spectrum) [20]:

$$S_\varsigma(\omega) = \frac{\alpha g^2}{\omega^2} exp\left[-\beta\left(\frac{g}{U\omega}\right)^4\right] \tag{1}$$

where $\propto = 0.0081, \beta = 0.74$ and U is the wind velocity at the height of 19.4 meters over the sea surface. In equation (1), the parameters α and β were determined via an interpolation process generated from analyses of wave spectra in the North Atlantic Ocean in 1963. Based on measured and simulated data, the wave frequency at the high sea state can be estimated from 0.05 to 0.3 Hz (Fig. 2).

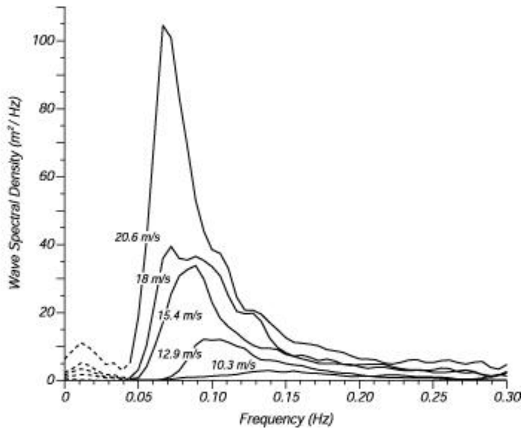

Fig. 2. The PM spectra with respect to wind velocity. The wind velocity of 20.6 m/s is equivalent to the sea state of 7 in Table 1. Reprinted from [20].

2.2. Model establishment

For a surface vessel, the hydrostatic stability is mostly related to the restoring forces which are usually referred to as metacentric stability. The metacentric height (GM) calculates the distance between the ship's center of gravity and the metacentre which is divided into the transverse metacentric height and longitudinal metacentric height (Fig. 3).

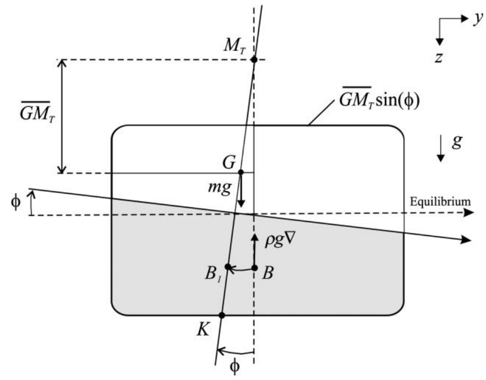

Fig. 3. Transverse metacentric stability. Reprinted from [21].

The motions of a ship can be expressed by Newton's laws, fluid dynamics, and restoring forces such as buoyancy force. However, because the ship structure is very complicated, together with the high nonlinearity of external forces including waves and wind, the simulation of ship motion can be classified by hydrostatic and hydrodynamics models. In the condition of a "dead ship", the manoeuvrings capability is not available, and the ship is hit by large waves at a certain ship speed, hence, the hydrostatic model in a short time is preferable to the hydrodynamic model. With the presence of the coming wave, the hydrostatic ship motion model is described by Eq. (2):

$$(I_x + I_{xx})\frac{d^2\Phi}{dt^2} + \delta\frac{d\Phi}{dt} + DgGZ(\Phi) = P(\alpha, t) \quad (2)$$

where I_x, I_{xx} are the inertial moment of the ship along an axis and the additional inertial moment with the presence of hitting waves, δ is the damping coefficient of water, D is the displacement mass of the ship, g is the gravity acceleration, $GZ(\Phi) = GM\sin(\Phi)$ is the upright level of restoring moment, or metacentric height, $P(\alpha, t)$ is the external moment which is given by the waves and wind.

