GREY-WOLF OPTIMIZATION BETTER ENHANCES THE DYNAMIC PERFORMANCE OF ROLL MOTION FOR TAIL-SITTER VTOL AIRCRAFT GUIDED AND CONTROLLED BY STSMC

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Abstract

This article presents the design of optimal sliding mode control (SMC) and optimal super-twisting sliding mode control (STSMC) for the roll motion of vertical take-off and landing (VTOL) unmanned air vehicle (UAV) in hovering flight under nonparametric uncertainty (gust and wind disturbance). The stability analysis of the controlled roll motion has been presented and asymptotic error convergence has been proven based on Lyapunov theorem. Accordingly, the control laws are developed for the aircraft system subjected to uncertainty. To avoid the try-and-error procedure in the selection of the design parameters and to improve the performances of SMC and STSMC, the grey-wolf optimization has been suggested for tuning purposes. Based on numerical simulation, a comparison study has been conducted between the optimal and non-optimal controllers, and also between optimal SMSTC and optimal SMC in terms of tracking error and chattering behaviour in control signals. The numerical simulation showed that the GWO could enhance the performances of SMC and STSMC. Also, the optimal STSMC has better dynamic performance than the optimal SMC in terms of tracking error and chattering effect in control signal.

Keywords: Grey-wolf optimization, Guidance, Roll motion, Stability analysis, Super-twisting sliding mode control, Tail-sitter VTOL aircraft.
1. Introduction

In the last years, the Unmanned Aerial Vehicles (UAVs) have acquired fast-growing popularity world-wide and experienced enormous development. Nowadays, these UAVs are largely applied in different critical military and defence purposes like reconnaissance, security reinforcement, and surveillance. However, in addition of military and defence applications of UAVs, the usage of these small aircraft has rapidly grown in many military and civilian applications, and they have incorporated in wide range of fields including disaster management, vegetarian monitoring, traffic surveillance, infrastructure inspection, and law enforcement [1].

There are other missions of UAV which extend the scope of conventional capabilities of small UAVs. The longer endurance of flight is not only the requirement of most missions, but also the vertical take-off and landing (VTOL) and hovering capabilities. Moreover, the capability of conversion from one configuration to another is the other required task of these small aircrafts, which became necessary demand recently. Combing the forward flight characteristics of fixed-wing aircraft with the take-off and landing capabilities of the helicopter will result in promising UAVs which characterized by unique flexible operation capabilities at low cost than other conventional UAVs [2].

The tail-sitter vehicle is one configuration of these convertible aircrafts. It is characterized by taking the vertical airframe attitude during landing and take-off, while takes the attitude of horizontal airframe in case of cruising just like conventional airplanes. However, the flight dynamics of tail-sitters are characterized by high complexity, particularly in hover mode, which makes them very difficult to control [2]. In what follows, some of the relevant and recent control strategies used for flight control of tail-sitter aircraft are briefly discussed.

In Barth et al. [3] applied model-free control scheme for stabilization of tail-sitter micro-aircraft attitude in the presence of wind disturbance. Four flight modes have been taken into account; particularly, vertical take-off, transitioning flight, forward flight, hovering and vertical landing. In Zhou et al. [4], established flight control design based on model predictive control (MPC) and successive linearization for VTOL tail-sitter aircraft in the hovering mode. In Wang et al. [5], presented a novel control design for tail-sitter aircraft actuated by twin rotors. The proposed control configuration is based on decreasing the distance between rotors and elevators, which results in maximizing the speed flow and in turn lead to generating the required control torque necessary to stabilizing the twin-rotor aircraft against disturbing wind.

Garcia-Nieto et al. [6] have presented the design of attitude tracking controller for unmanned flying-wing actuated by two tilting rotors to achieve VTOL manoeuvre in hovering operation. The efficiency of proposed controller has been tested based on both Hardware-In-the-Loop (HIL) simulation test and experimental test. Ge and Hou [7] have combined the design of fuzzy self-tuning PID control and L1-adaptive control for stabilization of longitudinal attitude of tail-sitter aircraft subjected to uncertainties. This study has applied the algorithm of double-Euler angle method in modelling to solve the singularity problem due to fuselage tilting. Abrougui et al. [8] has designed and developed a flight regulation control algorithm based on PID (Proportional, Integral and Derivative) controller for stabilization of roll motion for the VTOL aircraft during the operation of hovering.
flight. In Flores et al. [9], has presented a control design based on linear saturation functions and Lyapunov approach for manoeuvre in transition for tail-sitter drone.

