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MATHEMATICAL MODELS FOR TEMPERATURE DISTRIBUTION IN OIL WELLS

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Abstract

The oil temperature at the surface of the oil well is low compared to the oil temperature at the reservoir. The drop of temperature is due to heat transfer to the tubing wall. The temperature distribution of the oil from the reservoir to the surface and surrounding layers is studied in this paper. Two mathematical models for production and shut-in cases are developed to study the parameters affecting the temperature profile in oil wells. The natural flow of oil wells in Alwahat area located 70 Kilometres south of Marada area east of Libya in the Zaggut field called (6Q1-59) is taken as a study case. In production case, different mass flow rates in winter and summer seasons are studied. The temperature profile in the horizontal direction is estimated at different depths. In shut-in case, steady-state energy balance equation is solved using finite difference method, the results are compared with the actual date taken from the Waha company reports.

Keywords: Heat transfer, Mathematical model, Oil well, Reservoir, Temperature profile.

1. Introduction

Crude oil is a complex combination of hydrocarbons consisting predominantly of paraffinic, naphthenic, and aromatic hydrocarbons. To extract the oil from the earth, there are two ways, the natural flow, and the pumping operation. The pumping method is used to withdraw oil from the reservoir to the surface of the earth. This method is used when the difference in the pressure is not enough to make oil rises up because the gas melt proportion in the oil is not enough to make it flows naturally, and some of wells have high proportion of the wax mixed with oil. The pump design depends on the depth of the well and the oil specifications. When a fluid flows in a tubing, the properties of the fluid is affected by the rate of flow, tubing diameter, tubing roughness, and the properties of the surrounding formation. During production process, heat is usually conducted throughout tubing, to annulus fluid, casing, cement sheaths, and surrounding formation. Each of these media have different thermal properties, which make the process of heat transmission prediction more complex. There are two important contributory factors of heat transfer of the fluid, firstly, transfer of energy up inside the well is accomplished by the fluid flow, which it depends on fluid properties. Secondly, radial heat conduction from the well passing through the tubing cement sheaths, and throughout surrounding formation.

Mosavi et al. [1] have developed a mathematical model to calculate the temperature distribution under the laser hardening process on the surface and bulk of a steel plate. The model started from the basic heat equation, then was developed into a volumetric form and was related to the existing multiple solid phases. The model is based on two parameters of the laser hardening process that have a strong influence: laser spot velocity and irradiation time. The results were compared to the data from the experiments. An assessment of the temperature distribution on both the vertical and horizontal axes was provided by the volumetric model. Calculations have shown that the temperature profile has a horizontal x-and y-axis Gaussian distribution and presents an exponentially decreasing horizontal and vertical depth distribution.

Thomas [2] proposed an approximate approach for figuring out the temperature profile whilst injecting fluid is hot. This approximation is relevant to distinctly thin reservoir beds in which the injected fluid goes with the flow is high. Mathematical prediction models of temperature distribution through porous media in thermal flood process are the key factors in process design and production management [3]. Several models are proposed for temperature distribution in various geothermal and natural reservoirs [4] and well test analysis of thermal injection model [5]. Lawal and Vesovic [6] developed a model of one-dimensional heat transfer to predict temperature distribution for different conditions. Lawal [7] modified this model for a process of steam flooding. The author concluded that at any moment of the thermal flood a maximum of four zones (i.e., conduction, convection, and condensation) can be identified.

Ramey [8] proposed an approximate solution to the problem of transient heat conduction that involved moving hot fluids through a well bore. The heat transfer in the well bore was believed to be steady-state, and the heat transfer to earth was unstable radial convection and called the effect of thermal resistance in the well bore. The solution allowed the temperature estimation of the fluids, tubing and casing as a function of depth and time. Alves et al. [9] provided a general, unified equation for prediction of flowing temperatures. For compositional and black-oil fluid models, it has applied to pipelines and wells for production and injection, under single- and two-phase flows, over the entire angle of inclination from horizontal to vertical. Farshad et al. [10] proposed a novel approach to the use of artificial neural networks (ANNs) for temperature profiles prediction. Two artificial neural network models have been developed which predict the temperature of the flowing fluid in flowing oil wells at any depth. Back propagation was used in networking preparation. The networks were tested using temperature profiles obtained from 17 wells in the Gulf Coast region. The neural network models successfully mapped the general temperature-profile trends of naturally flowing oil wells.