The ship's natural frequency in Roll and Pitch angles are calculated based on the GM information as Eqs. (3) and (4).

$$\varpi_{roll} \propto \sqrt{\frac{\Delta\overline{GM_T}}{I_{xx}}} = 2\pi\frac{\sqrt{\overline{GM_T}}}{C_{nb}\times B} \quad (3)$$

$$\varpi_{pitch} \propto \sqrt{\frac{\Delta\overline{GM_L}}{I_{xx}}} = 2\pi\frac{\sqrt{\overline{GM_L}}}{C_{nl}\times L} \quad (4)$$

The detailed expression can be found in [21, 22].

However, because of the assumption of uncoupled waves, the linear hydrostatic model can be expressed in an engineering way as:

$$\ddot{\Phi} + \delta\dot{\Phi} + \frac{g\overline{GM}}{k^2} sin\Phi = \varepsilon_0 \sin\left(\varpi t - kx\right) \quad (5)$$

where $k = \frac{2\pi}{\lambda}$ is the wave number, λ is the wavelength. Considering the capsizing under one severe hit, the motion in space is not necessary. Therefore, the contribution of the space variable can be neglected. Equation (5) can be replaced by Eq. (6):

$$\ddot{\Phi} + \delta\dot{\Phi} + \frac{g\overline{GM}}{k^2} sin\Phi = \varepsilon_0 \sin\left(\varpi_e t\right) \quad (6)$$

where $\varepsilon_0 \sin\left(\varpi_{et}\right)$ is the encounter waves in sinusoidal form with zero phase lag.

From Eq. (6), a MATLAB SIMULINK model was built to express the linear vibration model at a particular frequency as shown in Fig. 4.

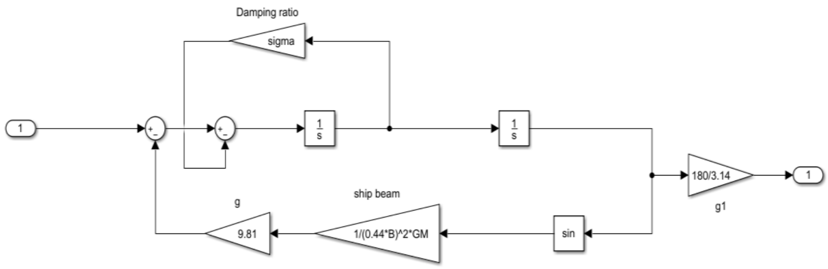

Fig. 4. The Simulink model to estimate the response of the ship.

The calculation procedure is expressed in the procedure below:

Step 1*:* Setting the boundary limits for the Roll and Pitch angle based on the statistical report. The maximum Roll angle before capsizing is determined as 40º to 45º, while the maximum Pitch angle is 8º followed by the 5 year-data of USS SPERRY aircraft carrier and SUMNER-class destroyer [23].

Step 2: Determine the damping ratio range

Step 3: Calculate the encounter frequency waves

Step 4: Expanse the encounter frequency waves range by a safety ratio based on experimental data

Step 5: Calculate the Roll/Pitch angle at extended encounter frequency waves

Step 6: Plot the data of the Roll/Pitch angle with respect to frequency.

2.3. Simulation results

A dataset of six Rumania coastal guard vessels (River Patrol, Venus, Minelayer, Fast Patrol, Corvette, Missile Patrol) (Table 2) was used for calculation. The parameters of six Rumania coastal guard vessels were collected from the official website.

Figure 5 shows the Roll angle of six vessels at a certain sea state level of 7 or at $H_{1/3}$ = 9 m and the damping ratio $\delta = 0.1$, River Patrol is the only survival with the Roll angle of 39.5º while all the others may capsize. If the ship was equipped with an anti-roll solution or carried additional heavy mass which lower the metacentre by 10 to 20%, the damping ratio can be increased [11, 22]. In the case of a high damping ratio $\delta = 0.4$, the chance for survival is tremendously increased [24] (Fig. 6). At that time, when the ship is hit by a 9 m height wave, Minelayer and Covertte may be in danger while the Roll angles of the four vessels are below 40°.

Table 2. Parameter design of six Rumania coastal guard vessels and a fisheries boat using for testing the model.

