

INVESTIGATION OF THE DISCHARGE COEFFICIENT FOR ORIFICE AND PIPE BEHAVIOR

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Abstract

A circular orifice is a hydraulic device used to measure and control the outflow from tanks, reservoirs and channels. The main purpose of this paper is to study the behaviour of flow through a circular opening of a certain length L and a diameter d and its subordination to the behaviour a pipe or orifice. The discharge coefficient is the factor that will determine which of the two behaviours to be followed for given L and d . To achieve this purpose, laboratory experiments were carried out to investigate the discharge coefficient C_{do} and C_{dp} variations with the circular opening diameter d and length L for each orifice and pipe behaviours respectively and developing equations for estimating C_{do} and C_{dp} . Four diameter values of 12.7, 19.05, 25.4, and 38.1 mm and eleven lengths of 2, 4, 7.5, 9.25, 20, 50, 100, 200, 400, 600, 1,000 mm were employed to achieve this purpose. The results show that C_{do} decreases but C_{dp} increases as L increases. Further, the discharge coefficient for each orifice and pipe behaviour C_{do} and C_{dp} are proportional to its diameter d and inversely proportional to the water head h . By using Buckingham Pi theory general chart and equations were developed to estimate the discharge coefficient for each orifice and pipe behaviour C_{do} and C_{dp} .

Keywords: Actual discharge, Discharge coefficient, Flow behaviour, Orifice flow, Pipe flow, Theoretical discharge.

1. Introduction

The orifice meter has significant importance in several fields, such as flow measurement devices, tanks, reservoirs, dams, medical instrumentations, piping systems, and fuel injection into combustion engines [1]. The orifice discharge coefficient C_d - a dimensionless number defined as the ratio of actual discharge to theoretical discharge - is a crucial parameter as it is considered to indicate the efficiency of the orifice flow. Previous research has argued that there is a relationship between the discharge coefficient and orifice geometric characteristics such as orifice thickness, orifice diameter, and orifice shape. The commonly accepted value of the discharge coefficient (i.e., as suggested by many hydraulics textbooks) is to use a single constant of $C_d = 0.60$. This value is also suggested in the design manual of urban drainage provided by the US Federal Highway Administration [2]. The Ontario Ministry of Environment uses a constant value for the discharge coefficient of 0.63 [3], while Bos [4] suggests a range for the discharge coefficient from 0.60 to 0.64, depending on the orifice diameter, d . Several studies have concentrated on the effects of viscosity, edge rounding, and plate roughness on the discharge coefficient.

The variation in the discharge coefficient for circular orifices in riser pipes for different values of the ratio of the orifice to the riser diameter (d/D) was investigated by Prohaska [5]. It was found that an increase in d/D leads to a decrease in the discharge coefficient, due to the associated contraction angle increasing with a higher d/D ratio. The results also showed that, in terms of the effect of head on the discharge coefficient, C_d increased at lower head values; a relationship was fitted to predict C_d as a function of d/D and h/d for discharge through orifices in a riser pipe.

Measurements in the design and use of orifice plates must be accurate and consistent. The upstream edge of the orifice must be square-sharped, and the minimum plate thickness must be based on the interior pipe diameter and orifice bore [6].

As noted above, the discharge coefficient has higher values under low head conditions [7]. Even though this result is widely known, the effort has been deficient in precisely quantifying this effect for design purposes. This is essential for applications such as stormwater detention facilities, where an orifice often discharges under low head conditions [8]. Parshad and Kumar. [9] investigated the effect of orifice diameter on the discharge coefficient and found that increasing the orifice diameter results in increasing the discharge coefficient. Ramamurthi and Nandakumar [10] investigated the discharge coefficient for small sharp-edged cylindrical orifices, such that the orifice diameter varied from 0.2 to 2 mm and the aspect ratio from 1 to 50. The flow characteristics were determined in the separated attached and cavitating flow regions, and also in the case of attached non-cavitating flow. It was found that the discharge coefficient is proportional to the Reynolds number and aspect ratio, and that the discharge coefficient and the onset of cavitation in the orifice were influenced by the orifice diameter. Eltoukhy and Alsaydalani [11] carried out experimental runs to study the characteristics of a hydraulic jump downstream of a sluice gate with an orifice for different orifice diameters and locations. It was found that, in the absence of any orifice, the hydraulic jump sequent depth ratio and jump height ratio increased with the initial Froude's number, with values less than those for the case with the presence of an orifice. Relative energy loss also increased with the initial Froude's number and

had greater values than those when there was an orifice present. Abdelrahman [12] studied the effect of geometric characteristics on the discharge coefficient for water flow through orifices, for four orifice shapes (circle, square, equilateral triangle, and rectangle), four areas of 78.54, 176.71, 314.16, and 490.87 mm², and five thicknesses of 2, 4, 6.75, 7.5, and 9.25 mm. It was found that the value of the discharge coefficient C_d was highest for the circular shape, then the equilateral triangle, square, and finally rectangle. The study also showed that the discharge coefficient decreased as the orifice thickness increased.

However, no known study has provided a general relationship between the discharge coefficients C_{do} and C_{dp} for orifice and pipe behaviour respectively as a function of its geometric characteristics such as diameter d and length L . The present paper developed experimentally equations for estimating the discharge coefficients C_{do} and C_{dp} for different diameters and lengths.