In Garcia et al. [10] developed a control algorithm based on separated saturation functions to stabilize the attitude of single-rotor actuated tail UAV affected by perturbation in the channel of vertical take-off and landing operation. Verling et al. [11] have proposed both structure design and control design of convertible VTOL tail-sitter aircraft. The development of UAV has combined the merits of technologies in both rotary and fixed wing UAV systems. A unified controller has been synthesized and designed to address the aircraft dynamics at any configuration and attitude like rotor-actuated scheme, fixed-wing scheme, or the transition from one configuration to another. This unified controller can achieve monotonic transition in switching from one mode to another without any discontinuity. Li et al. [12] presented the design of control algorithm based on MPC (Model Predictive Control) for motion control of tail-sitter VTOL aircraft during hovering flight operation. The control design of MPC is based on augmentation of linearized model. The disturbance rejection capability has been improved using an optimization technique. Çakici and Leblebicioğlu [13] have proposed the control design for stabilization of fixed-wing and multi-rotors actuated aircraft. Three modes of flight have been considered for the suggested UAV: Vertical-Take-off-Landing (VTOL), hovering and level flight mode. The control algorithm has been developed to switch between the modes of flight.

Like most aircrafts, the dynamics of tail-sitters encounter dramatic variations during their flight and the need of robust controller is a pre-requisition to solve this challenging problem. However, the use of conventional controller is awesome for these kinds of aerial systems. One of the drawbacks of conventional tracking controllers is that they are unable to cope with unknown load characteristics over a widely ranging of operating point. This study adopts the super-twisting sliding mode control (STSMC) design to cope with the uncertainty due to the gust wind in roll motion of the considered aircraft.

The STSMC is one new type of sliding mode control (SMC), which features with by the following merits [14-20].

- In case of STSMC, the chattering effect in control signal is less than SMC.
- The STSMC design has the ability to make the trajectories of system solution to reach the equilibrium states in finite time. In other words, the zeroing of both state and its derivative in finite time is possible with STSMC.
- In STSMC, there is need for the derivative of states, but only for the sliding variable or output variable. Therefore, STSMC differs from most SMC techniques in that the knowledge of state derivatives is not required. This in turn will lead to simple synthesized control law and to less effort of computation.
- Exact convergence and singularity avoidance can be achieved with STSMC.

According to the above features, the STSMC has been adopted to control the roll motion of VTOL aircraft.

A critical issue in control design of STSMC is how to set the values of design parameters arising throughout the stability analysis of the controlled system for developing the required control law. These design parameters play a vital role in performance of STSMC. Frequently, their setting is based on the try-and-error
procedure. This procedure is old, and it lacks the ability to find the optimal performance of STSMC. Recently, modern optimization techniques are incorporated for tuning these parameters such as to improve the dynamic response of the system controlled by STSMC in terms of minimum tracking error. In this study, the Grey Wolf Optimization algorithm (GWO) has been proposed to adjust the design parameters of STSMC. The GWO is a new meta-heuristic optimization algorithm proposed in 2014 [21]. It has been widely used in different applications due to its excellent performance [22, 23]. The contribution of this study can be summarized in the following points:

- Design of roll motion control for the tail-sitter VTOL aircraft based on STSMC.
- Design an optimization algorithm based on GWO technique to improve the performance of STSMC and SMC by tuning their design parameters and to replace the try-and-error procedure.
- Conducting a comparative study between optimal and non-optimal versions of SMC and STSMC. Also, a comparison study is made between optimal SMC and optimal STSMC for the tail-sitter VTOL aircraft system via numerical simulation.

2. The Dynamic Model of Tail-Sitter VTOL Aircraft

This part considers the development of dynamic model for the tail-sitter VTOL aircraft. Firstly, two assumptions are made to proceed in deriving the dynamic model:

**Assumption 1:** The aircraft operates within small local region. This will justify applying the model equation of flat earth [13].