Parameter	Term	Unit	River Patrol	Venus	Minelayer	Fast Patrol	Corvette	Missile Patrol	Fisheries ship for testing
Wave height at simulation/test condition	$H_{1/3}$	m	9.0	9.0	9.0	9.0	9.0	9.0	0.5-1
Maximum speed	V	kn	16	18	19	36	24	36	9
Length	Lt	m	45.7	38.7	79	36.8	92.4	56.1	16.5
Breath	B	m	8	5.4	10.6	7.6	11.7	11.5	4
Weight	M	t	320	129	1450	215	1500	455	55
Draught	D	m	2.3	3	5.4	2.7	4.7	3.8	1.5
Draft	T	m	1.5	2	3.6	1.8	3.1	2.5	2.1
Displacement volume	Delta	m^3	548.4	422.1	3014.6	503.4	3351.3	1612.9	130

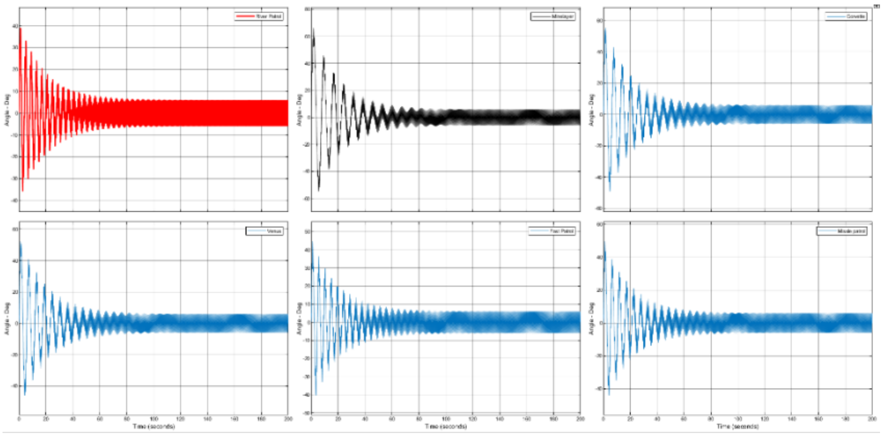

Fig. 5. The Roll angle simulation results of six surface vessels at a low damping ratio and sea state of 7 (wave height of 9.0 meters).

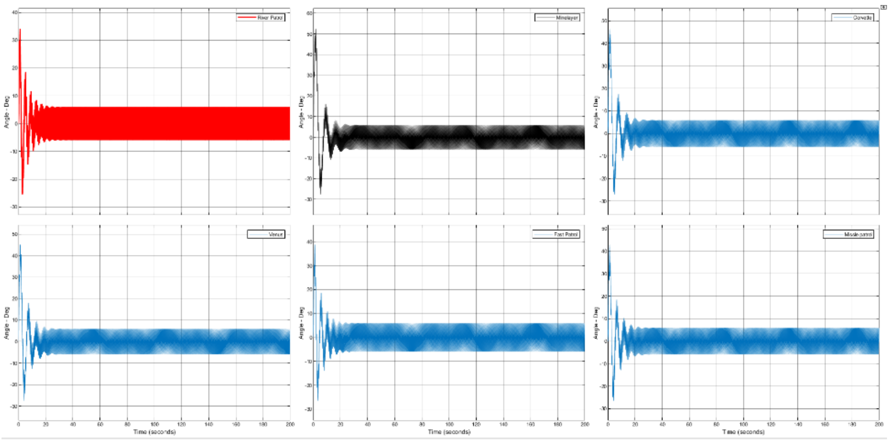

Fig. 6. The roll angle simulation results of six surface vessels at a high damping ratio and sea state of 7 (wave height of 9.0 meters).

