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INVESTIGATION OF CONCENTRATION INFLUENCE ON ELECTRONIC COEFFICIENTS OF HE:NE PLASMA BY PREDICTING A MATHEMATICAL MODEL

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Abstract

In this work a comprehensive investigation of specific electron transport coefficients in plasma state He-Ne gas mixtures has been carried out. Theoretical calculations and estimated data are presented that enable us to measure the influence of He:Ne concentration on plasma electronic coefficients based on the variation in a plasma field resistance situated in a varied electric field and under a thermodynamic equilibrium. The Boltzmann equation was used to calculate several concentrations of energy mobility and momentum frequency and varied electric fields. Utilisation of the BOLSIG+ simulation verified the results that the Boltzmann distribution analysis revealed. By using a simulation process, appropriated equations which indicate the variation of plasma electronic coefficients according to the variation of mixture concentration and reduced electric field (*E*/*N)* have been obtained. The applied reduced electric field has been chosen to be in the limited range of (0-100) *Td*, and for several concentrations in the limited range of (0.1-0.7) mol. The between unique information (utilizing BOLSIG+) and our estimated data. The results show a stark resemblance involving original data (using BOLSIG+) and our estimated data, our simulation data, where root mean square of ($\nu_{e\text{-}i}$ /*N*,*)* = 4.2×10^{-7}, root mean square of $\mu_{\varepsilon}N$ = 1.5×10^{-6} and root mean square of $<\varepsilon>$ = 9.4×10^{-5}. This improvement in electronic coefficients assume a significant part in the development of energy for He:Ne laser

Keywords: Elastic and inelastic collisions, Electron-ion momentum frequency, Energy mobility, Mean energy, Plasma electronic coefficients.

1. Introduction

Plasma, which represents one of the four essential states of matter had been firstly defined in the 1920s by Irving Langmuir as a 'mouldable substance' [1, 2]. It is primarily comprised of a gas of atoms and molecules bearing a small number of electrons within orbitals displaced, and free electrons [3]. Plasma is a state of matter wherein an electrical condensation conducted upon ionized gaseous to the point that long fields command the conduct of the matter by a range electric and attractive [4, 5].

Plasma medium possesses unbound moving negatively and positively charged particles, therefore it is an electrically neutralised buffer with a net charge of about nothing. Albeit these particles are unrestrained, they still have the capability of experiencing forces to the sense that they are not "free." An electrical current within a magnetic field is generated by the movement of a charged also, any development of a plasma molecule which is charged affects and is also directly affected by the created by the other present charges. In turn, this results in varying degrees of control collective behaviour [6, 7].

Plasma's properties are remarkably dependent upon its particle interactions and electron transport coefficients. Collective effects help to identify plasma's behaviour as being entirely unique towards that of liquids. Every charged molecule of plasma influence instantaneously alongside a substantial amount of related charged particles because of the vast scope of present electromagnetic powers. The resulting effects can be represented by elastic and inelastic collisions provides a rich variation of physical phenomena that occur in plasma [8, 9].

There has been widespread interest in plasma sciences over the past two decades and several breakthroughs regarding the field has been made. Developing advanced new sources of plasma was the core of this progress of new. This progress is generally the result of advancement of current plasma sources in light of plasma creation in electrical releases in vacuum, magnetrons and in gases with high or low pressures and is in many ways a part of the progress made. The plasma technique resulting from this processing is used in aerospace movement and largely within fields of nanoscience revolving around plasma and on particular in areas where meticulous control is necessitated. These applications involve, but are not limited to Plasma Chemistry, lighting systems revolving around Plasma, Plasma Spray and lighting systems [10-12].

The Boltzmann equation is one of the most powerful tools for investigating the plasma state, from the electron kinetics in weakly ionized gases [10] to fusion [13, 14] and astrophysical plasmas. Boltzmann equation introduced into physics the idea of probability, which was then used some years later in quantum physics [15]. The development of models and simulation techniques for electrical discharges has been going for more than five decades. In present paper, theoretical calculations for several electronic coefficients in a mixture of He:Ne gas (using several gas mixture concentrations) are presented by creating mathematical models utilizing Boltzmann distribution function and appropriated simulation process.