2. Dimensional Analysis

This paper aims to develop equations for estimating discharge coefficients for orifice and pipe behaviour C_{do} and C_{dp} as functions of its geometric characteristics (diameter d and length L). The Buckingham π - theorem of dimensional analysis was used to establish these relationships. According to White [13], "Basically, dimensional analysis is a method for reducing the number and complexity of experimental variables that affect a given physical phenomenon, by using a sort of compacting technique.

The discharge coefficients C_{do} and C_{dp} as dependent variables can be expressed as a function of all other relevant independent variables as follows:

$$C_{do} \text{ or } C_{dp} = f(\rho, g, h, v, d, \mu, L) \quad (1)$$

where ρ is the water density [ML⁻³], g is the acceleration of gravity [LT⁻²], h is the head above the centreline of the orifice or pipe [L], v is the outflow velocity from the orifice or pipe [LT⁻¹], d is the orifice or pipe diameter [L], μ is the water viscosity [ML⁻¹T⁻¹], and L is the orifice or pipe length [L]. Equation (1) may be written as:

$$C_{do} \text{ or } C_{dp} = f\left(\frac{h}{d}, \frac{d}{L}, \frac{v}{\sqrt{gh}}, \frac{\mu}{\rho h \sqrt{gh}}\right) \quad (2)$$

The first and second π groups are introduced to take into consideration the geometric characteristics of the behaviour of each orifice or pipe.

3. Experimental Setup and Methodology

To achieve the objective of this paper, experimental runs were conducted in the Water Resources and Hydraulics Laboratory, Faculty of Engineering at Shoubra, Benha University, Cairo, Egypt. The used apparatus consists primarily of two parts. The lower part is an Armfield hydraulic bench, and the upper part is a tank with 80 cm height, 80 cm length, and 60 cm width. The back 20 cm of the tank length has a baffle plate of 70 cm with a 10 cm opening with wire mesh and a gravel layer of 20 cm height to cool down the water entering the tank. The hydraulic bench was used to supply the upper part with water by pumping this from its reservoir. Actual discharge Q_{act} was estimated by dividing the collected water volume V by the corresponding time T ($Q_{act} = V/T$). The supplied water was controlled by the hydraulic bench control valve to adjust the water head h across values of 25, 30,

35, 40, 45, and 50 cm. The upper tank was formed from transparent acrylic plastic, as this is easy to laser-cut and allows visual observation of the water flow. The orifice was fitted to the front tank side using a groove for this purpose. The used diameter d has values of 12.7, 19.05, 25.4, and 38.1 mm, Lengths L of 2, 4, 7.5, and 9.25 mm were based on available plate thickness, and lengths of 20, 50, 100, 200, 400, 600, and 1,000 mm were achieved by fitting a standard UPVC pipe of the desired length (see Figs. 1 to 3). All diameters and lengths of the experimental runs were analysed for each orifice and pipe behaviour.

The experimental run procedure was devised to accurately determine the head-discharge relationship across multiple diameters and lengths. The following steps were followed for each experiment:

- i. Fit the plate with an opening diameter d of 12.7 mm (0.5 in) and length L of 2 mm.
- ii. Turn on the hydraulic bench pump and adjust the control valve to flow water into the upper tank through the flexible pipe to reach the first water head h of 25 cm.
- iii. Use the hydraulic bench to record the collected water volume V and the corresponding time T .
- iv. Close the control valve and turn off the hydraulic bench pump.
- v. Repeat steps 2- 4 for water heads 30, 35, 40, 45, and 50 cm.
- vi. Repeat steps 1- 5 for the same diameter of 12.7 mm (0.5 in) and with lengths of 4, 7.5, 9.25, 20, 50, 100, 200, 400, 600, and 1,000 mm.
- vii. Repeat steps 1- 6 for opening diameters of 19.05 mm (0.75 in), 25.4 mm (1 in), and 38.1 mm (1.5 in).

The experimental program is summarized in Table 1.

Table 1. The experimental program.

d (mm)	L (mm)	h (cm)	d (mm)	L (mm)	h (cm)	d (mm)	L (mm)	h (cm)	d (mm)	L (mm)	h (cm)
12.7	2	25,30,35, 40,45,50	19.05	2	25,30,35, 40,45,50	25.4	2	25,30,35, 40,45,50	38.1	2	25,30,35, 40,45,50
	4	25,30,35, 40,45,50		4	25,30,35, 40,45,50		4	25,30,35, 40,45,50		4	25,30,35, 40,45,50
	7.5	25,30,35, 40,45,50		7.5	25,30,35, 40,45,50		7.5	25,30,35, 40,45,50		7.5	25,30,35, 40,45,50
	9.25	25,30,35, 40,45,50		9.25	25,30,35, 40,45,50		9.25	25,30,35, 40,45,50		9.25	25,30,35, 40,45,50
	20	25,30,35, 40,45,50		20	25,30,35, 40,45,50		20	25,30,35, 40,45,50		20	25,30,35, 40,45,50
	50	25,30,35, 40,45,50		50	25,30,35, 40,45,50		50	25,30,35, 40,45,50		50	25,30,35, 40,45,50
	100	25,30,35, 40,45,50		100	25,30,35, 40,45,50		100	25,30,35, 40,45,50		100	25,30,35, 40,45,50
	200	25,30,35, 40,45,50		200	25,30,35, 40,45,50		200	25,30,35, 40,45,50		200	25,30,35, 40,45,50
	400	25,30,35, 40,45,50		400	25,30,35, 40,45,50		400	25,30,35, 40,45,50		400	25,30,35, 40,45,50
	600	25,30,35, 40,45,50		600	25,30,35, 40,45,50		600	25,30,35, 40,45,50		600	25,30,35, 40,45,50
	1000	25,30,35, 40,45,50		1000	25,30,35, 40,45,50		1000	25,30,35, 40,45,50		1000	25,30,35, 40,45,50