A parametric simulation process was run to find the rolling angle of the data set ship and the profile angle as shown in Figs. 7 and 8. From the simulation, the Roll natural frequency falls between 0.05 to 0.2Hz which is close to the wave frequency at sea state from 3 to 9. It explains that capsizing usually happens in Roll motion; in other words, the wave hit the ship at an angle of 90^o degrees. The Pitch natural frequency is from 0.3 to 0.5 Hz which is rather different from slamming. Therefore, it is not easy to capsize a Rumania surface vessel by a slamming hit at sea state 7. The simulation results reveal the most impact frequency region from 0.1 to 0.5 Hz at which the energy is concentrated. Based on the calculation from the data set of Rumania ships, a fisheries boat was used to test the model by an Inertial Motion Unit (IMU) – based device in the South China sea.

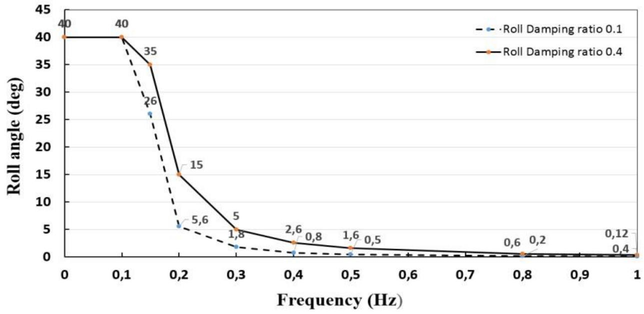

Fig. 7. The Roll angle profile of all six surface vessels at different damping ratios and encounter wave frequencies.

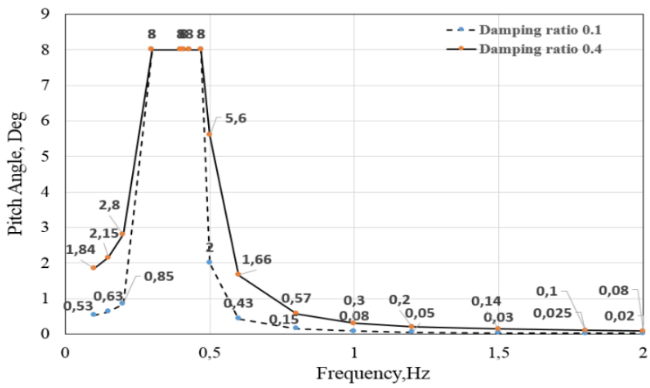

Fig. 8. The Pitch angle profile of all six surface vessels at different damping ratios and encounter wave frequencies.

3. Measurement Results

3.1. IMU-based measurement device

To test the model in a real situation, an IMU-based motion data logging device was developed. The device is composed of an IMU, a main control module, a real-time data logging module, and a data storage module. Of all the main modules, the IMU is the most important device that guaranty the performance of the measurement. The usage IMU was a high-performance GNSS-Aided Inertial Navigation System (GNSS/INS)

VN-210 (VECTORNAV, Texas, USA) which has an IMU data (sampling rate) of 800 Hz, the dynamic Pitch/Roll angle of 0.015°, and the noise density as < 0.04 mg/√Hz. The IMU VN-210 can provide an angular resolution up to 0.001° and a cross-axis sensitivity up to 0.05°. In the field environment, the data sampling frequency is set to 400 Hz. The diagram of the data acquisition device is shown in Fig. 9.

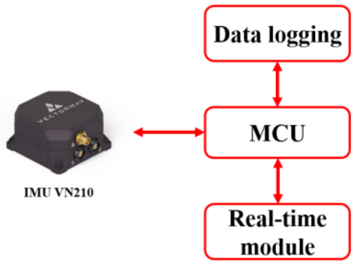

Fig. 9. The connection diagram of the motion data logging device

3.2. Implementation in the South China sea

A fisheries ship was used to test in a real sea condition. The ship parameter design is described in Table 2. The test was performed in a sea state 3 with the wave height $H_{1/3} = 0.5 \sim 1$ meters. The ship maneuvering path starts from a harbor to a test location at 20°32,102'N, 107^{O} 16,882'E (Fig. 10) at a constant speed of 7 knots. The data acquisition device was installed at the hull and used to acquire the Roll and Pitch angle in the time domain.

Fig. 10. The map of the test location in the South China sea (Vietnam territory).

