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A MODIFIED METHOD FOR SELECTING SINGULAR VALUES IN IMAGE COMPRESSION USING SINGULAR VALUE DECOMPOSITION

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Abstract

Most image compression techniques based on singular value decomposition (SVD) divide the image into blocks; some of these blocks contain the edges of an image, which represent an important part of human acceptance. SVD transform decomposes the matrix into three matrices, one contains the singular values of the matrix, and the others contain singular vectors. The compression is achieved by ignoring some singular values with the associated singular vectors, so the compression is lossy. The ignoring process starts from the smallest singular value until a predefined criterion is met such as a specified MSE or a specified percentage of the sum of singular values. The specified criteria's value is applied to all blocks. In this work, a range of percentage of the sum of the ignored singular values is specified and a percentage value is computed for each block lying within the range. The amount of the lost information in the edges' blocks is less than the other blocks. A comparison between the suggested method for selecting singular values where the percentage value is computed for each block and a constant percentage value for all blocks is presented. The Peak signal to noise ratio of the reconstructed image, compression ratio, and the average approximated rank are used as an evaluation criterion. The PSNR for the camera_man image is 27.74, and the average approximated rank is 1.119 for a constant percentage of 30 while the PSNR is 29.68, and the average approximated rank is 1.232 when the proposed method with a range of percentage is from 5 to 30 is used. The results show that the suggested method gave better image quality, especially the edges parts of the image, and better compression results.

Keywords: Edge preservation, Image coding, Image compression, Singular value decomposition.

1. Introduction

With the development of technology, digital information in multimedia is increasing daily which is mostly images in the form of either pictures or video frames. These images demand a massive memory and bandwidth in storage and transmission. Image compression solves these problems by reducing the storage space required for images taking into account the quality of the reconstructed images [1].

Image compression based on transform is flexible and more popular than direct and parametric techniques [2]. Transform coding is a reversible linear transform that maps the image into a set of transform coefficients. A significant number of these coefficients for most natural images have small values, which can be coarsely quantized or omitted with a low degradation in the reconstructed image quality. The image data can be transformed by many transformation techniques [3]. For any given image the best energy packing efficiency is the singular value decomposition transform. A large computation effort is required for computing the eigenvalues and eigenvectors of a big image matrix, which limits the benefit of the SVD transform [4]. The encoder must transmit the singular values and the corresponding singular vectors of the transform [5]. The SVD transform can be used to find a good approximated image when the image has a low rank or can be approximated well by a low- rank matrix, the representation of the low- rank matrix needs less storage space than the original image [6].

In most applications based on transform coding, the images are divided into blocks. In general, the computational complexity and the level of compression increase as the block size increases. The most common block sizes are 8*8 and 16*16 [3]. Many compression techniques apply SVD transform on blocks of the image [2, 5, 7-9], some of these blocks contribute to the edges of the image. The edges in images are significant for the human acceptance of reconstructed images. The subjective quality of the decoded image is decreased when the edges are not well represented despite the signal-to- noise ratio may give a good decoding result [10, 11].

The SVD transform is combined with other transforms in many works. The Discrete Cosine Transform (DCT) and SVD are combined in [5, 7], the DCT is applied to blocks of the divided source image having high correlation, while SVD transform is used on blocks having greater high-frequency contents. In [9], the 64*64 blocks of the divided image are compressed by either SVD or Discrete Wavelet Transform (DWT). In [12], the image is first compressed by SVD then it is compressed again by wavelet Difference Reduction(WDR). In [2] a modified singular value decomposition is used as a preprocessing for the Adaptive Set Partitioning in Hierarchical Tree. In [13], SVD is used with an optimal choice of singular values and an adaptive partitioning of the blocks. The DWT, DCT, and SVD transforms are combined to obtain satellite image compression [14].

In SVD transform, the image can be well represented by keeping a few largest singular values and corresponding singular vectors [9]. Different criteria are utilized for selecting the kept singular values such as the Mean Square Error (MSE) of the block or a percentage of the sum of singular values [2, 9]. The criteria's value used for stopping the discarding of singular values is the same for all blocks. In this work, the quality of the reconstructed image is improved by keeping more information about the blocks containing edges. A range of percentage, of the sum of the ignored singular values, is specified and a percentage value is computed for each block depending on the edginess of the block.