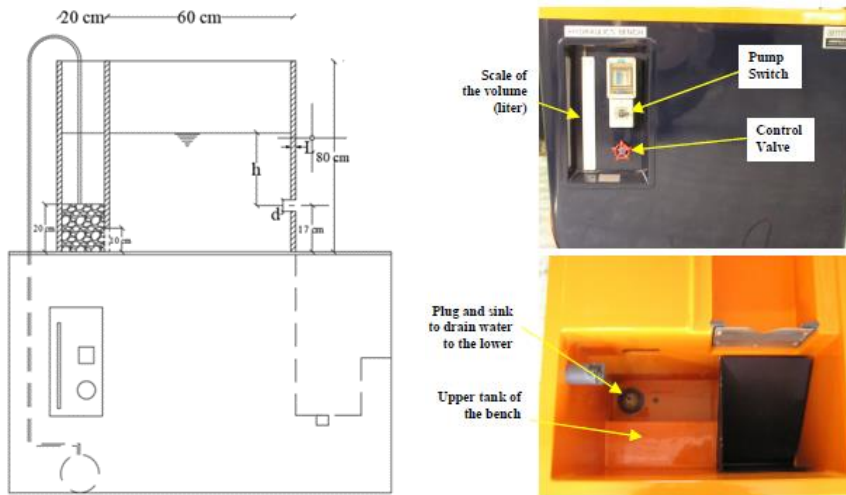


Fig. 1. Apparatus layout.



Fig. 2. Opening: $d = 12.7$ mm and $L = 4$ mm.



Fig. 3. Model of the plate and pipe used.

4. Results and Discussion

4.1. Pipe roughness

First, an experimental run was carried out to determine the pipe roughness (ε). Water was pumped through a pipe with a length of 1.0 m and a diameter of 25.4 mm on which a manometer was installed between two points 60 cm apart. The manometer deflection was recorded to calculate the pressure head loss h_L and the actual discharge by dividing the collected water volume V to the corresponding time T . The mean velocity v was calculated by dividing the actual discharge by the pipe area, allowing us to calculate the Reynolds number R_e as follows:

$$R_e = \frac{\rho v d}{\mu} \quad (3)$$

By solving the head loss equation, the pipe friction coefficient f may be calculated as:

$$h_L = f \frac{L}{d} \frac{v^2}{2g} \quad (4)$$

From the Moody chart, using the obtained values of the Reynolds number R_e and the pipe friction coefficient f , the roughness ratio $\frac{\varepsilon}{d}$ can be estimated. Here, pipe roughness ε was found to equal 0.002 mm; this was assumed to be the same for all experimental runs.

4.2. Effect of diameter on discharge coefficient for each orifice and pipe behaviour

Some 264 experimental runs were conducted to study the variation in discharge coefficient with water head ratio h/d and the opening length ratio d/L for different pipe diameters (17.2, 19.05, 25.4, and 38.1 mm). Each experimental run was carried out according to the procedure described above. At the end of the run, actual discharge Q_{act} was calculated as the ratio of the collected volume V to the corresponding time T for each applied water head h (25, 30, 35, 40, 45, and 50 cm).

Theoretical discharge may be calculated for the behaviour of each orifice Q_{tho} and pipe Q_{thp} for all lengths L by applying Bernoulli's equation, as follows:

$$Q_{tho} = \frac{\pi}{4} d^2 \sqrt{2gh} \quad (\text{Orifice behavior}) \quad (5)$$

$$Q_{thp} = \frac{\pi}{4} d^2 \sqrt{\frac{2gh}{C_e + f \frac{L}{d} + 1}} \quad (\text{Pipe behavior}) \quad (6)$$

where, C_e is the entrance loss coefficient was assumed to be 0.5 for a sharp entrance.

The friction coefficient f was estimated as follows:

For each experimental run, after calculating the theoretical discharge for pipe behaviour Q_{thp} , the mean flow velocity was calculated by dividing Q_{thp} by the pipe area $\frac{\pi}{4} d^2$. Reynolds number R_e was calculated using Eq. (3). The pipe friction coefficient f was estimated by using Moody chart with the values of R_e and the roughness ratio ε/d .

The discharge coefficient was calculated for each orifice C_{do} and pipe C_{dp} behaviour as:

$$C_{do} = \frac{Q_{act}}{Q_{tho}} \quad (\text{For orifice behaviour}) \quad (7)$$

$$C_{dp} = \frac{Q_{act}}{Q_{thp}} \quad (\text{For pipe behaviour}) \quad (8)$$

Analysis of the results shows that for all lengths and diameters, the discharge coefficients C_{do} and C_{dp} for each orifice and pipe behavior respectively, decreases with increasing flow head h (see Figs. 4 to 11). This is because the theoretical discharges Q_{tho} and Q_{thp} for orifice and pipe behaviors increase with the flow head by values more than that corresponding to the actual discharge Eqs (4) and (5). It was found that the values of C_{dp} are larger than those of C_{do} for the same head ratio h/d . For example, for d of 12.7 mm and d/L of 6.35, h/d varies from 19.68 to 39.37, resulting in variation in C_{do} from 0.5987 to 0.5439 and variation in C_{dp} from 0.6856 to 0.621. This may be because, the consideration of entrance and friction losses for the case of pipe behaviour. The same results were obtained when using the four diameters for a fixed length ratio L/d of 15.75 (see Fig. 12) and the case of experimental runs for all diameters with a fixed length L of 50 mm (see Fig. 13).