Gomes et al. [11] developed mathematical models to predict the temperature profile in the steel pipe, to investigate the causes of defect formation and to allow a better control of the cooling conditions. Two-dimensional (2D) and tridimensional (3D) models were developed to simulate the heat transfer during the cooling of the pipe. The models were tested and validated using experimental temperature data collected in plant trials to validate the three-dimensional model. In order to compare the 2D and 3D methods, multiple simulations were performed, and the effect of the pipe's angular speed was investigated. The findings showed that there were different temperatures calculated using the 2D and 3D models. Liu et al. [12] developed a mathematical model for constant temperature electrical heating of dual-horizontal-well steam-assisted gravity drainage (SAGD). A finite element simulation based on a standard SAGD block was performed to validate the model, which showed an excellent agreement between the model solution and the effects of the simulation. During the multi-well process, underground temperature distribution and energy-related parameters were collected. The proposed model could be used to predict key indexes such as the distribution of underground temperature, parameters of energy consumption, and accumulated quantities of energy, water and fuel conserved versus steam heating.

Samsuri et al. [13] studied the variation of original oil due to porosity estimation technique. It concluded that original oil in place estimation is significantly influenced by technique used for porosity determination in an interbedded reservoir as compared with a clean sand reservoir. Valberg [14] proposed three methods for measuring the temperature of the well. The methods were; an analysis of a stable state equation, a stand-alone Fortran program and a simulation and modelling application of a commercial stable state operation. The project's goal was to apply these three entirely different methods for calculating temperature on two case studies, and to compare their efficiency. The case studies included water injection into an onshore pipe, and gas extraction from one of the Ormen Lange gas fields.

In the available temperature profile mathematical model studies, models are developed for production case in general. Hence, in the current study, two mathematical models are developed to predict temperature profile along the wellbore from the reservoir to the surface. Temperature distribution in the wall tubing of the oil well and casing, cement sheaths, and surrounding formation are studied. The thermal resistance has been calculated based on the different layers around a wellbore.

2. Mathematical Model

The temperature of the surrounding formation (T_∞) is computed at different depths (z) from the following equation [15].

$$T_\infty(z) = \left(\frac{T_{bh} - Tsur}{Total\ depth}\right) \times Depth + T_{sur}(z) \tag{1}$$

The thermal resistance (*Rth*) is calculated using the following equation for an oil well section as shown in Fig. 1.

$$Rth_{total} = \sum_1^n \left(\frac{1}{h.(\Pi.d.\Delta z)}\right)_1 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k\Delta z}\right)_2 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k\Delta z}\right)_3 + \cdots + \left(\frac{ln(\frac{ro}{ri})}{2\pi k\Delta z}\right)_n \tag{2}$$

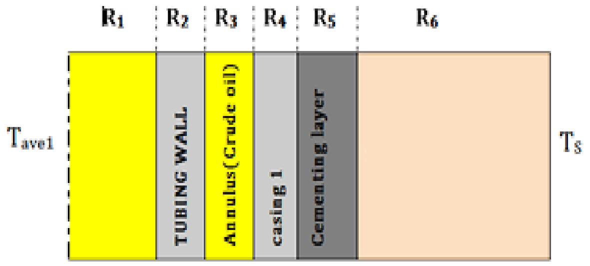

Fig. 1. Thermal resistance in vertical section of oil well.

The following assumption are made in order to simplify the derivations of the mathematical model; (1) the flow is assumed to be fully developed incompressible single phase, (2) the fluid properties are assumed to be constant with temperature variation and (3) the heat transfer by conduction in vertical direction (z) is negligible.