**Assumption 2:** The masses of blade and elevators have been neglected [24].

Referring to Fig. 1, let the coordinate system \((X, Y, Z)\) represents the body fixed frame \((b-frame)\) and the north-east-down (NED) coordinate system \((x, y, z)\) represents the inertial reference frame \((n-frame)\). Based on Fig. 1, the kinematic equations describing the position, the forces and the moments are represented by [25]:

\[
\begin{align*}
\dot{p} &= R(e) V \\
\dot{\Theta} &= H(e) \Omega \\
m \dot{V} &= -\Omega \times V + F \\
J \dot{\Omega} &= -\Omega \times J \Omega + \tau
\end{align*}
\]

where \(p = [p_n \ p_e \ p_d]^T\) represents the positions of mass centre of rigid body relative to the \((n-frame)\), \(e = [e_o \ e_1 \ e_2 \ e_3]^T\) represent the quaternion of the current attitude and it is defined by \(e = e_o + e_1 i + e_2 j + e_3 k\), \(\Theta = [\phi \ \theta \ \psi]^T\) represents the orientation of the VTOL aircraft in the \((n-frame)\), where the Euler angles \(\phi, \theta \) and \(\psi\) defines the roll, pitch and yaw angles, which are commonly used in aerodynamic applications. The vector \(\Omega = [P \ Q \ R]^T\) represents the angular velocity in the body fixed frame \((b-frame)\), \(F\) is the vector of external thrusts applied to mass center of the VTOL aircraft, the torque vector \(\tau = [\tau_r \ \tau_m \ \tau_n]^T\) combines the torque components exerted to the centre of VTOL aircraft mass in the body frame and \(J\) is the inertia matrix of the flying aircraft, which is given by

\[
\begin{align*}
\dot{p} &= R(e) V \\
\dot{\Theta} &= H(e) \Omega \\
m \dot{V} &= -\Omega \times V + F \\
J \dot{\Omega} &= -\Omega \times J \Omega + \tau
\end{align*}
\]
\[ J = \begin{bmatrix} J_x & J_{xy} & J_{xz} \\ J_{yx} & J_y & J_{yz} \\ J_{zx} & J_{zy} & J_z \end{bmatrix} \] (5)

If it is assumed the body axis \( xz \)-plane of the configuration of tail-sitter VTOL aircraft is coincident with the plane of symmetry, then the products of inertia \( J_{xy} \) and \( J_{yz} \) vanishes. Also, the tail-sitter configuration has a plane of symmetry in the \( yz \)-plane, this leads that the product of inertia \( J_{xz} \) equal to zero. Then the inertia matrix and its inverse become

\[ J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix}, \quad J^{-1} = \begin{bmatrix} 1/J_x & 0 & 0 \\ 0 & 1/J_y & 0 \\ 0 & 0 & 1/J_z \end{bmatrix} \] (6)

The transformation matrix \( H \) given in Eq. (2), transforms the components of the angular velocity, generated by Euler rotations, from the body frame to the inertial frame is given by:

\[ H(e) = \begin{bmatrix} 1 & \tan(\theta) \sin(\phi) & \tan(\theta) \cos(\phi) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \] (7)

Therefore, the kinematic Eq. (2) can be written as follows:

\[ \dot{\phi} = P + \tan \theta (Q \sin(\theta) + R \cos \phi) \] (8)

\[ \dot{\theta} = Q \cos \phi - R \sin \phi \] (9)

\[ \dot{\psi} = (Q \sin \phi + R \cos \phi)/\cos \theta \] (10)

Using the inertia matrix given by Eq. (6) and using the thrust moments \( T_i, T_m \) and \( T_n \) as indicated in Fig. 1, Eq. (4) can be rewritten by:

\[ \dot{P} = (J_y - J_z) QR/J_x + T_i/J_x \] (11)

\[ \dot{Q} = (J_z - J_x) PR/J_y + T_m/J_y \] (12)

\[ \dot{R} = (J_x - J_y) PQ/J_z + T_n/J_z \] (13)

![Flight dynamics of tail-sitter VTOL aircraft](image)

Fig. 1. Flight dynamics of tail-sitter VTOL aircraft [25].
In order to extract the roll dynamic, it is assumed that the yaw and pitch rates are set to zero; that is, \( P = Q = 0 \). Based on this assumption, the configuration of tail-sitter VTOL aircraft can be shown in Fig. 2. Therefore, based on Eqs. (6) and (9), the rotational dynamics can be represented for the roll angle using the following simple dynamic equation:

\[
\ddot{\phi} = \frac{T_l}{J_x} \tag{14}
\]

where the exerted torque \( T_l \) can be calculated as follows:

\[
T_l = F \cdot d - C_l \dot{\phi} \tag{15}
\]

where \( F = F_1 - F_2 \) represents the resultant force between the force due to right rotor and left rotor, \( d \) is the distance between the mass center and reach rotor. The term \( C_l \dot{\phi} \) is the drag force, which represents the aerodynamic moment that works to oppose the rolling moment with damping coefficient of \( C_l \).