In present paper, theoretical investigation of electron transport coefficients in plasma state He-Ne gas mixtures has been introduced using BOLSIG+ simulation method as a Boltzmann equation solver often employed in plasma modelling community. Origin program has been utilized in a simulation process resulting appropriated equations for theoretical calculations of electronic coefficients. Each

of mean energy $<\varepsilon>$, energy mobility μ and momentum frequency v at several concentrations of He-Ne gas mixture and under varied reduced electric field has been theoretically calculated.

2. The Boltzmann Equation

The Boltzmann condition portrays the factual way of behaving of a nonequilibrium thermodynamic framework designed in 1872 by Ludwig Boltzmann. What defines the Boltzmann equation in modern day and particularly its uses in a wider sense are any alteration of a perceptible amount within a system of thermodynamics for example, particle number, charge, energy. might be utilized to decide how actual amounts change when a fluid is in transport, such as heat energy and momentum [16, 17].

Certain behaviours trademark of liquids like consistency, heat-based conductivity, and electric-powered conductivity can also be extracted (through influencing the bearers of charges within a liquid like a gas). The mathematical unknown of the equation is a function of likelihood amount within the space revolving around particle position and momentum. This means that the equation becomes a nonlinear integral differential equation [18].

Boltzmann equation gives the ratio of number per unit volume (the numeral mass) of molecules, ions or atoms, *denoted by* N_2, in a certain level of energy comparable to the numeral mass within another lower energy level which shows as N_1, and it is given by [19]:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{E}{KT}} \tag{1}$$

where, g_1 and g_2 represent the multiplicity of the two energy levels, in another ward the degeneracies of the energy levels with the same energy, E is the required energy to excite particles, K represents Boltzmann constant and T represents thermodynamic temperature, so as T increases a greater number of particles will be excited [20].

Boltzmann kinetic equation

The kinetic Boltzmann equation highlights the distribution function of gas molecules f (v, r, t) in this case v becomes the velocity and r is coordinates (as mathematic equations of time shown as t) which showcases non-equilibrium operations within low density. The mathematical process of f helps to showcase the mean molecules with speed between the scope of v to v, and include the coordinate between the scope of r to r. This Boltzmann (kinetic) equation has the form only when the distribution function x and the velocity component of v. speeds inside a little reach from v *to* $v + \Delta v$ and facilitates inside a little reach from one r *to* $r + \Delta r$. In the event that the dispersion work relies just upon the direction x and the speed part v_x, then the Boltzmann (active) condition has the structure [21]:

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v_x} = \left(\frac{\partial f}{\partial t}\right)_{collission} \tag{2}$$

where m is the mass of the molecule. The pace of progress of the appropriation work is addressed by the fractional subordinate $\partial f/\partial t$. The second term in the situation assesses the adjustment of f because of the development of particles in space. Third term in this situation decides the adjustment of dissemination work that is alluding to impact of an outer power F.

The right term represents changes in the function of distribution which is referring to particle collision. This term is reliant on each of *f* as well as the idea of interaction powers among particles and this is show by [21]:

$$\frac{\partial f}{\partial t})_{collission} = (f' f_1' - f f_1) \left| v_1 - v_2 \right| d\sigma \, dv_1 \quad (3)$$

where, f, f_1, and f', f_1' represent functions of particle distribution prior to and following collision; v_1, v_2 represent the velocities of the particles before collision; and $d\sigma = \sigma d\Omega$ is the differential powerful dissipating cross area into the strong point $d\Omega$ (depending on coordinate system of laboratory) and it depends on the law of molecular interaction [22].

The kinetic or transport equations are generalizations of the Boltzmann equation and showcase electron gas behaviors in crystal lattice phonons and in metals [23, 24].

3. Mean Energy

3.1. Influence of reduced electric field and concentration on mean energy

The concentration affects obviously on discharge process can be noticed in Fig. 1. and it can be also noticed the dependence of mean energy $<\varepsilon>$ on reduced electric field (E/N), where the increasing of mean energy according to the increasing of (E/N) is observed also note that the mean energy values increase when moving from the low concentration (0.1) mole to the high concentration (0.9) mole with respect to the confined area from (0-68) *Td* as in Table 1, while the behavior becomes reversed in relation to the confined region between (69 - 100) *Td* as in Table 2. This is because the cross-section area is directly influenced by the reduced electric area - this impact is high form low to high concentrations (0,1 to 0,7) mole for when E over N is (0 - 68) *Td*. This behavior becomes opposite in the case of E over N= (69 - 100)*Td*, wherein all of the mean energy is highly sensitive to the concentration of the gas mixture, and as a result it will increase the rate of elastic-inelastic collisions.