2.1. Production case

The conservation of energy equation for the steady flow of a fluid in a tubing [16] as shown in Fig. 2 can be expressed as:

$$\dot{Q} = \dot{m} * Cp * (T_{m.o} - T_{m,i}) \tag{3}$$

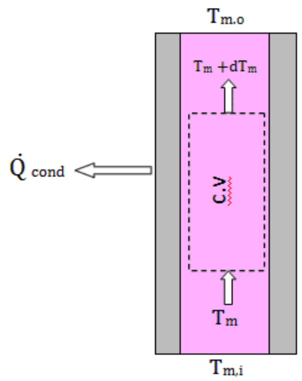

Fig. 2. control volume for internal flow in oil well.

Appling this equation to the differential control volume (c.v), we obtained:

$$d\dot{Q} = \dot{m}\ Cp\ dT_m \tag{4}$$

$$d\dot{Q}_{cond} = \frac{(T_S - T_m)}{Rth_{total}} \tag{5}$$

$$\frac{(T_S - T_m)}{Rth_{total}} = \dot{m}\ Cp\ dT_m \tag{6}$$

$$Rth_{total} = \frac{1}{\Delta z}\sum\left(\frac{1}{h.(\Pi.d)}\right)_1 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_2 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_3 + \cdots + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_n \quad (7)$$

$$Rth_{total} = \frac{R_i}{\Delta z} \quad (8)$$

where $Ri = \sum\left(\frac{1}{h.(\Pi.d)}\right)_1 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_2 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_3 + \cdots + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_n$ (9)

$$\dot{m}\, Cp\, dT_m = \frac{(T_s - T_m)}{R_i}\Delta z \quad (10)$$

$$\frac{dTm}{dz} = \frac{(T_s - T_m)}{\dot{m} * Cp * R_i} \quad (11)$$

Let $Fi = \frac{1}{\dot{m} * Cp * Ri}$ (12)

where (F_i) is constant in the same part but it varied from part to another.

$$\frac{dT_m}{dz} + F_i T_m = F_i T_s \quad (13)$$

Equation (13) is a first order non-homogeneous linear ordinary differential equation (*ODE*) which is in the form of the following Bernoulli differential equation [17].

$$\bar{y} + P(z)y = r(z) \quad (14)$$

$$y(z) = e^{-N}\int e^{N}.r(z)\, dz + c\, e^{-N} \quad (15)$$

where $N = \int P(z)dz$

Let $P = F$, $r(z) = F . Ts$, and $Ts = bz + a$

The general solution for first - order *ODE* Eq. (15) is given by the following equation:

$$Tm = bz - \frac{b}{F} + a + c\,.e^{-Fz} \quad (16)$$

Equation (16) is used in all parts from the bottom to the top to compute the mean temperature of the oil for every point at any part by using the thermal resistance.

2.2. The static (Shut - in) case

In this case, we will consider the oil as solid body inside the pipe, and it is formed only from the crude oil which is considered as steady state, homogeneous material with no heat generation. The energy balance equation as shown in Fig. 3 is [18]:

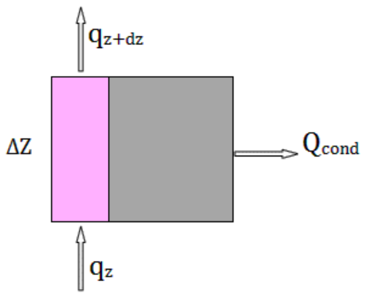

Fig. 3. Schematic of the heat-transfer in wellbore.

$$q_z - (q_{z+dz}) - q_{cond} = 0 \quad (17)$$

$$q_z - q_z - \frac{\partial q_z}{\partial z}\, dz - q_{cond} = 0 \tag{18}$$

$$-\frac{\partial}{\partial z}\left(-KA\frac{\partial T}{\partial z}\right)dz - q_{cond} = 0 \tag{19}$$

$$\left(\frac{d^2T}{d\,Z^2}\right)dz - \frac{q_{cond}}{kA} = 0 \tag{20}$$

Using Eq. (5) for q_{cond}

$$\left(\frac{d^2T}{d\,Z^2}\right)dz - \frac{(T_z - T_S)}{k.A\,.Rth_{tot}} = 0 \tag{21}$$

$$Rth_{tota} = \frac{1}{\Delta z}\sum\left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_1 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_2 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_3 + \cdots + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_n \tag{22}$$

$$Rth_{tot} = \frac{R}{dz} \tag{23}$$

where $R = \sum\left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_1 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_2 + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_3 + \cdots + \left(\frac{ln(\frac{ro}{ri})}{2\pi k}\right)_n$ (24)