Combining Eq. (14) and Eq. (15) and taking into account the effect of gust wind as uncertainty \( \zeta(t) \) applied to the roll dynamic system to have,

\[
\ddot{\phi} = \left( -C_l \phi + F \cdot d \right) / J_x + \zeta(t) \tag{16}
\]

Equation (16) can be rewritten as

\[
\ddot{\phi} = f_o + b u + \zeta(t) \tag{17}
\]

where \( f_o \), \( b \) and \( u \) are represented by \( f_o = -C_l \dot{\phi} / J_x; b = d / J_x; u = F \).

Remark 1: \( \zeta(t) \) is the gust wind, the derivative of uncertainty satisfies \( |\dot{\zeta}(t)| \leq \gamma \), where \( \gamma \) is the upper bound of the derivative of the uncertainties, which is a positive constant.

3. Super Twisting Sliding Mode Control for Tail-Sitter VTOL Aircraft

Let \( e \) defines the tracking error between the actual roll angle (\( \phi \)) and the desired roll angle (\( \phi_d \)).

\[
e = \phi - \phi_d \tag{18}
\]

The time derivative of error is given by

\[
\dot{e} = \dot{\phi} - \dot{\phi}_d \tag{19}
\]

The sliding surface can be defined by

\[
s = \dot{e} + \lambda e \tag{20}
\]

Taking the time derivative of Eq. (20), one can have
\[
\dot{s} = \ddot{\phi} + \lambda \dot{\phi} = \dot{\phi} - \ddot{\phi}_d + \lambda \dot{\phi} \quad \text{or} \quad \dot{s} = f_o + \zeta(t) + b u - \ddot{\phi}_d + \lambda \dot{\phi} \tag{21}
\]

In the sense of sliding control theory, the control law is composed of two parts: equivalent part and switching part; that is,

\[
u = \frac{1}{b}(u_e + u_s) \tag{22}
\]

By setting \(s = \dot{s} = 0\), one can easily obtain the equivalent part of control law. Accordingly, based on Eq. (21), the equivalent component can be found

\[
u_e = -f_o + \dot{\phi}_d - \lambda \dot{\phi} \tag{23}
\]

The switch part is proposed, based on STSMC, to be

\[
u_s = -c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt \tag{24}
\]

Therefore, the control law can now be determined as follows

\[
u = \frac{1}{b} \left( -f_o + \dot{\phi}_d - \lambda \dot{\phi} - c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt \right) \tag{25}
\]

Accordingly, Eq. (21) becomes

\[
\dot{s} = -c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt + \zeta(t) \tag{26}
\]

The asymptotic stability of the tail-sitter VTOL aircraft, controlled by super-twisting SMC, can be assured by choosing the following Lyapunov function is chosen,

\[
V = \frac{1}{2} s^2 \tag{27}
\]

Combining the time derivative of Eq. (27) with Eq. (26) to have

\[
\dot{V} = s \left( -c_1 \sqrt{|s|} \text{sgn}(s) - c_2 \int \text{sgn}(s) \, dt + \zeta(t) \right) \tag{28}
\]

Based on the elementary linear algebra and the mathematical fact \(s \text{sgn}(s) = |s|\), Eq. (28) can be expressed by

\[
\dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \int c_2 \, dt + |\zeta(t)| \, s \]
\[
\dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \int c_2 \, dt + |s| \int |\dot{\zeta}(t)| \, dt \tag{29}
\]

As we assume in remark 1 that \(|\dot{\zeta}(t)| \leq \delta\), where \(\delta\) is a positive constant and it represents the upper bound of the derivative of the uncertainty, then the previous equation can be written as

\[
\dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \int c_2 \, dt + |s| \int \gamma \, dt \]
\[
\dot{V} \leq -c_1 \sqrt{|s|} |s| - |s| \left( \int c_2 \, dt - \int \gamma \, dt \right) \tag{29}
\]

Remark 2: The stability of tail-sitter VTOL aircraft, controlled by STSMC, can be guaranteed if the value of \(c_2\) satisfies \(c_2 > \gamma > |\dot{\zeta}(t)|\).