2. SVD Coding

The SVD is a linear mechanism that decomposes the image matrix into its eigenvectors and eigenvalues. This transform can find a low-rank approximation of the image, which makes it widely used for image compression [15].

An image is represented as a matrix with numbers reflecting the intensity values of pixels in the image. SVD transform decomposes a matrix denoted by $A_{m\times n}$ into three matrices denoted by U, Σ, and V, the matrices U and V are orthogonal [12, 16]. U is a matrix including left singular vectors of A. V is a matrix including right singular vectors of A. Σ is a diagonal matrix containing the singular values of the matrix A [17].

The eigenvectors of the matrix AA^T are the columns of U, where the eigenvectors of the matrix A^TA are the columns of V. The square roots of the eigenvalues of AA^T or A^TA are the diagonal elements of Σ in descending order sort [13, 18]. Let σi represent the diagonal elements of Σ, then $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_N \geq 0$, $\sigma_i = \sqrt{\lambda_i}$,

where λ_i for $i = 1, 2, \ldots\ldots, N$ are the eigenvalues of the matrix A[14, 17].

Assume the size of the image matrix A is $m\times n$, the sizes of U are $m\times m$, Σ is $m\times n$, and V is $n\times n$. The number of non-zero elements on the diagonal of Σ represents the rank of the input image. Compression is achieved by ignoring small singular values to obtain the approximation of the input matrix. Equation (1) is showing the mathematical representation

$$\Sigma_{m\times n} = \begin{bmatrix} \bar{\Sigma}_{p\times q} & 0 \\ 0 & \ddots \end{bmatrix} \quad p \leq m \ and \ q \leq n \tag{1}$$

The number of rows and columns in $\bar{\Sigma}$ is less than Σ, so to do the matrix multiplication the U and V can be described as in Eqs (2) and (3).

$$U_{m\times m} = [\overline{U}_{m\times p} \quad \widetilde{U}_{m\times(m-p)}] \tag{2}$$

$$V_{n\times n} = [\overline{V}_{n\times q} \quad \widetilde{V}_{n\times(n-q)}] \tag{3}$$

However, the image can be reconstructed as follows :

$$A_{m\times n} = \overline{U}_{m\times p}\overline{\Sigma}_{p\times q}(\overline{V}_{n\times q})^T \tag{4}$$

Because the diagonal matrix Σ contains the singular values in descending sort order, the visual quality of the reconstructed image will not be affected significantly when ignoring low singular values. This property makes the SVD suitable for image compression [12, 19].

Assume a matrix A of size $m*n$ is to be compressed using p singular values, p left singular vectors from U, and p right singular vectors from V, then A require $p(1+m+n)$ memory cells. The data compression can be calculated as follows

$$d = \frac{m\times n}{p(1+m+n)} \tag{5}$$

where $m\times n$ is the memory cells required for storing uncompressed matrix A, $p(1+m+n)$ is the memory cells required for storing compressed matrix A [20].

3. The Proposed Method

Compression techniques based on SVD are lossy. Many techniques divided the image into blocks and the SVD transform is used on some or all blocks. Some of these blocks contain the edges of the image, which represent noticeable parts of the vision, and the degradation in these parts reduces the subjective quality of the reconstructed image.

In this work, a range of the loss of information in the blocks depending on the edginess of the block is suggested, so the loss of information allowed on the edge blocks is reduced. The edginess of the block is computed as in [10] depending on the variance measure Eq (6) inside the block x.

$$variance = \frac{1}{N}\sum_{k=1}^{N} x_k^2 - \bar{x}^2 \quad (6)$$

where $\bar{x}$: is the mean of the block
N: number of pixels in the block.

As mentioned in section 2, the SVD transform break down the block into U, Σ, and V matrices. The Σ matrix is a diagonal matrix sorted from the biggest to the lowest value. In most previous works, the SVD is applied to the block, the singular values of each block are ignored progressively beginning from the lowest value until a predefined criteria value is met, such as a MSE [9] or a percentage of the sum of singular values of the block [2].

In this work, a range of a percentage of the sum of the ignored singular values is defined. For each block, a percentage of the sum of the ignored singular values is computed depending on the edginess of the block. The block with the highest variance gets the lowest value of the specified range.