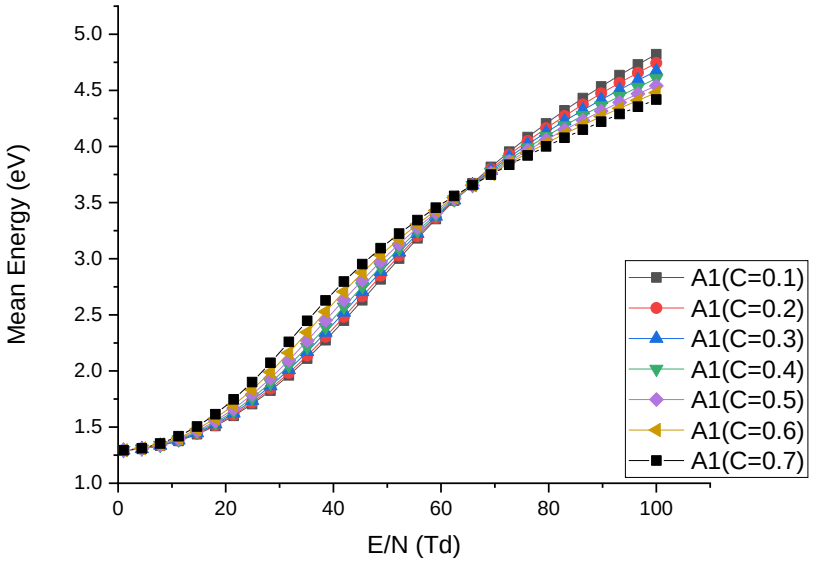

Fig. 1. Mean Energy as a function of reduced electric field (*E*/*N*).

Table 1. The correlation between mean energy and *Ne* concentration (*E*/*N*= (0-68) *Td*).

E/*N*=45.38 *Td*	*E*/*N*= (0-68) *Td*						
Concentration (Mol.)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Mean Energy (*eV*)	2.63	2.665	2.71	2.75	2.81	2.87	2.95

Table 2. The relationship between mean energy and concentration of *Ne*, *E*/*N*= (69-100) *Td*.

E/*N*=100 *Td*	*E*/*N*= (69-100) *Td*						
Concentration (Mol.)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Mean Energy (*eV*)	4.88	2.665	2.71	2.75	2.81	2.87	2.95

3.2. Mean energy modelling

For all concentrations, the fitting relationship could be obtained as shown in Fig. 1. This relationship addresses a matching between the BOLSIG+ program values and present model (Fig. 2) that based of utilizing strategic capacity and it can be represented by Eq. (4):

$$< \varepsilon >= A_1 + \frac{(A_1 - A_2)}{\left[1+\left\{\frac{\left(\frac{E}{N}\right)}{x_o}\right\}\right]^p} \quad (4)$$

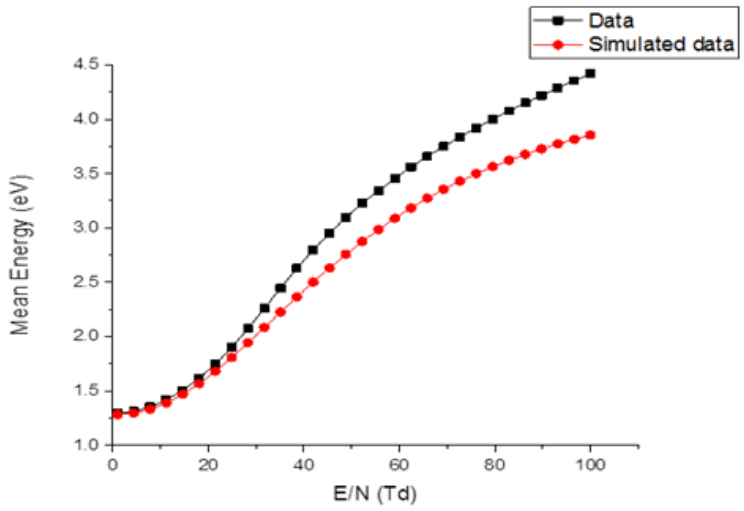

Fig. 2. Represents percent and BOLSIG+ data (using mathematical model) of mean energy.