The schematic diagram of roll motion-controlled VTOL aircraft based on is indicated in Fig. 3.
Other control strategies can be suggested to extend this study for future work. The active disturbance rejection control, observer-based control, backstepping control, adaptive control, RISE control, nonlinear PD control, model predictive control, and extremum seeking control can be included for roll stabilization and control of tail-sitter VTOL such that a comparison study in performance can be conducted between the proposed controller and one of these control schemes [26-38].

4. Classical Sliding Mode Control for Tail-Sitter VTOL Aircraft

The switch part in the classical SMC is proposed as [39, 40],

$$u_{sw} = k \, sgn(s)$$

Therefore, one can write the control law as follows:

$$u = \frac{1}{b} \left( -f_o + \dot{\phi}_d - \lambda s - k \, sgn(s) \right)$$

Accordingly, Eq. (21) becomes

$$\dot{s} = -k \, sgn(s) + \zeta(t)$$

To establish the stability analysis of VTOL aircraft based on classical SMC, one can choose a Lyapunov function of the following form,

$$V = \frac{1}{2} s^2$$

Again, taking the time derivative of Eq. (33) and applying Eq. (32) to have

$$\dot{V} = s \left( -k \, sgn(s) + \zeta(t) \right)$$

Based on the elementary linear algebra and the mathematical fact $s \, sgn(s) = |s|$, Eq. (34) can be expressed by

$$\dot{V} \leq -k \, |s| + |\zeta(t)| \, s ; \dot{V} \leq -k \, |s| + |s| \int |\zeta(t)| \, dt$$

$$\dot{V} \leq |s| \left( -k + \int |\zeta(t)| \, dt \right)$$

**Remark 3:** The stability of the tail-sitter VTOL aircraft, controlled by SMC, is guaranteed if the value of $k$ satisfies $k > \int \gamma \, dt$. 

**Fig. 3. Schematic diagram of super-twisting sliding mode attitude-controlled VTOL aircraft.**
5. GWO-Based STSMC

In this part, GWO algorithm is used to find the optimal values of these parameters in terms of better performance of tacking based on STSMC of tail-sitter VTOL aircraft [40-44].

5.1. Social hierarchy

Grey wolves are social animals, and they have a strict social hierarchy. The population is divided into four levels, namely \( \alpha, \beta, \delta \) and \( \omega \). Corresponding to the GWO algorithm, we denote the individual with the highest fitness as \( \alpha \). The two suboptimal individuals are \( \beta \) and \( \delta \). \( \omega \) represents the remaining individuals. The optimization process of GWO is mainly guided by the three individuals with the best fitness in the population.

5.2. Encircling prey

The grey wolf usually approaches and surrounds the prey gradually, and the process of grey wolves surrounding their prey can be presented as

\[
D = |C \cdot X_p(t) - X(t)|
\]

where \( t \) is the current iteration, \( X_p \) is the location of the prey, \( X \) is the position of the grey wolf, and \( C = 2 \cdot r_1 \) and \( r_1 \in [0,1] \) is a random number.

\[
X(t + 1) = X_p(t) - A \cdot D
\]

where \( A = 2a r_2 - a \), \( a \) gradually decrease from 2 to 0, \( r_2 \in [0,1] \) is a random number.

5.3. Hunting

Assume that namely \( \alpha, \beta \) and \( \delta \) can track the prey effectively. At every iteration, the positions of the other grey wolves are updated based on the position information of wolves \( \alpha, \beta \) and \( \delta \). The mechanism of wolf position updating is shown in Fig. 4. In Fig. 4, \( D_\alpha, D_\beta \) and \( D_\delta \) are the distance between wolves \( \omega \) and wolves \( \alpha, \beta \) and \( \delta \) which have the best fitness.

