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A REVIEW OF THE TRANSFORMATION TECHNIQUES IN THE OFDM SYSTEM

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Abstract

Orthogonal frequency division multiplexing (OFDM) system has been favourably applied in various aspects of wired and wireless communication systems due to its flexibility, high spectral efficiency, efficient implementation, and robustness against the different types of interferences. In this paper, all the transformation techniques used in the OFDM system were reviewed. Major transformations used in the OFDM system such as Fourier Transform (FT), lapped transform (LT), cosine transform (CT), wavelet transform (WT), and curvelet transform (CvT) were surveyed. The computational complexity of all these transformations techniques was also addressed. The results show that the discrete wavelet transform (DWT), the complex wavelet packet transform (CWPT), and the dual-tree complex wavelet transform (DTCWT) give lower order of computational complexity compared to other types of the transformation techniques. At the same time, the discrete cosine transform (DCT) gives lower order of computational complexity compared to fast Fourier transform (FFT), modulated lapped transform (MLT), wavelet packet transform (WPT) and CvT.

Keywords: Wavelet transform, Curvelet transform, Discrete cosine transform, Modulated lapped transform, Orthogonal frequency division multiplexing.

1. Introduction

Multicarrier modulation (MCM) schemes, such as orthogonal frequency division multiplexing (OFDM), are used in current communication systems due to their flexibility to frequency selective channels [1-4]. In this paper, an attempt is made to collect all the developments that occurred in the OFDM system as a result of using different transformation techniques. In general, as shown in Fig. 1, MCM systems can be broadly classified into a wired system (digital subscriber line (DSL)) and wireless systems (OFDM). The wireless MCM systems can be categorized into following systems according to the type of the block-transform used. fast Fourier transforms (FFT) based systems, modulated Lapped transform (MLT) based systems, discrete cosine transform (DCT) based systems, wavelet transform (WT) based systems, and the fast discrete curvelet transform (FDCT) based systems. WT-based systems can be classified into different five systems: discrete wavelet transforms (DWT) based systems, wavelet packet transform (WPT) based systems, complex wavelet packet transform (CWPT) based systems and dual-tree complex wavelet transform (DTCWT) based systems. On the other hand, the FDCT-based systems themselves can be categorized into two types: FDCT via unequispaced fast Fourier transform (USFFT) based systems and FDCT via Wrapping systems.

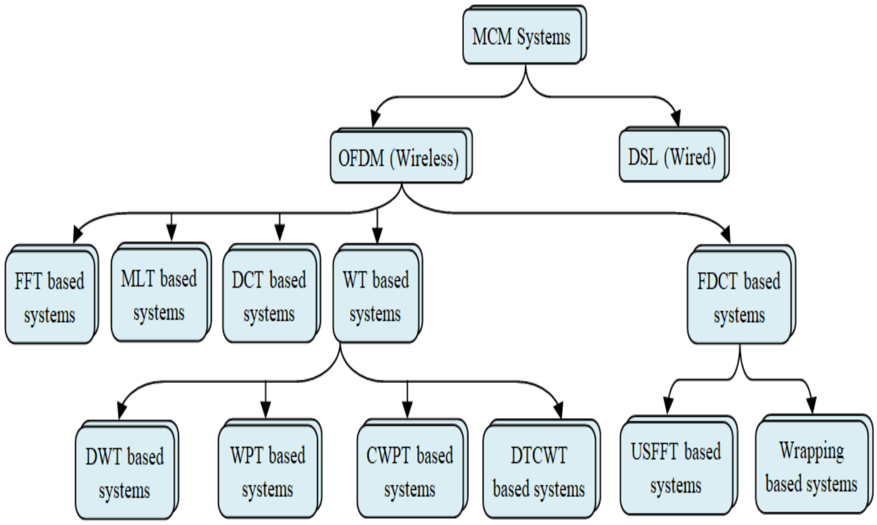

Fig. 1. Classification of the MCM systems.

One can say that the idea of OFDM comes from MCM transmission technique. The principle of MCM is to partition the input bit stream into numerous parallel bit streams and then use them to modulate several subcarriers as shown in Fig. 2. Each subcarrier is separated by using the guard band to prevent the subcarrier from overlapping with each other. On the receiver side, band-pass filters are used to separate the spectrum of individual subcarriers. OFDM is a particular form of MCM technique of spectral efficiency, which uses orthogonal and multispectral interfering spectra. Figure 3 shows the block diagram of the traditional OFDM system.