From Eq. (4) for all concentrations, parameters (A_1, A_2, X_0, P) can be shown through calculation of function of logistic and polynomial to determine the required concentration calculated in Eq. (4). The equations of these parameters:

$$A_1 = 1.3148 - 0.02493C - 0.03214 \quad (5)$$

$$A_2 = 6.31` - 2.141C - 0.663C^2 \quad (6)$$

$$X_o = 68.848 - 19.24C - 5.973\,C^2 \quad (7)$$

$$P = 2.482 - 0.075C - 0.0635\,C^2 \quad (8)$$

where C is concentration.

4. Energy Mobility

4.1. Influence of reduced electric field and concentration on energy mobility

The relationship between energy mobility $\mu\varepsilon N$ and E/N is shown in Fig. 3., where sharp decreasing in energy mobility can be observed particularly in the limited region between E/N= (0-50) *Td*. This decreasing is referred to energy losing by ionization and excitation processes inside He:Ne mixture which is highly influenced by reduced electric field. While in the limited region between E/N= (50-100) *Td* energy mobility is almost steady.

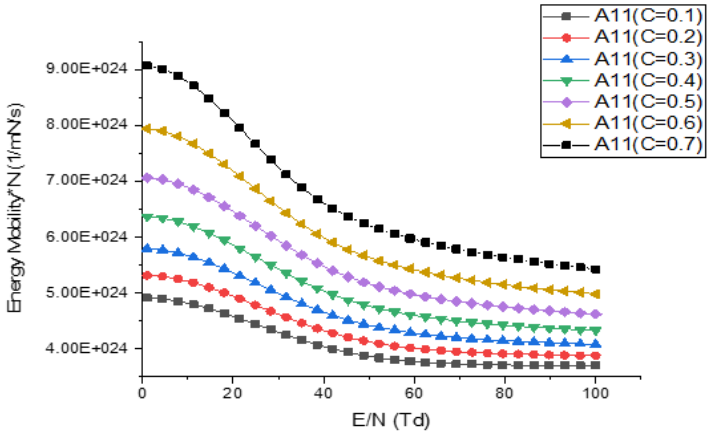

Fig. 3. Energy mobility as a capacity of decreased electric field (E/N).

Dependences of energy mobility on gas mixture concentration can be observed, where increasing of mobility according to the increasing of E/N can be noticed as in Table 3, the reason for this, is the increase electron drift by increasing the concentration with increasing the reduced electric field [10].

Table 3. The correlation between energy mobility *N and mass of *Ne*.

	E/N =65.86*Td*						
Concentration (Mol.)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Energy mobility *N×10^{24} ($1/m/V/s$)	3.74	3.97	4.23	4.53	4.89	5.32	5.84

4.2. Energy mobility modeling

The following Eq. (9) illustrates the behavior of energy mobility according to the change of *E*/*N* based of using logistic function:

$$N = B_2 + \frac{(B_1 - B_2)}{\left[1+\left\{\left(\frac{\left(\frac{E}{N}\right)}{r_o}\right)\right\}\right]^r} \quad (9)$$

where B_1, B_2, r_0, and r are parameters, C is gas mixture concentration. This relationship addresses a close link between the BOLSIG+ programming numbers and percent model Fig. 4.

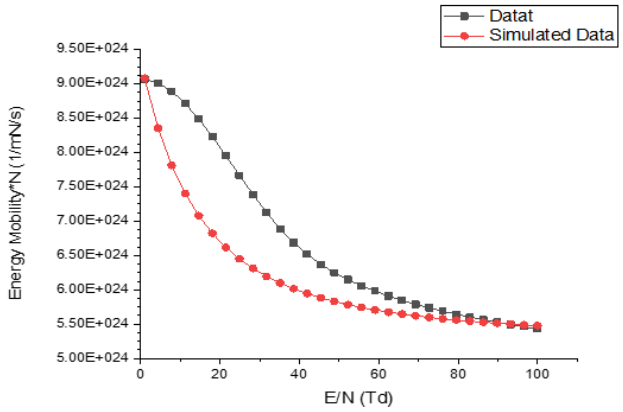

Fig. 4. Estimated/simulated data (using mathematical model) of Energy mobility and calculated data.