![Fig. 4. Wolf hunting mechanism [41].](image)

The mathematical description of the hunting process is

\[
D_{\alpha} = |C_1 \cdot X_\alpha - X|
\]

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\[ D_\beta = |C_2 \cdot X_\beta - X| \]
\[ D_\delta = |C_3 \cdot X_\delta - X| \]
\[ X_1 = X_\alpha - A_1 \cdot D_\alpha \]
\[ X_2 = X_\beta - A_2 \cdot D_\beta \]
\[ X_3 = X_\delta - A_3 \cdot D_\delta \]
\[ X(t + 1) = \frac{X_1 + X_2 + X_3}{3} \]  

(38)

where \( X_\alpha, X_\beta \) and \( X_\delta \) are the positions of \( \alpha, \beta \) and \( \delta \), respectively. \( C_1, C_2 \) and \( C_3 \) are three random vectors, and \( X \) is the current location of the grey wolf. The list of algorithm steps for grey-wolf optimization is given below:

**Optimization algorithm of grey-wolf technique**

1. Initialize the grey wolf population \( X_i \) (\( i = 1, 2, \ldots, n \))
2. Initializtion of \( a, A, \) and \( C \)
3. Calculation of fitness for each search agent
4. Set \( X_\alpha \) for the best search agent
5. Set \( X_\beta \) for the second-best search agent
6. Set \( X_\delta \) for the third best search agent

**While** (stopping criteria has not been reached) do

1. for each search agent
2. Update the position of the current search agent using Eq. (38)
3. end for
4. Update the parameters: \( a, A, \) and \( C \)
5. Calculation of the fitness for all search agents
6. Update the variables \( X_\alpha, X_\beta, \) and \( X_\delta \)

**end while**

Output the best solution found

Other optimization techniques like cuckoo optimization algorithm, sine-cosine algorithm, whale optimization, spider social optimization, and monkey-spider optimization can be suggested to tune the design parameters of both SMC and STSMC and to establish a comparison study in performance with grey-wolf optimization [41-51].

6. Computer Simulation

In this section, the effectiveness of STSMC and SMC have been evaluated via numerical simulation using MATLAB programming software. The numerical simulation has used Ode45 as a numerical solver. Table 1 gives the numerical values of parameters for the considered tail-sitter VTOL aircraft.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_x )</td>
<td>x-axis moment of inertia</td>
<td>0.0144 kg.m(^2)</td>
</tr>
<tr>
<td>( C_L )</td>
<td>Roll damping coefficient</td>
<td>0.36</td>
</tr>
<tr>
<td>( d )</td>
<td>The rotor distance from mas center</td>
<td>0.2 m</td>
</tr>
</tbody>
</table>

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The optimization algorithm based on GWO results in optimal design parameters for STSMC and SMC and hence an optimal STMSC and optimal SMC will be obtained. Table 2 lists the values of optimal design parameters given by GWO. The simulation results have been initiated by showing the behaviors of cost functions w.r.t the optimization iterations, as illustrated in Fig. 5.

**Table 2. The optimal setting of design parameters based on GWO algorithm.**

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC</td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>3</td>
</tr>
<tr>
<td>$k$</td>
<td>3</td>
</tr>
<tr>
<td>STSMC</td>
<td></td>
</tr>
<tr>
<td>$c_1$</td>
<td>2.5</td>
</tr>
<tr>
<td>$c_2$</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>3</td>
</tr>
</tbody>
</table>

In the present work, two scenarios have been presented. The first scenario presents the uncertainty-free case, while the uncertainty has been accounted for in the second...
scenario. The tracking behaviors of the roll angle for VTOL aircraft system in the
certainty-free case system are shown in Fig. 6, based on optimal and non-optimal
SMCs. On the other hand, Fig. 7 shows the tracking behaviour of the roll angle based
on optimal and non-optimal STSMC. The behaviour error ($e$) for all designed
controllers are shown in Figs. 8 and 9, where $e$ represents the difference between the
desired signal and the output. Table 3 shows the tracking performance of tail-sitter
VTOL aircraft based on the controllers. Figure 10 shows the behaviour of control law
under this situation. Based on Table 3, one can deduce that the optimal STSMC has
better performance than the optimal SMC. In addition, the chattering shown in control
signal in case of STSMC is lower than that based on SMC.

![Fig. 6. Behavior of roll angle SMC.](image)

![Fig. 7. Behavior of roll angle STSMC.](image)
In the next scenario, it is assumed that the aircraft is subjected to rotational gust behaviour, which represents a nonparametric uncertainty. The rotational gust component $\xi(t)$ was modeled as a sin-function of amplitude 0.1 and frequency 30 Hz. The behaviours of aircraft roll angle under gust uncertainty for both optimal

Fig. 8. Behavior of tracking errors with SMC.