The effect of the diameter d on the discharge coefficients C_{do} and C_{dp} for fixed-length L of 50 mm and water head h of 35 cm was also investigated. The results showed that the discharge coefficients increase as the opening diameter increases for each orifice and pipe behaviour (see Fig. 14). It was also found that the discharge coefficient values are more responsive to opening diameter in the case of pipe behaviour than for orifice behaviour. For example, on changing the opening diameter from 12.7 to 38.1 mm (200%), the discharge coefficient values changed from 0.54 to 0.624 (15.6%) and from 0.71 to 0.77 (8.5%) for orifice and pipe behaviour respectively. However, the effect of diameter on the discharge coefficient increases slightly when investigated at fixed h/d and L/d , Fig. 15.

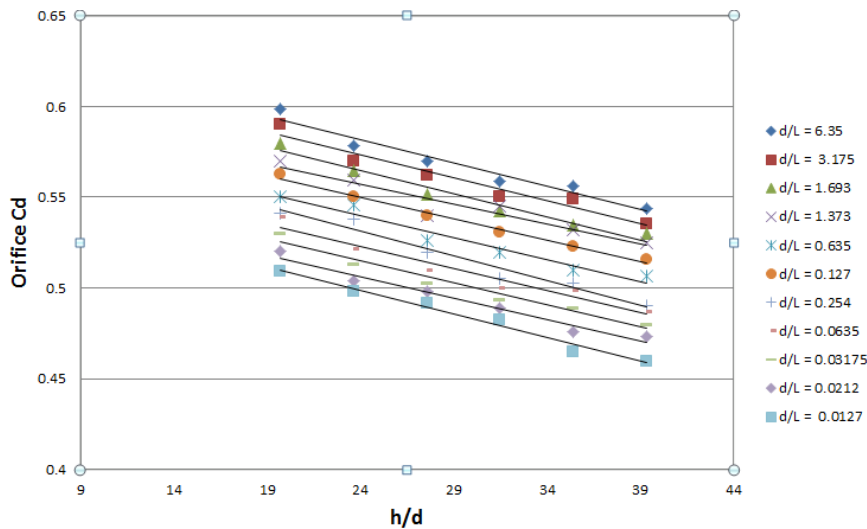


Fig. 4. Orifice discharge coefficient C_{do} for different d/L for $d = 12.7$ mm.

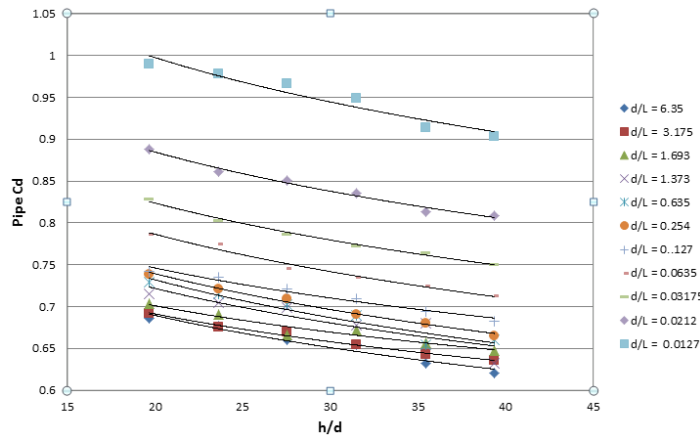


Fig. 5. Pipe discharge coefficient C_{dp} for different d/L for $d = 12.7$ mm.

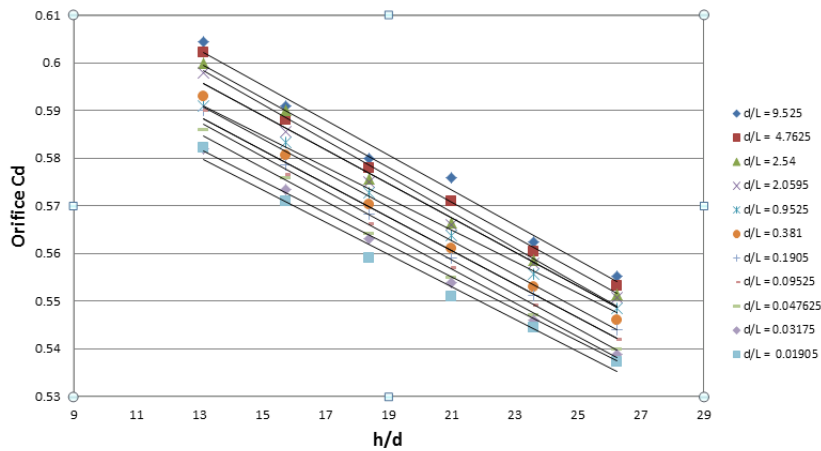


Fig. 6. Orifice discharge coefficient C_{do} for different d/L for $d = 19.05$ mm.