4. Discussion

The simulation result shows that for the Rumania coastal guard ships, the typical Roll resonance frequency is between 0.05 to 0.2Hz and the Pitch resonance frequency is between 0.3 to 0.5Hz. In the case of large-scale surface vessels, Roll and Pitch resonance angle calculation was between 0.05 to 0.12 Hz and 0.1 to 0.18 Hz respectively. Compared to the experimental data, the typical warship characteristics have periods in the same range (10.1 to 15.8 s with Roll angle and 6.5 to 8.8 s with Pitch angle) [3, 13].

Some reports have published equivalent results by different mathematical methods [24-26]. Ueng [24] analysed the hydrostatic motion based on the stability theory of ship. The simulation results suggested that the ship has the Roll angle of 30 degrees at the wave height of 6 meters. Dostal et al. [25] calculated the probability density of stability for large ship and summarized the Pitch angle response ranging from 0.25 to 0.6 rad/s. Maki et al. [26] discuss the dynamic of the capsizing of a vessel in seas and derived the characteristic of the equation of the ship motion in hydrostatic model. The authors utilized the basis numbers of ship parameters with nonlinear differential equations to describe the behavior of ship in the sea and obtained the similar capsizing possibility.

In the case of the testing fisheries boat, the boat is much smaller in dimension and runs at a slower speed than coastal guard vessels. Under the calm sea state of 0.5 to 1.0-meter wave height, the Roll and Pitch angle was simulated as 0.25° and 0.15° respectively (Figs. 11 and 12). However, the real data shows that the Roll and Pitch angle was 0.22° and 0.12°. It is obvious that the simulation model is based on linear theory on hydrostatic and uncoupled waves, therefore, the theoretical model is needed to upgrade. The measured frequency range fits with the Roll – Pitch angle profile (Figs. 7 and 8) which shows that the simplified model can express partly the behavior of the ship.

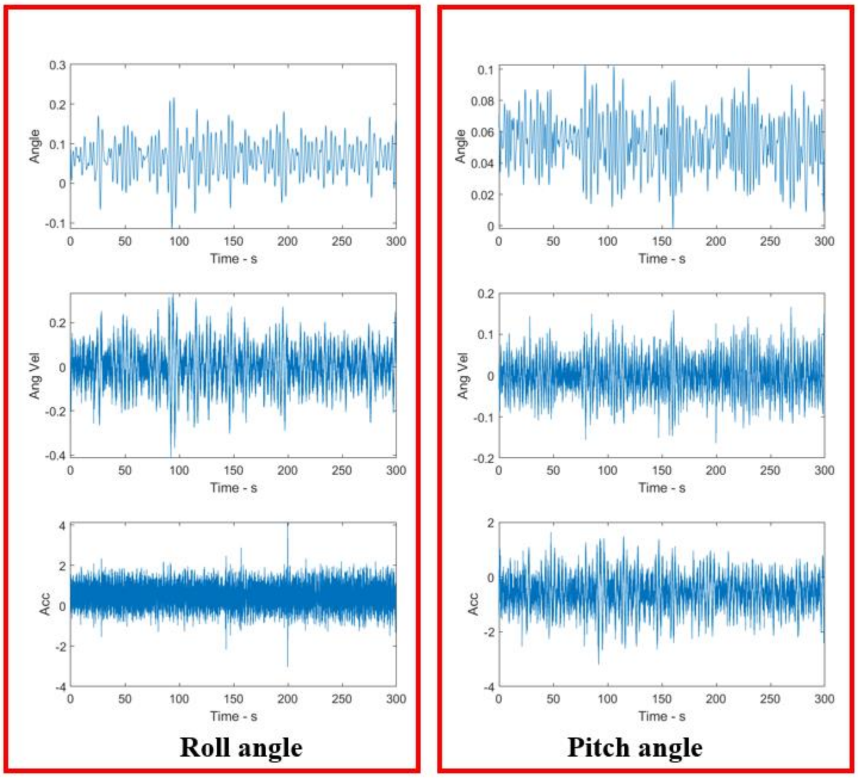

Fig. 11. Time domain plot of roll and pitch angle.