As the variance decreases the computed percentage value, of the ignored singular value, grows toward the maximum value of the range. The ignoring of singular values starts from the lowest value (σ_8), the suggested formula for computing the value of the percentage, of the ignored singular values, of each block is as follows:

$$block\_percentage = maxp - (Vr/maxv)\,(maxp - minp) \quad (7)$$

where
minp: minimum value of the specified range.
maxp: maximum value of the specified range.
maxv :maximum variance for all blocks.
Vr: current block variance.

Assume block size is 8×8, after computing the block percentage of the sum of the ignored singular values, the approximated rank is the first q satisfying the following formula:

$$\frac{\sigma_8+\sigma_7+\sigma_6+\ldots\ldots+\sigma_q}{\sigma_1+\sigma_2+\sigma_3+\sigma_4+\ldots\ldots+\sigma_8} * 100 \geq block\_percentage \quad (8)$$

According to the proposed formula, we obtain a reduction in the average rank and an improvement in the decoded image quality by preserving the sensitive features, edges, of the image. Figure 1 shows the flowchart of the proposed method.

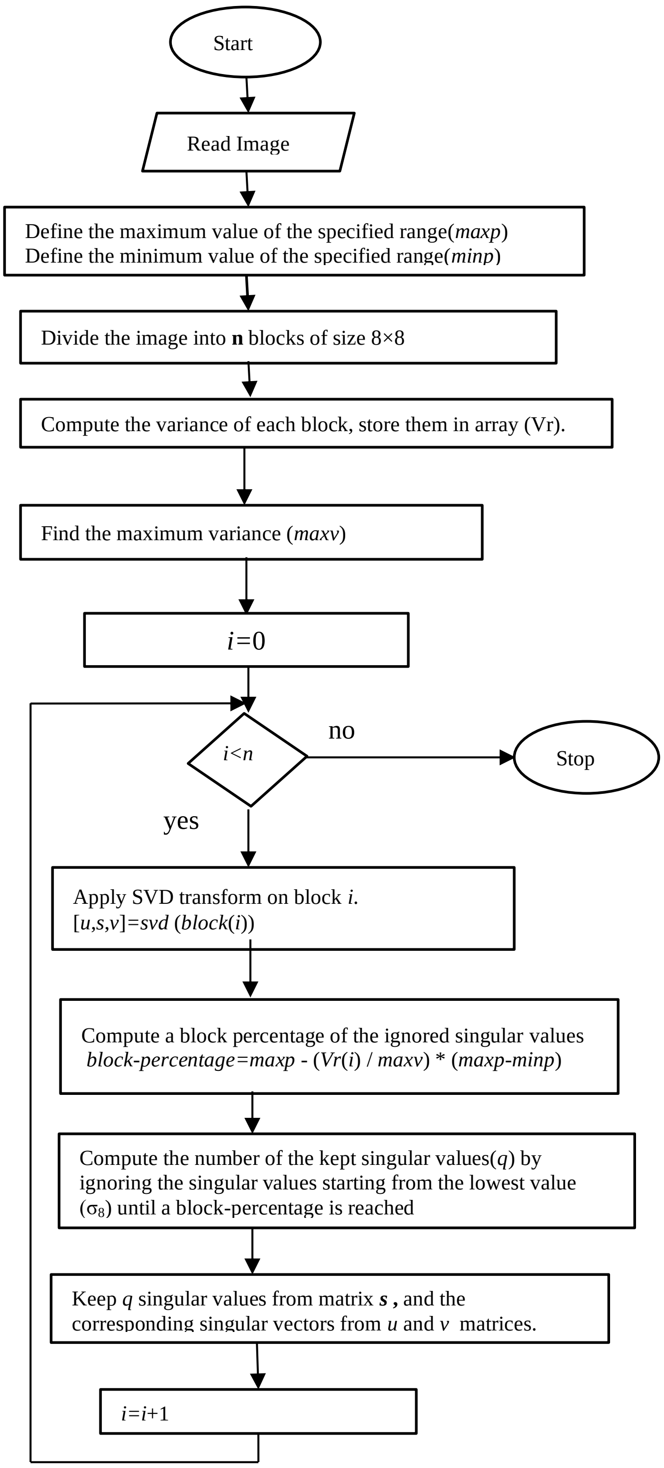


Fig. 1. Flowchart of the SVD image compression using the proposed method for selecting the singular values.