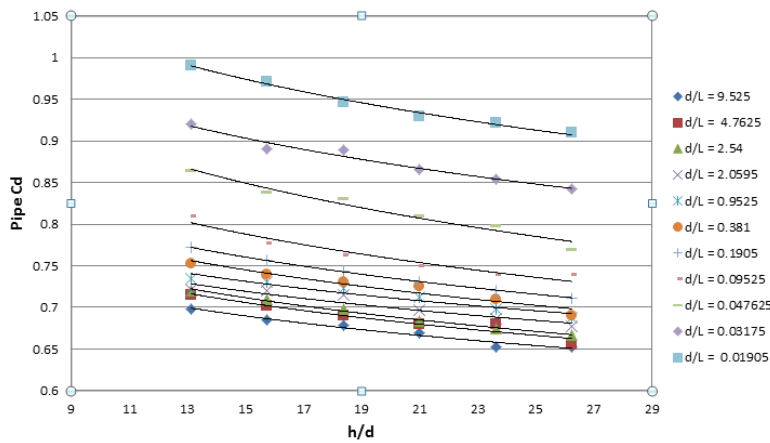


Fig. 7. Pipe discharge coefficient C_{dp} for different d/L for $d = 19.05$ mm.

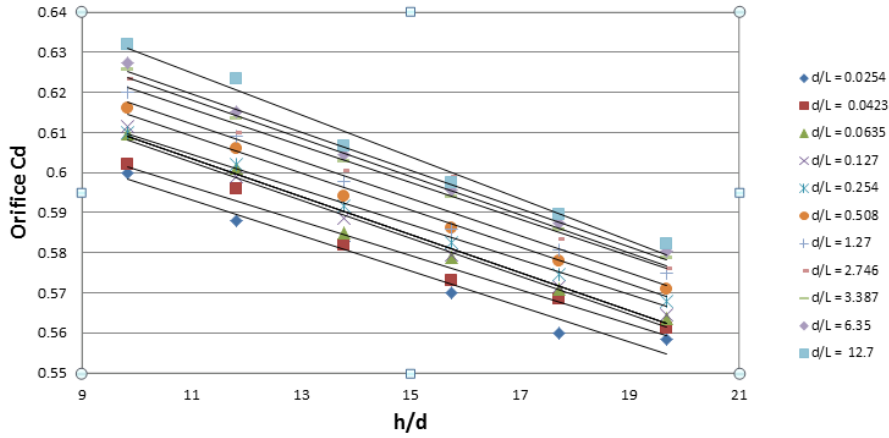


Fig. 8. Orifice discharge coefficient C_{do} for different d/L for $d = 25.4$ mm.

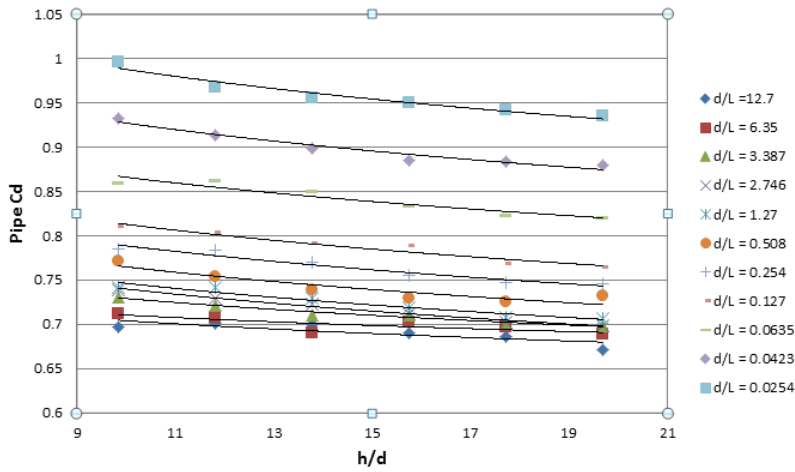


Fig. 9. Pipe discharge coefficient C_{dp} for different d/L for $d = 25.4$ mm.

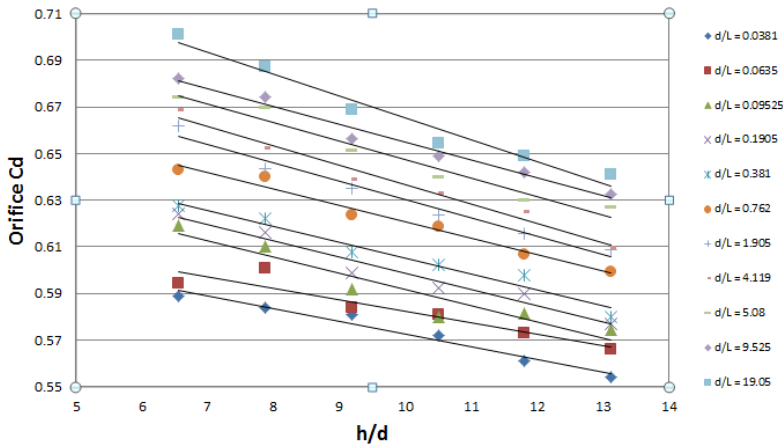


Fig. 10. Orifice discharge coefficient C_{do} for different d/L for $d = 38.1$ mm.