Substituting for Rth_{tot} in Eq. (21) we get:

$$\left(\frac{d^2T}{d\,Z^2}\right) - \frac{(T_z - T_S)}{k.A\,.R} = 0 \tag{25}$$

By use the finite difference method to solve equation we get [19, 20]:

$$\left(\frac{d^2T}{d\,Z^2}\right) = \frac{(T_{z+1} - 2\,T_z + T_{z-1})}{\Delta z^2} \tag{26}$$

$$\left(\frac{(T_{z+1} - 2\,T_z + T_{z-1})}{\Delta z^2}\right) - \frac{(T_z - T_S)}{k.A\,.R} = 0 \tag{27}$$

$$(T_{z+1} - 2\,T_z + T_{z-1}) - \frac{\Delta z^2}{k.A\,.R}.(T_z - T_s) = 0 \tag{28}$$

let $\lambda = \frac{\Delta z^2}{k.A\,.R}$ (29)

$$(2+\lambda)\,T_z = T_{z+1} + T_{z-1} + \lambda T_s \tag{30}$$

Equation (30) is used in all parts from the bottom to the top to compute the mean temperature of the crude oil.

2.3. Temperature distribution in the layers

After the medium temperatures of the crude oil and the surrounding formation are computed for different depths, the rate of heat transfer is computed from the oil to the layers through the tubing wall and the casing by:

$$\dot{Q} = \frac{T_{ave} - T_S}{Rth_{Tot}} \tag{31}$$

$$\dot{Q}_i = \frac{T_{(i)} - T_{(i+1)}}{R_i} = \dot{Q} \tag{32}$$

R_i= thermal resistance of layer (i)

$$T_{(i+1)} = T_{(i)} - \dot{Q} * R_i \tag{33}$$

where i the layers number of i=1, 2, 3,..

3. Case Study

To calculate the temperature distribution in the wellbore at different depths, the wellbore is divided into four main parts. Every part is divided to different sections based on the oil well surrounding formation as shows in Fig. 4. The wellbore is divided into four main parts based on the number of layers surrounding the tubing. There are four casing; the casings begin around the production tubing from the surface to different depths. The depths of each casing are determined by the geological conditions, and safety requirements. All casings and the production tube are designed from carbon steel. Figure 5 shows the wellbore schematic and dimensions of the casing and tubing.

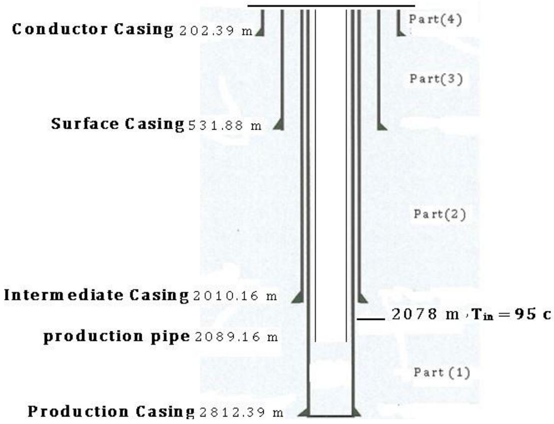

Fig. 4. Schematic well bore diagram (6Q1-59).

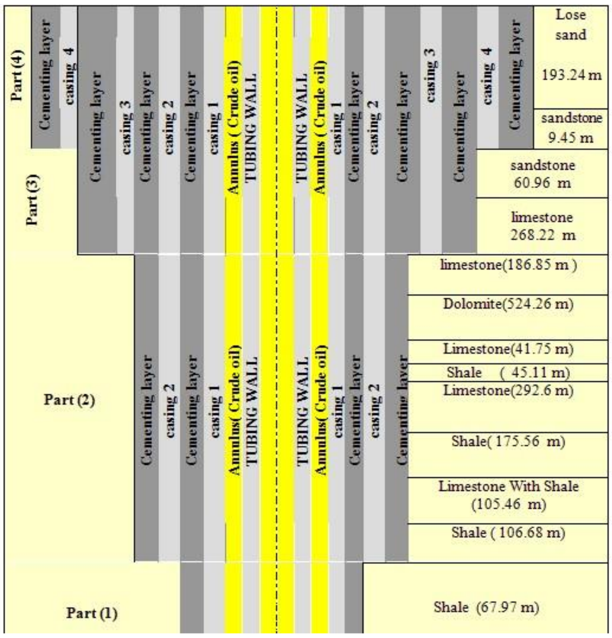

Fig. 5. Vertical section of the oil well and surrounding formation.