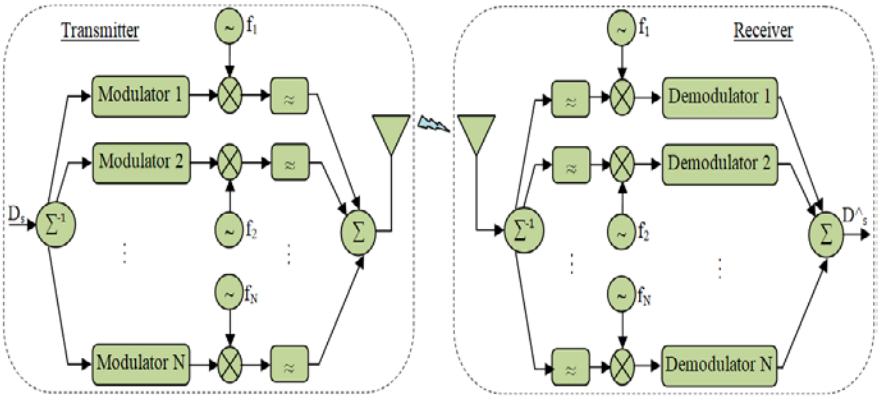

Fig. 2. Block diagram of a generic MCM transceiver.

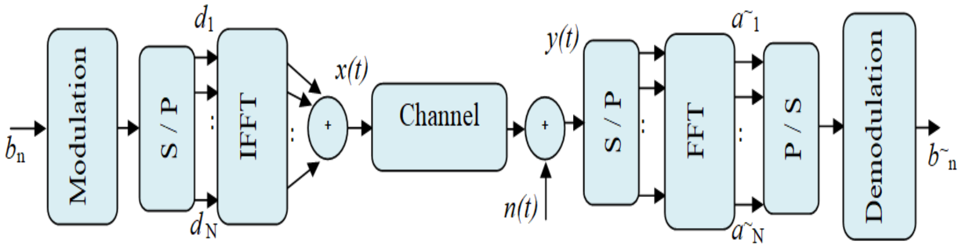

Fig. 3. Block diagram of the traditional OFDM system.

2. Discrete Fourier Transform Based System

The story of parallel data transmission and the use of frequency-division multiplexing began in the early 1950s [5, 6]. As a result of the great growth and great development in the field of communications at that time, the need to find systems with efficient use of bandwidth became urgent. The linear characteristics and frequency stability of the available transmitter equipment are helped to improve the frequency spectrum utilization [7, 8]. Here the idea of sending several orthogonal data simultaneously appeared through the linear bandwidth with a maximum data rate and without interference between the channels or interference between the symbols [9]. Figure 4 shows the block diagram of parallel data transmission using a discrete Fourier transform (DFT).

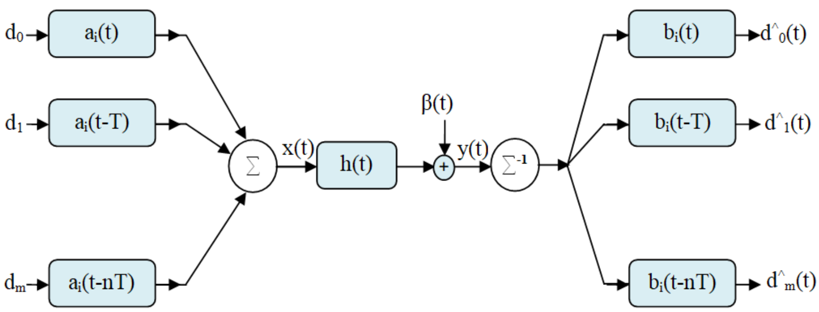

Fig. 4. Block diagram of parallel and orthogonal data transmission.