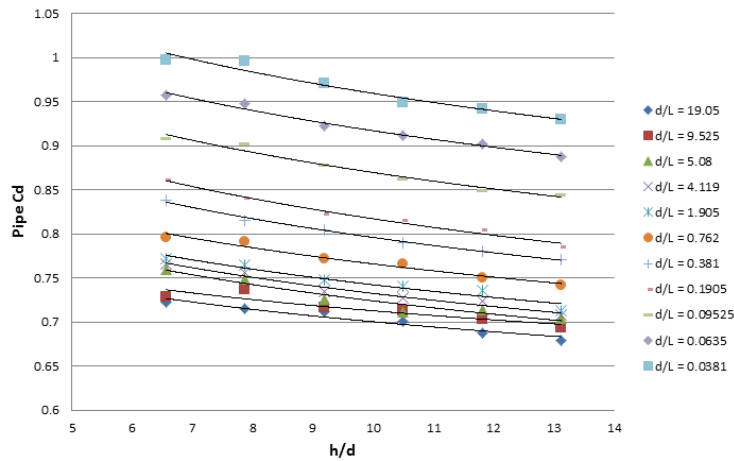


Fig. 11. Pipe discharge coefficient C_{dp} for different d/L for $d = 38.1$ mm.

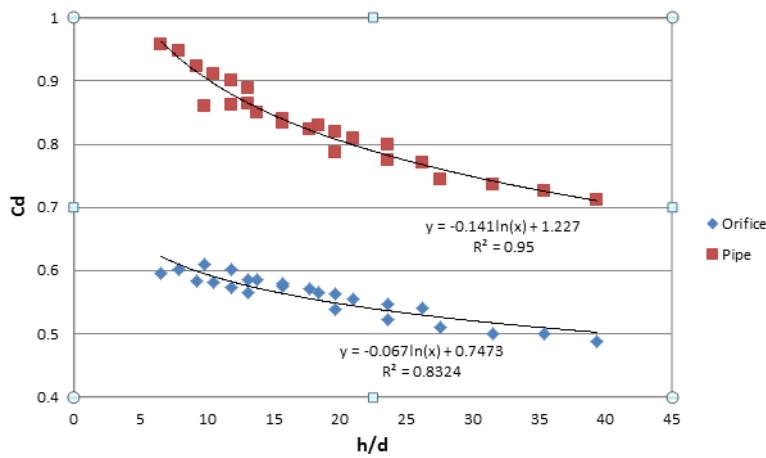


Fig. 12. Orifice and pipe discharge coefficients C_{do} and C_{dp} for length diameter ratio of $L/d = 15.75$.

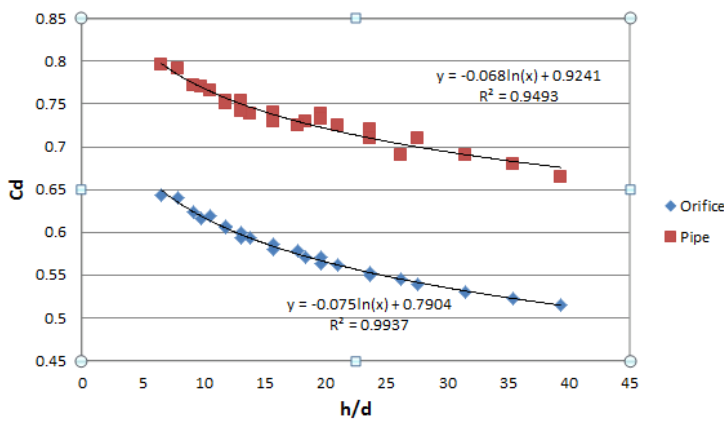


Fig. 13. Orifice and pipe discharge coefficients C_{do} and C_{dp} for length L of 50 mm.

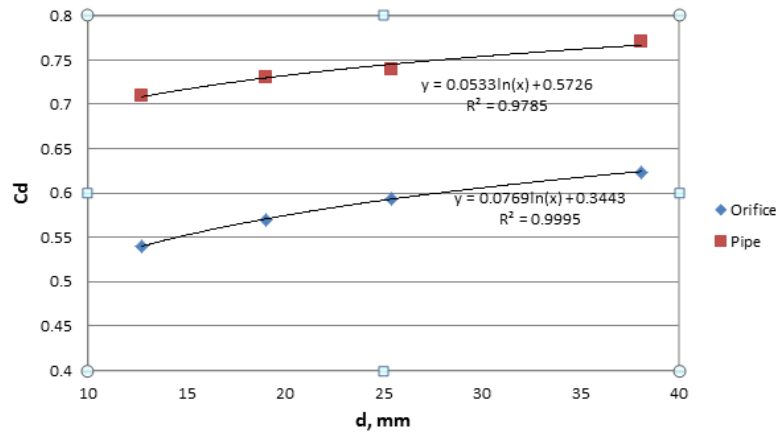


Fig. 14. Orifice and pipe discharge coefficients C_{do} and C_{dp} for $h = 35$ cm and $L = 50$ mm.