4. Results and Discussion

The mean temperature of the crude oil along the wellbore in winter and summer seasons are shown in Fig. 6. The temperature of the crude oil decreases as the oil rises from the reservoir to the surface. The temperature distributions are slightly higher in summer than the temperature distributions in winter.

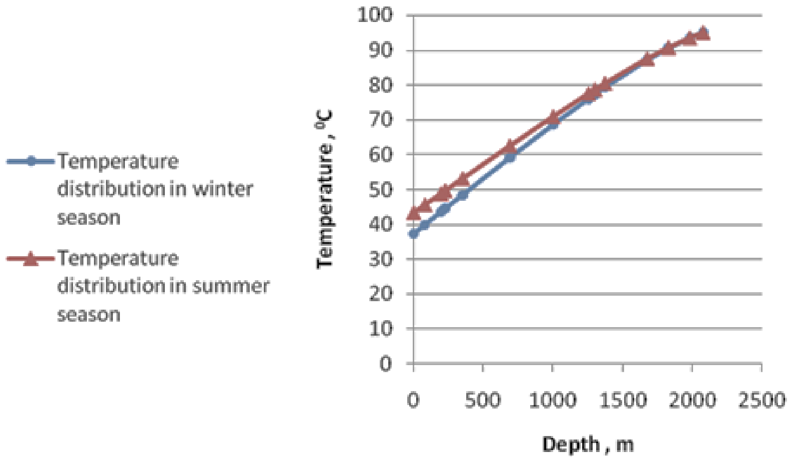

Fig. 6. Oil temperature distribution in winter and summer seasons at $\dot{m} = 1.36$ kg/s.

Vertical section and temperature distribution profile for oil well wall and the surrounding layers at the depth of (2076 m) are shown in Fig. 7. Figure 8 explains the temperature distribution through the third layer surrounding the production tube (annulus). Temperature drops from (T_2) to (T_3) as a result of high thermal resistance of this layer. After this layer, the temperature continues to drop but with small difference between every layer and another.

Figure 9 illustrates the temperature distribution in the well wall and the surrounding layers at different depths. The temperature profiles at all depths are similar and the temperature decreases as the distance from the centre of the oil well increases. The temperature drops sharply outside the well pipe. Temperature decreases as the depth from the surfaces decreases.

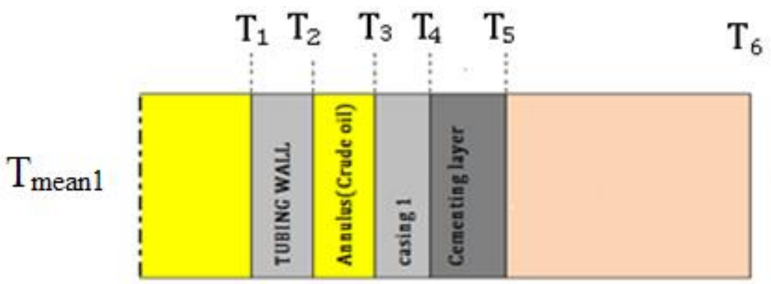

Fig. 7. Vertical section of the part (1).