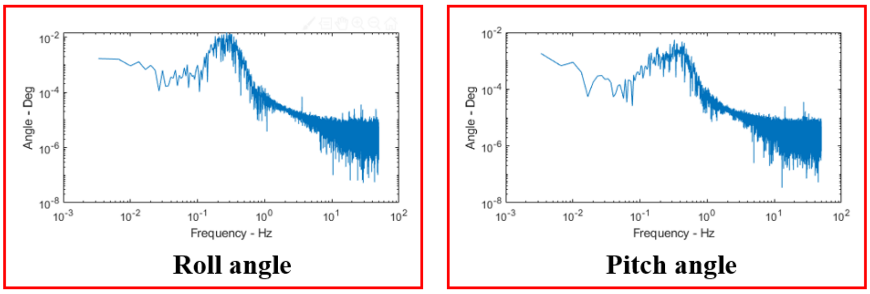

Fig. 12. Frequency domain of roll and pitch angle.

The research results can be applied in a variety of applications in marine operations such as automatic identification system (AIS) used by vessel traffic services (VTS). By the data such as type, length, breadth, draught, speed, and the estimated sea state, the Ocean warning like the Indian Ocean Tsunamic Warning (IOTW) system will announce to reduce the casualty. It is also important for surveillance, fisheries to survive in the case of hitting large waves in storms or typhoons.

5. Conclusion

The study focuses on establishing a linear hydrostatic model of surface vessels under severe sea state conditions. The reason for applying the theory to surface vessels is that they are designed to be fully functional in storms, typhoons, or even tsunamis. The delicate devices such as weapons, light-of-sight systems, radar, communication are able to operate in such severe conditions. Six Rumania surface vessels were used to apply the model.

The calculation showed that only two ships can survive in the sea state of 7 while others may capsize. Based on that estimation, a profile of safe operating condition in Roll and Pitch angle of these six ships were plotted in which the resonance frequency of the Roll and Pitch angle falls between 0.05 to 0.2Hz and 0.3 to 0.5 Hz respectively in the condition of sea state 7 (wave height of 9 meters). To verify the model, a fisheries boat was tested under a calm and good sea in the South China sea and provided information by the use of the model.

In conclusion, based on fundamental and engineering parameters such as the length, breadth, draught, draft, and displacement volume, the response of the vessels in the most dangerous condition of Roll - Pitch angle can be estimated. Therefore, the capsizing phenomenon can be predicted, and loss caused in such heavy conditions can be reduced. The study can be upgraded to an automatic warning system in marine engineering worldwide.

References

1. O'Riourke, R. (2022). Navy Large Unmanned Surface and Undersea Vehicle: Background and Issues for Congress. *Congressional Research Service*. p. 43.
2. Zhou, X.; Jia, Y.; Zhao, Q.; and Yu, Q. (2016). Experimental Validation of a Compound Control Scheme for a Two-Axis Inertially Stabilized Platform with

Multi-Sensors in an Unmanned Helicopter-Based Airborne Power Line Inspection System. *Sensors*, 16(3), 366.