If $x[n]$ is the discrete-time version of a continues time-domain (TD) signal $x(t)$ so the transformation of $x[n]$ in the frequency domain (FD) using DFT, i.e., $X(k)$ is given by:

$$X(k) = \sum_{n=-0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}} \quad , \quad k = 0,1,2,\cdots,N-1 \tag{1}$$

The transformation back of $X(k)$ into discrete *TD* ($x[n]$), i.e., IDFT can be obtained using the following equation:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} \quad , \quad n = 0,1,2,\cdots,N-1 \tag{2}$$

Let $d_0, d_1, \ldots, d_m$ are a sequence of transmitted data, $a_i(t)$ the transmitter filter's impulse response, $h(t)$ the transmission channel's impulse response, $w(t)$ the channel's noise, $b_i(t)$ the receiver filter's impulse response, and $d^\wedge_0, d^\wedge_1, \ldots, d^\wedge_m$ are a sequence of received data. Then, the transmitted signal will be:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} a_k e^{j\frac{2\pi kn}{N}} \tag{3}$$

The received signal at the final output is:

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} h(n,l)x(n-l) + w(n) \tag{4}$$

The received signals $y(t)$, i.e., $y_1(t), y_2(t), \ldots y_m(t)$ are orthogonal to each other means:

$$\int_{-\infty}^{\infty} w_i(t) w_i(t-kT)dt = 0 \quad , \quad k = \pm 1, \pm 2, \cdots \tag{5}$$

where, $w_i(t) = \int_{-\infty}^{\infty} d_i h(t-\tau) a_i(\tau) d\tau$. The relationship between d_i, a_i and h(t) will be convolution in the TD and it will be converted to multiplication in the FD. Assuming noise elimination and perfect channel estimation, the received signal is:

$$Y_m = X_m H_m + w_m \tag{6}$$

If $a_i(t)$ is proportional to $b_i(t)$, i.e., $a_i(t) = Ae^{-j\alpha t}$ and $b_i(t) = (1/A)e^{j\alpha t}$, it can be deduced that $a_i(t)$ and $b_i(t)$ are the inverse and forward DFT respectively [7, 8]. And j is the square root of (-1) then, the estimated received signal is:

$$\hat{d}(t) = \hat{d}_0 + \hat{d}_1 + \cdots \hat{d}_m \tag{7}$$

Figure 3 shows the block diagram of conventional OFDM, i.e., OFDM system based on the FFT. At the transmitter side, the IFFT is used to modulate the transmitted data and at the receiver side, the FFT is used to demodulate the received signal [9-12]. For the comparison purpose, Fig. 5 shows the block diagram of the OFDM based on various transformation techniques.

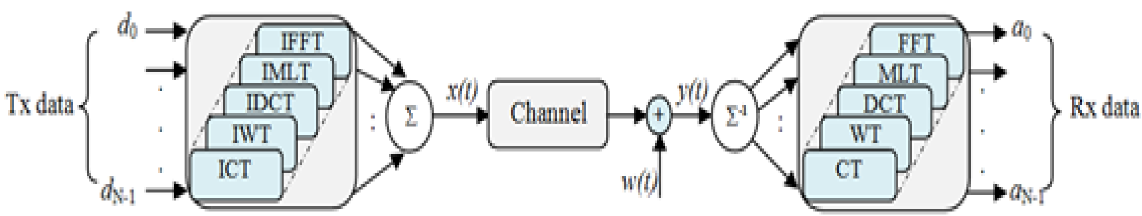

Fig. 5. Block diagram of the OFDM based on various transformation techniques.

For a number of N subcarriers, the computational complexity of DFT is $O(N^2)$ which is too high for a large number of subcarriers. But when using the fast Fourier transform (FFT), the computational complexity now will be:

$$O(N \log N) \tag{8}$$

which is less computational complexity compared to that for the DFT. And that is the reason behind using the FFT in the real implementation of the OFDM system instead of using the DFT [12-14].

3. Modulated Lapped Transform Based System

The OFDM system based on the FFT considered that a large number of subcarriers are spectrally overlapped. Hence, the guard intervals which are then are padded by the cyclic prefix (CP) is used to mitigate the problem of inter symbol interference (ISI). On the other hand, the OFDM system is very sensitive to frequency offset leads to destroying the orthogonality between the subcarriers that achieved using the FFT this problem is known as the problem of the inter-channel interference (ICI).