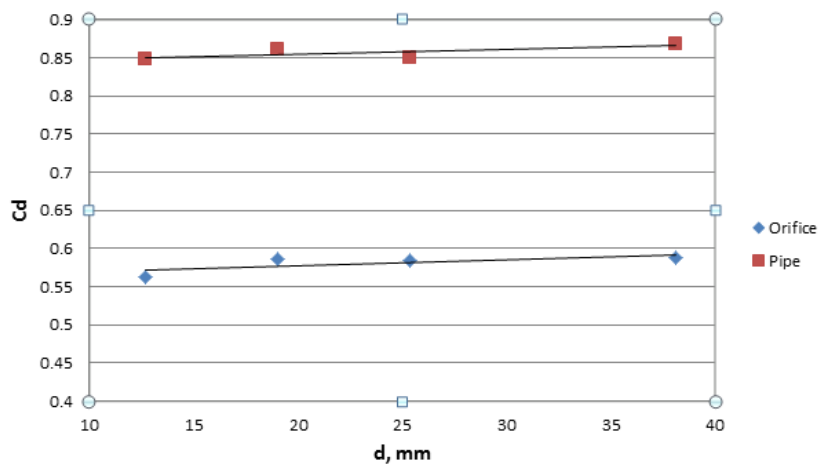


Fig. 15. Orifice and pipe discharge coefficients C_{do} and C_{dp} for $h/d = 13$ and $L/d = 15.75$.

As shown in Figs. 4 to 11, the effect of varying length L on the discharge coefficient was investigated, and the results showed that the discharge coefficient C_{do} for the orifice' behaviour decreases as the length increases. This may be due to the pipe effect as a result of increasing the length. Only when length L is 2 mm does the discharge coefficient C_{do} takes a value close to its commonly assumed value of 0.6; thus, to ensure orifice behaviour, the length should not exceed 2 mm, and the opening length L should be constructed with a smooth length edge of up to 2 mm, followed by a 45-degree bevel. In contrast to these results, the discharge coefficient C_{dp} for pipe' behaviour increases as the length increases and converges to unity. This may be due to taking into consideration entrance and friction losses. The region in which the discharge coefficient C_{dp} is less than one is the hydrodynamic entrance region, where the thickness of the boundary layer increases in the flow direction until the boundary layer reaches the pipe centre and thus fills the entire pipe, hence the flow becomes uniform (see Fig. 16). The discharge

coefficients C_{do} and C_{dp} distribution along the length is shown in Fig. 17, which confirms the results mentioned above.

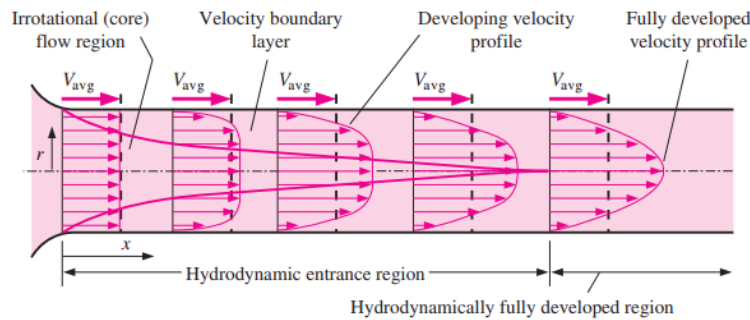


Fig. 16. Development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in the laminar flow, as shown, but somewhat flatter or fuller in the turbulent flow.

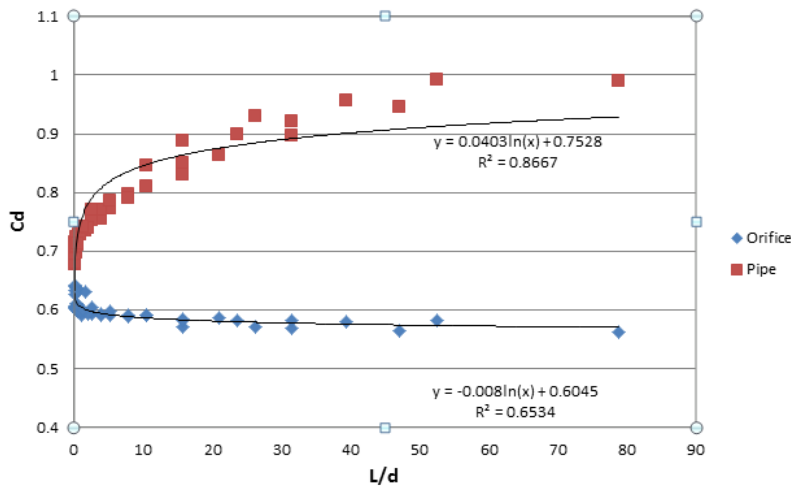


Fig. 17. Orifice and pipe discharge coefficients C_{do} and C_{dp} along the pipe length.

4.3. Discharge coefficient general equations for each orifice and pipe behaviour

Based on dimensional analysis and multiple regression analysis, the opening diameter d , opening length L , and the water head h played an important role in the discharge coefficient values C_{do} and C_{dp} . These exhibit complex behaviour. Using non-linear statistical analysis, many forms of equations for discharge coefficients C_{do} and C_{dp} were tested by introducing different combinations of these parameters and evaluating the effect of such combinations on the estimation of the discharge coefficient. Out of these trials, two equations were obtained for discharge coefficients C_{do} and C_{dp} (for orifice and pipe behaviour respectively):

$$C_{do} = 0.00467 \left(\frac{h}{d}\right) + 0.0055 \ln\left(\frac{d}{L}\right) + 0.6639 \quad (9)$$

$$C_{dp} = \left(0.007 \ln\left(\frac{d}{L}\right) - 0.0558\right) \ln\left(\frac{h}{d}\right) - 0.057 \ln\left(\frac{d}{L}\right) + 0.9008 \quad (10)$$

The values of each C_{do} and C_{dp} may be estimated using the developed Eqs (9) and (10) respectively. The average values for R^2 were 0.98 and 0.976 for Eqs (9) and (10) respectively.