From Eq. (9) for all concentrations, parameters (B_1, B_2, r_0, r) can be shown mathematically by function of numerical and mathematical to determine the required concentration calculated in Eq. (9). The equations of these parameters are:

$$B_1 = (4.65 \times 10^{24} \pm 4.8 \times 10^{22}) + (1.97 \times 10^{24} \pm 3.2 \times 10^{23})C + (5.8 \times 10^{24} \pm 4.4 \times 10^{24})\,C^2 \quad (10)$$

$$B_2 = (3.55 \times 10^{24} \pm 1.76 \times 10^{22}) + (8.02 \times 10^{24} \pm 1.12 \times 10^{23})C + (2.078 \times 10^{24} \pm 1.61 \times 10^{23})C^2 \quad (11)$$

$$r_o = 28.75 + 18.982\,C - 19\,C^2 \quad (12)$$

$$r = 3.0832 - 2.275\,C - 1.7\,C^2 \quad (13)$$

where C concentration.

5. *e-i* Momentum Frequency

5.1. Influence of reduced electric field and concentration on *e-i* momentum frequency

The increasing of *E*/*N* produce sharp decreasing of ($\nu_{e\text{-}i}$) particularly in the limited region between (0-63) *Td* as shown in Fig. 5. It is clearly approved that the electron-ion momentum frequency ($\nu_{e\text{-}i}$) values increase when changing position from the greater concentration (0.1) mole to the reduced concentration (0, 9) mole with

respect to the confined area from (0-63) *Td* as in Table 4, while the behavior becomes reversed in relation to the confined region between (63 - 100) *Td* as in Table 5. This is because the electron-ion momentum frequency has an inverse proportion with temperature ($\nu_{e\text{-}i} = 2.9 \times 10^{-6}\, n_{cm} ln\Lambda / T^{3/2} ev$).

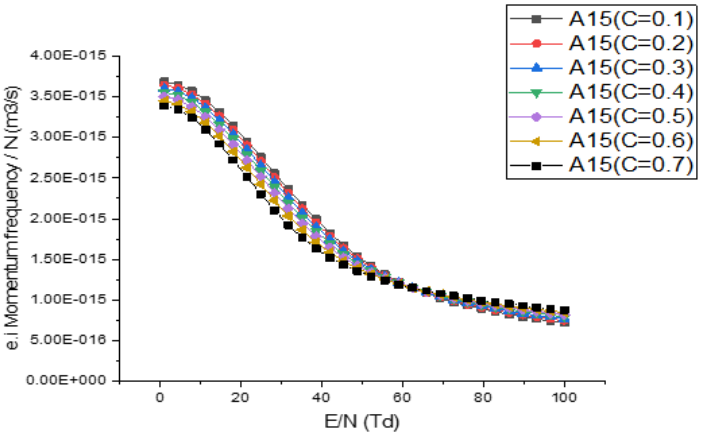

Fig.5. *e-i* momentum frequency /N as part of an equation of reduced electric field.

Table 4. The relationship between *e-i* momentum frequency /*N* and concentration of Ne *E*/*N*= (0-63) *Td*.

E*/*N*= 48.79 *Td	***E*/*N*= (0-63) *Td***						
Concentration (Mol.)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
e-i momentum frequency /*N*×10^{-15} (m^3/s)	1.53	1.51	1.48	1.45	1.42	1.39	1.35

Table 5. The relationship between *e-i* momentum frequency /*N* and concentration of Ne *E*/*N*= (65-100) *Td*.

E*/*N*=100*Td	***E*/*N*= (65-100) *Td***						
Concentration (Mol.)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
e-i momentum frequency /*N*×10^{-16} (m^3/s)	7.2	7.45	7.7	7.95	8.2	8.47	8.75

5.2. *e-i* momentum frequency /N modeling

The following Eq. (14) illustrates the behavior of *e-i* momentum frequency ν/N according to the change of *E*/*N* based of using logistic function:

$$\nu_{ei}/N = G_2 + (G_1 - G_2) / [1 + \left\{\frac{\left(\frac{E}{N}\right)}{g_0}\right\}]^g \quad (14)$$

where, G_2, G_2, g_0, g are parameters, C is gas mixture concentration.