3. Huang, S.; Liu, W. ; Luo, W.; and Wang, K. (2021). Numerical simulation of the motion of a large scale unmanned surface vessel in high sea state waves. *Journal of Marine Science and Engineering*, 9(9), 982.
4. Rutting, J.; and Tazartes, D. (2017). System Performance of an Inertially Stabilized Gimbal Platform with Friction, Resonance, and Vibration Effects. *Journal of Nonlinear Dynamics,*1-20
5. Lin, Y.-H.; and Lin, C.-W. (2019). Numerical simulation of seakeeping performance on the preliminary design of a semi-planing craft. *Journal of Marine Science and Engineering,* 7(7), 199
6. Andres, F.; Rodriguez, A.; Sande, J.; Peña E.; and Rabuñal, J.-R. (2018). Dynamical study of a moored vessel using computer vision. *Journal of Marine Science and Engineering,* 26(2), 240
7. Wang, M.H.; Yu, Y.Q.; and Zeng, B. (2011). Roll Damping and Anti-Capsizing System for Ships Using Fuzzy-Extension Control Strategy. *Applied Mechanics and Materials*, 84-85, 144-154
8. Jiao, J.; Ren, H.; Sun, S.; and Adeyna, C.A. (2016*).* Experimental investigation of wave-induced ship hydroelastic vibrations by large-scale model measurement in coastal waves. *Shock and vibration*, 1-14
9. Do, T.S.; and Tran, C.V.(2016). Determination of Added Mass and Inertia Moment of Marine Ships Moving in 6 Degrees of Freedom. *International Journal of Transportation Engineering and Technology*, 2(1), 8-16
10. Heins, P.H.; Jones, B.L.; and Taunton, D.J. (2017). Design and validation of an unmanned surface vehicle simulation model. *Applied Mathematical Modelling*, 48, 749-774
11. Newman, J.N. (2017), Marine Hydrodynamics, *The MIT Press*
12. Lin, T.R.; Pan, J.; Peter, J.O.; and Mechefskec, C.K. (2009). A study of vibration and vibration control of ship structures. *Marine Structures,* 22(4),730-743.
13. Vishnubhotla, S.; Falzarano, J.; and Vakaskis, A. (2000). A new method to predict vessel/platform critical dynamics in a realistic seaway. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences,* 358(1771),1967-1981
14. Thu, A.M.; Htwe, E.E.; and Win, H.H. (2015). Mathematical modeling of a ship motion in waves under coupled motions. *International Journal of Engineering and Applied Sciences,* 2(12), 97-102.
15. Hu, L.-F.; Zhang, K.-Z.; Li, X.-Y.; and Chang, R.-X. (2019). Capsizing Probability of Dead Ship Stability in Beam Wind and Wave for Damaged Ship. *China Ocean Engineering*, 33(2), 245-251.
16. Sanchez, L. (2008). Capsize prevention system for power vessels, *US Patent,* US20090228162A1
17. Wang, L.; and Han, S. (2018). Visual simulation of a capsizing ship in stormy weather conditions. *The Visual Computer,* 35(12), 1855-1868

18. Rahaman, M.M.; Islam, H.; Akimoto, H.; and Islam, M. R. (2017). Motion predictions of ships in actual operating conditions using potential flow-based solver. *Journal of Naval Architecture and Marine Engineering*, 14(1),65-76.
19. Pang, L.; Chen, X.; and Li, Y.-L. (2013). Long-Term Probability Prediction on the Extreme Sea States Induced by Typhoon of the South China Sea. *Advanced Material Research*, 726-731,833-841.
20. Wikiwaves.(2012). Ocean-Wave Spectra from: https://wikiwaves.org/Ocean-Wave_Spectra
21. Fossen, T.I. (2011) *Handbook of marine craft hydrodynamics and motion control.* John Wiley & Sons.
22. Adrian, B.; and Ruben, L.-P. (2014). *Ship Hydrostatics and Stability*. Elsevier.
23. Buchmann, E.; and Connell, M. (1959). Environmental conditions of ship motions and vibrations for design of radar systems on destroyers and aircraft carriers. David Taylor Model Basin report
24. Ueng, S.-K. (2013). Physical models for simulating ship stability and hydrostatic motions. *Journal of Marine Science and Technology*, 21(6), Article 8.
25. Dostal, L.; Kreuzer, E.; and Namachchivaya, N. S. (2013). Stochastic Averaging of Roll-Pitch and Roll-Heave Motion in Random Seas, *Procedia IUTAM*, 6. 132-140.
26. Maki,A.; Miino, Y.; Umeda, N.; Sakai, M.; Ueta, T.; and Kawakami, H. (2022) Nonlinear dynamics of ship capsizing at sea. *Nonlinear theory and its applications, IEICE.* 13(1). 2-24.