Gary et al. [15] proposed a new OFDM system based on the Lapped transform (LT) in order to overcome the ICI problem. In this mode, the number of transmitted subcarriers (L) is not restricted as it is in the number of transmitted subcarriers (M) for the conventional OFDM system. Let K be the overlapping factor, the number of transmitted subcarriers (L) is given by:

$$L = 2KM \tag{9}$$

Assume that the modulated lapped transform (MLT) use a $2M$ tap low pass filter model given by $\beta[n]$, the modulated vector $p_l[n]$ can be given by the following equation [15, 16]:

$$p_l[n] = \beta[n]\sqrt{\frac{2}{M}}\cos\left[\frac{\pi}{M}\left(\frac{2n+2k+M+2}{2}\right)\right] \tag{10}$$

where $0 \le n \le 2M$ & $0 \le k \le M-1$, $F[n]$ will be given by the equation:

$$\beta[n] = -\sin\left[\frac{(2n+1)\pi}{4M}\right] \tag{11}$$

The forward and the inverse transformation of the MLT of the signal $x(t)$ are given respectively by:

$$X(k) = \sum_{n=0}^{M-1} x(n)p(n,k) \quad \& \quad x(n) = \sum_{k=0}^{M-1} X(k)p(n,k) \tag{12}$$

As the case of the conventional OFDM, Fig. 5 depicted the block diagram of the OFDM based on the modulated Lapped transform (MLT). The inverse MLT (IMLT) and the MLT are used instead of the IFFT and FFT, respectively.

At the transmitter side of the OFDM system based on the MLT, the IMLT is used to modulate the input data then the transmitted signal $x(t)$ is sent through the channel $h(t)$. at the receiver side, the received signal $y(t)$ is passed the forward MLT do demodulate the transmitted data again to its original format. The transmitted signal $x(n)$ at the output of the inverse MLT:

$$x(n) = \sum_{i=0}^{N} \widetilde{W}_i \, d_i(n) \tag{13}$$

At the receiver side, the received signal $y(n)$ is described by:

$$y(n) \; = \; h\;(n)x(n) + w(n) \tag{14}$$

where *h(n)* is the channel response and *w(n)* is the discrete version of continues noise signal *n(t)*, the estimated received data a_i at the output of the forward transform of the FDCT-USFFT or the FDCT-wrapping is:

$$a_i(n) \; = \sum_{i=0}^{N} W_i\{y(n)\} = \sum_{i=0}^{N} W_i\{h(n)x(n)\} + \sum_{i=0}^{N} W_i\{w(n)\} \tag{15}$$

Assuming perfect channel estimation and absence of noise, in the output of the forward transform of the FDCT-USFFT or the FDCT-wrapping, the final output can be given by:

$$a_i(n) \; = \hat{d}_i(n) = \sum_{i=0}^{N} W_i \widetilde{W}_i\{d_i(n)\} \tag{16}$$

The computational complexity of the MLT algorithms for computing a number of *N* subcarriers is [17]:

$$O(N \log N) \tag{17}$$

4. Discrete Cosine Transform Based System

Instead of using DFT, discrete cosine transform (DCT) can be used in order to reduce the complexity and increase the throughput of the DFT based OFDM system due to efficient bandwidth utilization of the DCT [18]. As the cases of the conventional OFDM and MLT based OFDM system, multi-tone modulation (MTM) using DCT is shown in Fig. 5.

The DCT is outperformed the DFT in terms of bandwidth utilization, it required half of that for DFT [18-21], assuming that $d_0, d_2, \ldots, d_{N-1}$ are an N-length sequence of transmitted data, $x(k)$ is the discrete version of the continuously transmitted signal *x(t)* at the output of inverse discrete cosine transform (IDCT) given by

$$x(k) \; = \; \sum_{n=0}^{N-1} d_l D_l(n) \quad , \; n = 0,1.\cdots, N-1 \tag{18}$$

While the forward DCT is given by $d_l \; = \; \sum_{n=0}^{N-1} x(k) D_l(n)$ where,

$$D_l(n) \; = \begin{cases} \sqrt{\frac{1}{N}} & k = 0 \\ \sqrt{\frac{2}{N}} \cos \pi\, n \left(\frac{2k+1}{2N}\right) & k = 1,2,\cdots, n-1 \end{cases} \tag{19}$$

The total transmitted signal of an *N*-length is

$$x(k)^l \; = \; x(k)\;, \quad 1 \le k \le N/2 \tag{20}$$

$$x(k)^h \; = \; x(k)\;, \quad N/2 + 1 \le k \le N \tag{21}$$

If the channel impulse response is h_m and *M* is the OFDM samples, then the received signal can be given by

$$y(k) \; = \; \sum_{i=\infty}^{\infty} \sum_{m=0}^{M-1} h_m x(k - m - iN) + w(n) \tag{22}$$