Fig. 9. Behavior of tracking errors with STSMC.

Fig. 10. The control response with uncertainty-free case.
SMC and optimal STSMC are in Fig. 11. The responses of tracking errors are shown in Fig. 12. The STSMC gives less variance in error as compared to the classical SMC. The behaviour of control law resulting from this situation is illustrated in Fig. 13. Again, the STSMC could successfully eliminate the chattering level as compared to the SMC.

Table. 3 Tracking performance of tail-sitter VTOL aircraft based on the controllers.

<table>
<thead>
<tr>
<th>Controller</th>
<th>RMSE</th>
<th>(t_s) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC-Without GWO</td>
<td>3.36</td>
<td>0.0128</td>
</tr>
<tr>
<td>SMC-With GWO</td>
<td>2.04</td>
<td>0.0131</td>
</tr>
<tr>
<td>STSMC-Without GWO</td>
<td>3.13</td>
<td>0.0148</td>
</tr>
<tr>
<td>STSMC-With GWO</td>
<td>2.19</td>
<td>0.0113</td>
</tr>
</tbody>
</table>

Fig. 11. Behavior of roll angle with gust wind uncertainty.

Fig. 12. Behavior of tracking errors with gust wind uncertainty.
7. Conclusion

In this study, two schemes of sliding mode controllers are presented, STSMC and SMC, to stabilize and control the roll motion of tail-sitter VTOL aircraft. The stability of controlled aircraft system has been analyzed and proved based on Lyapunov theorem. Accordingly, the control law has been established. The GWO is invoked to find the optimal setting of design parameters for both SMC and STSMC and accordingly to further improve the dynamic performance of controlled systems. A comparison study in terms of steady-state error and chattering level in control signal has been conducted for uncertainty-free case and uncertainty case. The computer simulation showed that the GWO could enhance the performances of proposed controllers. Moreover, the optimal STSMC has better performance than optimal SMC in terms of tracking error and chattering behaviour. This study will be extended in the future work to consider more degree of freedom in dynamic model of aircraft, and also taking into account the motion in three-dimensional space. One can suggest other optimization techniques like PSO, SSO, or WOA to conduct a comparison in performance with GWO in terms of minimum tracking error and computation effort indices.

Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1, k_1, k_2$</td>
<td>Design parameters of SMC</td>
</tr>
<tr>
<td>$c_2, k_3, k_4$</td>
<td>Design parameters of STSMC</td>
</tr>
<tr>
<td>$C_L$</td>
<td>The roll damping coefficient</td>
</tr>
<tr>
<td>$d$</td>
<td>The rotor distance from the mass centre</td>
</tr>
<tr>
<td>$J_x$</td>
<td>Moment inertia in the x-axis</td>
</tr>
<tr>
<td>$T_1, T_m, T_n$</td>
<td>Thrust moments</td>
</tr>
<tr>
<td>$u$</td>
<td>Control law</td>
</tr>
<tr>
<td>$u_{eq}$</td>
<td>Equivalent part of control law</td>
</tr>
<tr>
<td>$u_{sw}$</td>
<td>Switching part of control law</td>
</tr>
<tr>
<td>$V(.)$</td>
<td>Lyapunov function</td>
</tr>
</tbody>
</table>

Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>is a positive constant and it represents the upper bound of the derivative of the uncertainty</td>
</tr>
<tr>
<td>$\zeta(t)$</td>
<td>The gust wind</td>
</tr>
</tbody>
</table>

Fig. 13. The control response with gust wind uncertainty.

\[ \theta_d \] Desired angular position
\[ s_p \] Sliding Surface
\( k, \lambda_1 \) Design parameters of SMC.
\( c_1, c_2, \lambda \) The design parameters of STSMC.
\( \varphi \) The disturbances

**Abbreviations**

GWO  Grey-Wolf Optimization  
HIL Hardware-In-Loop  
INDI Incremental Nonlinear Dynamic Inversion  
MPC Model Predictive Controller  
PID Proportional Integral Derivative  
PSO Particle Swarm Optimization  
SIL Software-In-Loop  
SMC Sliding Mode Control  
SSO Spider Social Optimization  
STSMC Super Twisting Sliding Mode Control  
UAV Unmanned Air Vehicle  
VTOL Vertical Take-off Landing

**References**