4. Results and Discussion

The test images used in this section are the camera_man and car of size 256×256 with 256 gray levels. The image is divided into blocks of size 8×8 and the SVD is applied to each block. The peak signal to noise ratio (*PSNR*) Eq (9) is used to measure the distortion based on the *MSE* [9]:

$$\mathrm{PSNR} = 10\log_{10}\frac{(255)^2}{\mathrm{MSE}}\,\mathrm{dB} \tag{9}$$

The average approximated rank Eq. (10) is computed as follows [21]:

$$\mathrm{Average(q)} = \frac{\sum_{i=1}^{nblocks} q_i}{nblocks} \tag{10}$$

The compression ratio (*CR*) is computed as follows

$$CR = \frac{m\times n}{\sum_{i=1}^{nblocks} q_i(x+y+1) \quad +nblocks} \tag{11}$$

where q_i represents the kept singular values for block *i*, *nblocks* represents the number of image blocks, *m* and *n* represent the size of the image, *x* and *y* represent the size of the block.

The results of applying the proposed algorithm which compute a different percentage of the ignored singular values for each block, are collected in Table 1. Different maximum values of the range are specified with a minimum value is 5. Table 2 shows the results when a constant percentage of the ignored singular values is used for stopping the ignoring process on all blocks.

Table 1. A range of percentage, of the ignored singular values, is specified with a minimum value of 5. The image is partitioned into 8*8 blocks.

image_ name	Max-range of the percentage	*PSNR*	Average approximated rank (*q*)	Maximum q_i	Compression Ratio
Camera_man	10	35.8007	1.8633	6	1.9586
Camera_man	15	33.1789	1.5576	5	2.3290
Camera_man	20	31.5493	1.3799	5	2.6167
Camera_man	25	30.5072	1.2939	5	2.7830
Camera_man	30	29.6829	1.2324	4	2.9156
Car	10	38.1994	1.5908	4	2.2821
Car	15	35.5180	1.2588	3	2.8572
Car	20	34.0094	1.1270	3	3.1749
Car	25	32.8678	1.0752	3	3.3198
Car	30	32.0281	1.0469	2	3.4048

Table 2. A constant percentage of the ignored singular values. The image is partitioned into 8*8 blocks.

Image name	Constant percentage	*PSNR*	Average approximated rank (*q*)	Maximum q_i	Compressi on Ratio
Camera_man	5	39.7159	2.4629	6	1.4929
Camera_man	10	34.7161	1.7920	6	2.0341
Camera_man	15	31.8879	1.4727	5	2.4582
Camera_man	20	29.8170	1.2783	4	2.8155
Camera_man	25	28.5043	1.1738	4	3.0542
Camera_man	30	27.7443	1.1191	4	3.1959
Car	5	42.2967	2.3252	5	1.5791
Car	10	37.4874	1.5547	4	2.3332
Car	15	34.5492	1.2275	3	2.9266
Car	20	32.7491	1.0918	2	3.2719
Car	25	31.4872	1.0420	2	3.4199
Car	30	30.5994	1.0137	2	3.5102

An improvement in the PSNR results is obtained when specifying a range of percentage compared with a constant percentage value. Figure 2 shows the original camera_man image. Figure 3 shows the reconstructed images, and it can be seen the boundaries of the head, shoulder, coat, camera, etc., are more obvious in Fig. 3(a) when a range of percentage is used than in Fig. 3(b) that uses a constant percentage. Figure 4 shows the edges improvement in the reconstructed image of the proposed method. Figures 5, 6, and 7 show the original, decoded, edges respectively of the car image.

Figures 8 and 9 show the improvement in PSNR and the average approximated rank of the proposed method using camera_man image results. The improvement in PSNR is obtained by improving the preservation of edges of the image, which represent very important parts for the human acceptance of the reconstructed image.

Fig. 2. Source camera_man image 256*256.

(a) A range of percentage from 5 to 30. (b) a constant percentage 30.

Fig. 3. Decoded camera_man images.