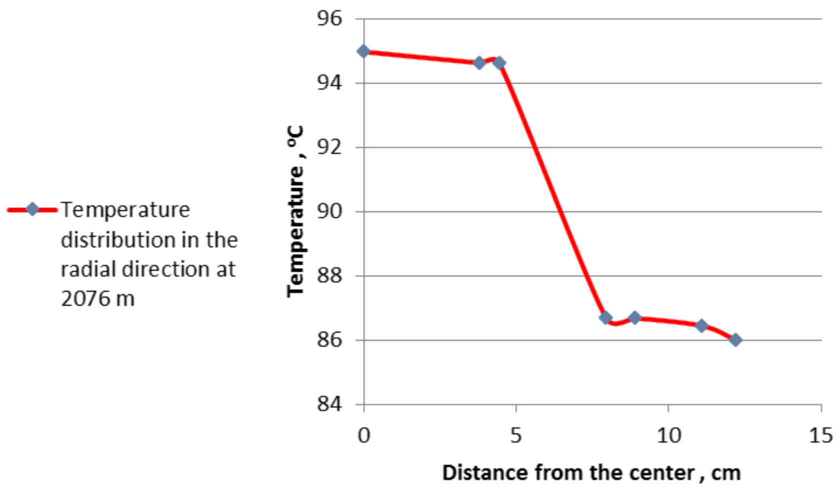

Fig. 8. The temperature distribution in the well wall and the surrounding at depth (2076 m)

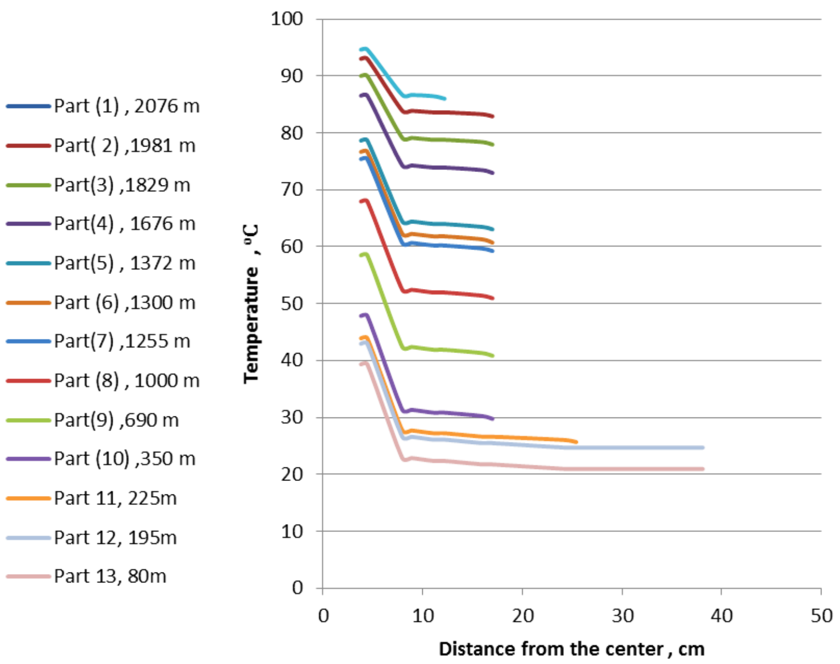

Fig. 9. The temperature distribution in the well wall and the surrounding at different depths.

Figure 10 shows comparison between temperature distribution profile Waha company data and calculation data in shut-in case. The temperature profile of the crude oil obtained from the present calculation seems to behave quite the same with the measured data from the company. Present calculated results are lower than the measured data and the difference is justified by the model assumptions.