This relationship showcases a correlation between the BOLSIG+ values and percent model (Fig. 6.):

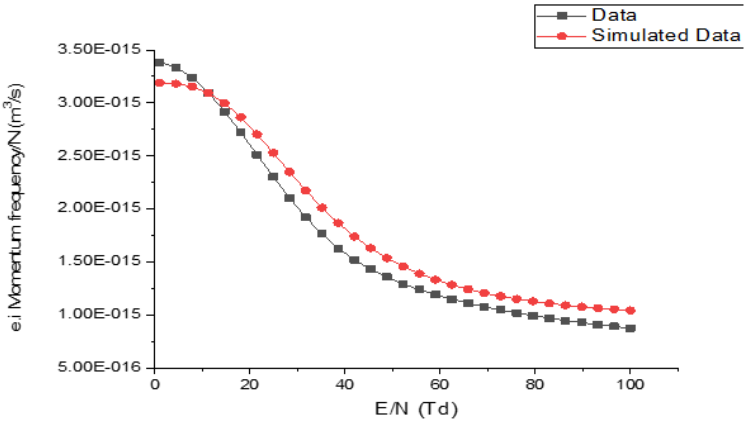

Fig. 6. Assessed/recreated information (utilizing numerical model) of *e-i* Momentum Prevalence and calculated data.

From Eq. (14) for all concentrations, parameters (G_2, G_2, g_0, g) can be shown mathematically by function of numerical and mathematical to determine the required concentration calculated in Eq. (14). The equations of these parameters are:

$$G1 = (3.7 \times 10^{-15} \pm 2.3 \times 10^{-18}) + (-3.3 \times 10^{-16} \pm 1.5 \times 10^{-17})C + (-1.84 \times 10^{-16} \pm 2.1 \times 10^{-17})C^2 \quad (15)$$

$$G2 = (3.65 \times 10^{-16} \pm 1.34 \times 10^{-18}) + (5.352 \times 10^{-16} \pm 8.782 \times 10^{-18})C + (-1.93 \times 10^{-17} \pm 1.22 \times 10^{-17})C^2 \quad (16)$$

$$g_0 = (-2.18 \times 10^{-15} \pm 7.91 \times 10^{-16}) + (10 \pm 5.17 \times 10^{-15})C + (-7.6 \times 10^{-15} \pm 7.24 \times 10^{-15})C^2 \quad (17)$$

$$g = (3.69 \times 10^{-15} \pm 2.26 \times 10^{-18}) + (-3.28 \times 10^{-16} \pm 1.48 \times 10^{-17})C + (-1.83 \times 10^{-16} \pm 2.07 \times 10^{-17})C^2 \quad (18)$$

Figure 6 represents the high matching between BOLSIG$^+$ data and our estimated data, where root mean square is in the range of (4.2×10^{-7}). Also, Eqs. (15-18) are used to calculate concentration in Eq. (14).

6. Conclusions

A comprehensive investigation of specific electron transport coefficients in plasma state He-Ne gas mixtures has been carried out using the solution of Boltzmann equation, adopting BOLSIG$^+$ program. Through this investigation the influence of mixture concentration on the electronic coefficients ($\nu_{e\text{-}i}/N$, $\mu_\varepsilon N$, and $<\varepsilon>$) can be observed. It can be also observed that by changing the reduced electric field, each of versatile inelastic crashes and cross-area of collisions has a significant role in discharge process.

In addition to, this investigation introduced appropriated equations of electronic coefficients of He:Ne plasma field utilizing a simulation process for each of these

coefficients. Also simplified equations for concentration calculation have been introduced. In general, the obtained equations through our simulation process are shown high matching between BOLSIG$^+$ data and our simulation data, where root mean square of ($\nu_{e\text{-}i}$ /N,) = 4.2×10^{-7}, root mean square of $\mu_\varepsilon N = 1.5\times10^{-6}$ and root mean square of $<\varepsilon> = 9.4\times10^{-5}$.

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Nomenclatures

E/N	Reduce electric field, 10^{-21} Vm^2
F	External force, N
$f, \acute{f}, f_1, f_1^{\acute{}}$	Distribution function before and after collision, (e.V)$^{-3/2}$
$g_1\ g_2$	Multiplicity of two energy level
K	Boltzmann constant, J/K
m	Particle mass, kg
N_e	Density of charge, m^{-3}
$N_1\ N_2$	Number of densities, m^{-3}
T	Thermodynamic temperature, K
v	Velocity of electron, m/s^2

Greek Symbols

$d\sigma$	Differential effective scattering
$d\Omega$	Solid angle
$\partial f/\partial t$	Rate of the change of the distribution rate of the change of the distribution function
$<\varepsilon>$	Mean energy, e.V
μ	Mobility, m^2 /vs.
ν/N	Momentum frequency, m^3/s

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