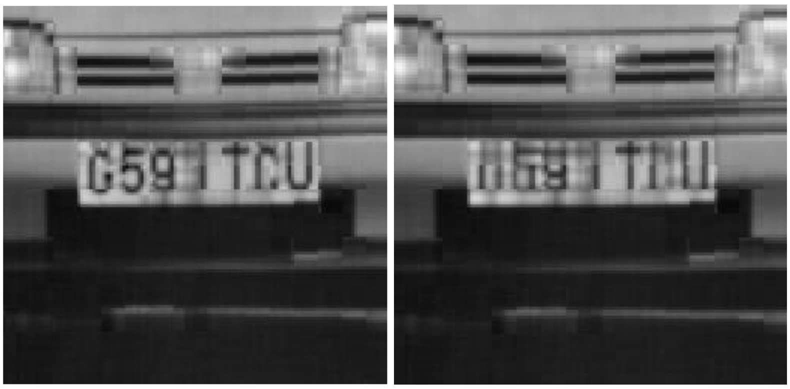
(a) A range of percentage from 5 to 30. **(b) A constant percentage 30.**

Fig. 4. Decoded camera_man images, showing the improvement in edges.

Fig. 5. Source car image 256*256.

(a) A range of percentage from 5 to 30. **(b) A constant percentage 30.**

Fig. 6. Decoded car images.



(a) A range of percentage from 5 to 3. (b) A constant percentage 30.

Fig. 7. Decoded car images, showing the improvement in edges.

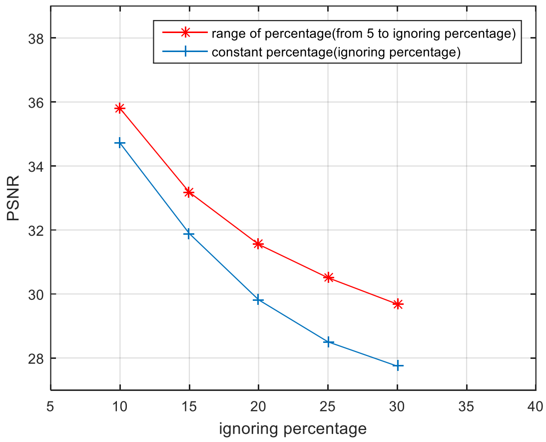


Fig. 8. A comparison between a constant percentage of the ignored singular values and a range of percentage from 5 to ignoring percentage for the camera_man image.

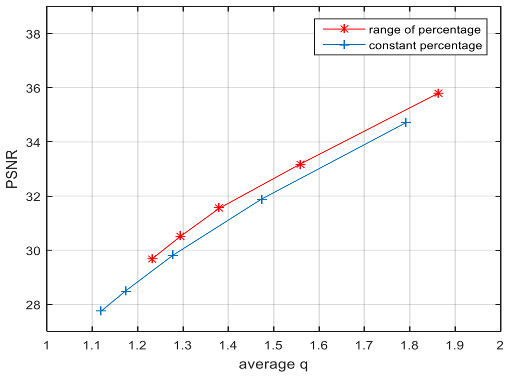


Fig. 9. A comparison between a constant percentage of the ignored singular values and a range of percentage with respect to PSNR and average approximated rank (*q*) for the camera_man image.

5. Conclusions

This paper proposes a modified method for choosing the singular values which are kept with the associated singular vectors in image compression using SVD. This method is concerned with preserving the edges in the image which represent a very noticeable part for human eyes. This is achieved by allowing a range of percentage, of the sum of the ignored singular values, instead of a constant percentage. The variance is used as a measure of block edginess and allowing the block with the maximum variance gets the lowest percentage in the specified range. For the car image the value of PSNR is 32.028 and the compression ratio is 3.405 when the proposed method is used with a range of percentage of the ignored singular values is 5 to 30 while the PSNR is 30.5994 and compression ratio is 3.510 for a constant percentage of 30. The obtained results show that the proposed method gave better results for the quality of the reconstructed image, especially the edges with a better compression result than the constant percentage value of the ignored singular values.

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Abbreviations

DCT	Discrete Cosine Transform
DWT	Discrete Wavelet Transform
MSE	Mean Square Error
PSNR	Peak signal to Noise Ratio
SVD	Singular value decomposition

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