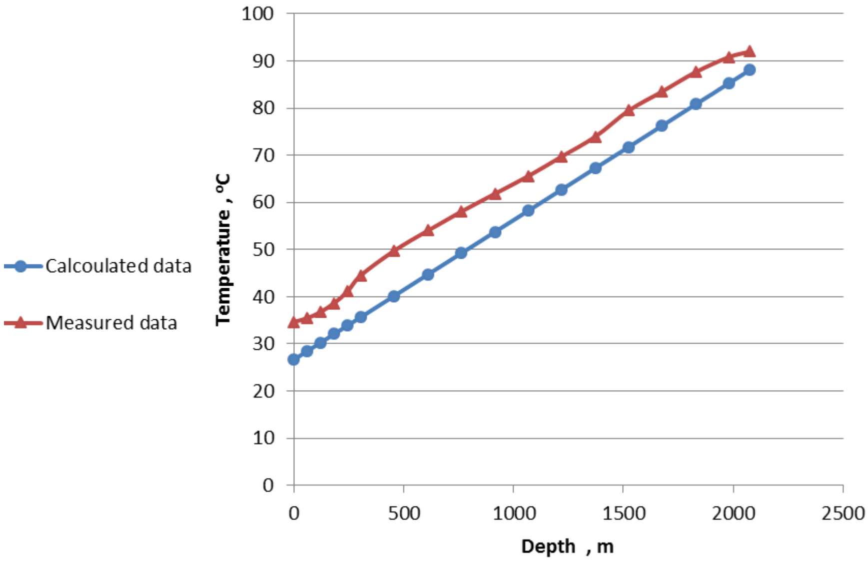

Fig. 10. Comparison between present calculated data and measured data (static case).

Figure 11 shows the temperature distribution at different mass flow rates in winter season. The figure illustrates that with increasing of mass flow rate, the crude oil temperature at the surface is increased. Temperature profile at the depth 1000 m with difference mass flow rates in summer season is taken as an example.

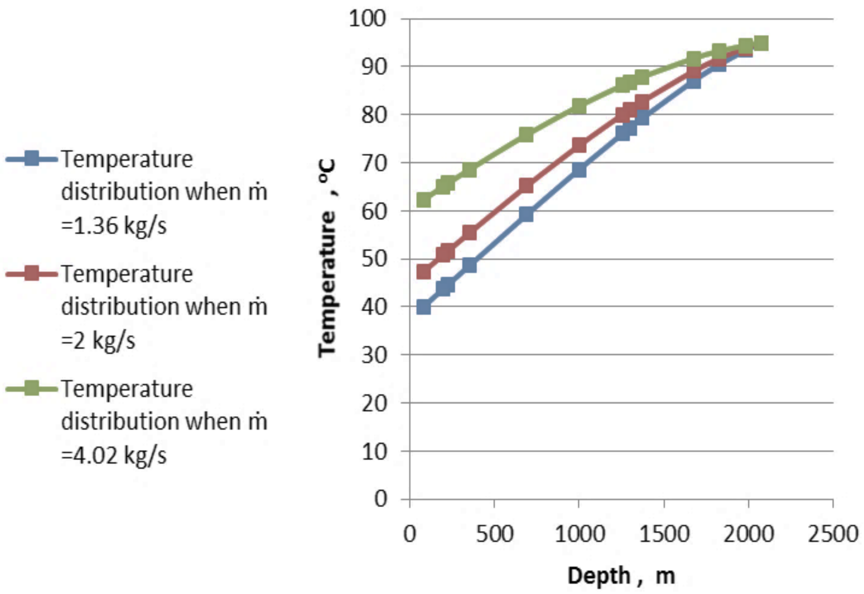

Fig. 11. Temperature distribution for different mass flow rates.

5. Conclusions

Two mathematical models have been developed and solved to predict temperature distribution at production and shut-in cases in oil wells, the temperature distributions along the tubing from the reservoir to the surface are calculated and plotted at different depths. As a conclusion, the following inference are drawn:

- The created models include the total thermal resistance and convection heat transferring mechanism were accounted. Hence temperature differences are not large, radiation heat transfer is not taken in a consideration.
- The developed mathematical models can solve the temperature distribution at production case in oil wells, as well as the temperature distributions along the tubing from the reservoir to the surface.
- The present results are found to be closed and slightly lower than the measured data.

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Nomenclatures

Cp	Specific heat, J /kg K
$\dot{Q}$	The heat transfer rate, J/m
Rth_{total}	Total thermal resistance
$\mathcal{R}i$	Inner radius, m
$\mathcal{R}o$	Outer radius, m
TG	Thermal gradient, °C
T_{ave}	The average temperature of the Crude oil at every part, °C
T_{bh}	The bottom hole temperature, °C
T_f	The formation temperature, °C
T_m	The mean temperature of the fluid, °C
T_s	The surface temperature, °C
T_{sur}	The surface temperature, °C
T_z	The oil temperature at depth (z) m
T_∞	Formation temperature, °C

Greek Symbols

Δ	Difference

Abbreviations

C.V	Control Volume
ODE	Ordinary Differential Equation

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