Equations (9) and (10) estimate C_{do} and C_{dp} , with maximum error less than 5% and 11% respectively. Figures 18 and 19 show the computed and the actual values of the discharge coefficients C_{do} and C_{dp} respectively, in which good fit was observed. The coefficients of determinate R^2 were 0.9774 and 0.8421 for Figs. 18 and 19, respectively.

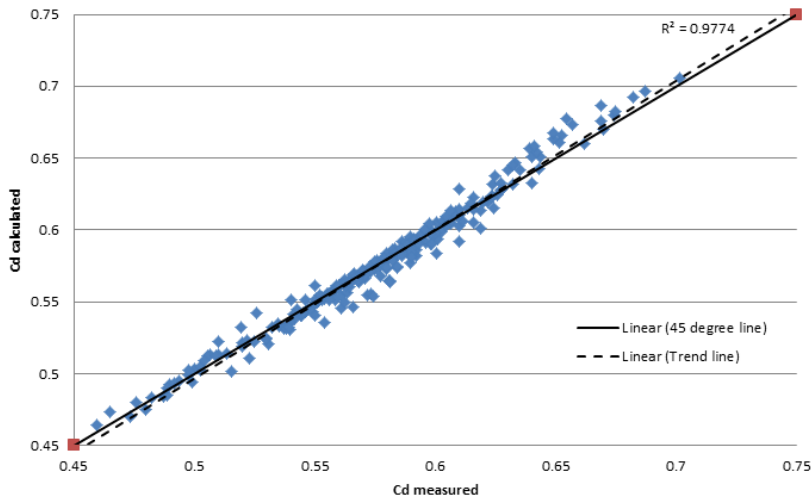


Fig. 18. Comparison between measured and computed C_{do} .

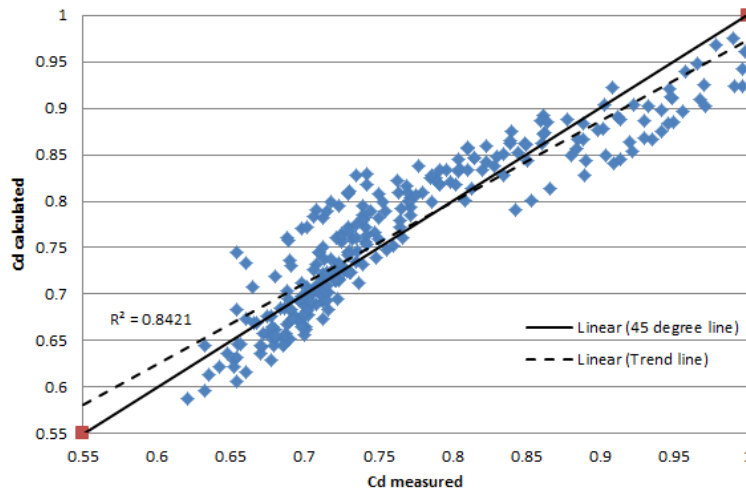


Fig. 19. Comparison between measured and computed C_{dp} .

5. Conclusions

Based on the analysis of experimental run results to investigate the discharge coefficient for orifice and pipe behaviour, the following conclusions were obtained:

- There are variations involved when considering a circular opening as an orifice or a pipe.
- The discharge coefficients C_{do} and C_{dp} decreases as the water head increases at a constant opening diameter and length.
- The obtained values of the discharge coefficient C_{dp} for pipe behaviour were greater than those of C_{do} for orifice behaviour.
- Increasing the opening diameter results in increasing the discharge coefficients C_{do} and C_{dp} .
- The discharge coefficient increases along the opening length in the case of pipe behaviour but decreases along the length for orifice behaviour.
- Equations (9) and (10) are suggested for calculating the discharge coefficient for orifice and pipe behaviour respectively.
- Only for length L up to 2 mm is the discharge coefficient close to its commonly assumed value of 0.6; thus, to ensure orifice behaviour, the length should not exceed 2 mm, and the opening length L should be constructed with a smooth length edge of up to 2 mm followed by a 45-degree bevel.

Nomenclatures

C_d	The discharge coefficient
C_{do}	The discharge coefficient for orifice behaviour
C_{dp}	The discharge coefficient for pipe behaviour
C_e	Entrance coefficient of losses = 0.5
D	The opening diameter, m
f	Friction coefficient
g	Gravitational acceleration, m/s^2
h	Water head above the opening centreline, m
h_L	Head losses, m
L	The opening length, m
P	Water density, kg/m^3
Q_{act}	Actual discharge, m^3/s
Q_{tho}	Theoretical discharge for orifice behaviour, m^3/s
Q_{thp}	Theoretical discharge for pipe behaviour, m^3/s
Re	Reynolds number
T	The time corresponding to the collected volume, s
V	Collected volume, m^3
v	Mean velocity, m/s

Greek Symbols

ε	Opening surface roughness, m
μ	Dynamic viscosity, Ns/m^2

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