Assume perfect channel estimation and noise elimination, at the output of the DCT, the final received signal $a_i(t)$ will be

$$A_i(k) \; = \begin{cases} \frac{\sqrt{2}}{N}\; d_l, & k = 0 \\ \sqrt{\frac{2}{N}} \; \sum_{k=1}^{N-1} x(k) \cos \pi\, n \left(\frac{2k+1}{2N}\right) \end{cases} \tag{23}$$

Computational complexity of the DCT for computing of *N* subcarriers is [22-24]:

$$O(N/2 \log N) \quad (24)$$

5. Wavelet Transform Based System

The wavelet transform has a set of features such as higher suppression of side lobe due to extender basis function, good adaptation, larger flexibility, and good performance for different channel effects. These advantages enabled the wavelet transform to be a strong candidate to be an alternative of Fourier transform (FT) in the OFDM system. As result, different types of wavelet transform, i.e., discrete wavelet transforms (DWT), wavelet packet transform (WPT), complex wavelet packet transform (CWPT), and dual-tree complex wavelet transform (DTCWT) was proposed to be used in conventional OFDM instead of FT. The following subsections will explore how these different types of wavelet transformations have been used in the OFDM system.

5.1. Discrete wavelet transform based system

Figure 6 shows the wavelet decomposition and reconstruction for two stages transform. By using the DWT, the discrete time signal $x(n)$ can be represented in the discrete wavelet domain as:

$$x_j(n) = \sum_{k\in Z}^{\infty} a_{0,k}\varphi_{0,k}(n) + \sum_{k\in Z}^{\infty} d_{j,k}\psi_{0,k}(n) \quad (25)$$

Where the two coefficients $a_{o,k}$ and $d_{j,k}$ are given by:

$$a_{0,k} = \sum_{k\in Z}^{\infty} x(n)\varphi_{0,k}(n) \quad \& \quad d_{j,k} = \sum_{k\in Z}^{\infty} x(n)\psi_{0,k}(n) \quad (26)$$

Figure 6 shows two stages of wavelet decomposition and reconstruction [25]. *h(n)* and *g(n)* are low and high pass filters respectively given as:

$$h(k) = \frac{1}{\sqrt{2}}\left\langle \varphi\left(\frac{t}{2}\right), \varphi(t-k)\right\rangle \quad \& \quad g(k) = \frac{1}{\sqrt{2}}\left\langle \psi\left(\frac{t}{2}\right), \varphi(t-k)\right\rangle \quad (27)$$

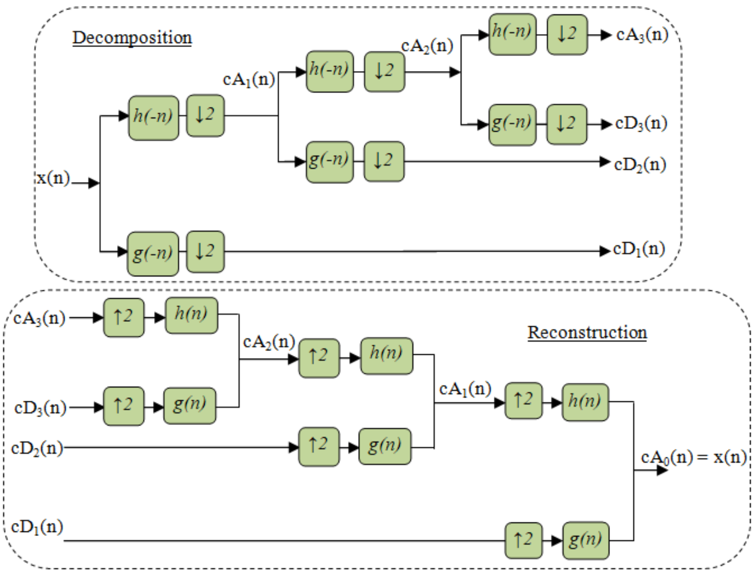

Fig. 6. Two stages wavelet decomposition and reconstruction.

The first ($cA_1(n)$ and $cD_1(n)$) and the second ($cA_2(n)$, and $cD_2(n)$) levels decomposition output for filters of length L can be found by the following equations:

$$cA_1(n) = \sum_{k=0}^{L} h(k)x(2n+k) \quad \& \quad cD_1(n) = \sum_{k=0}^{L} g(k)x(2n+k) \tag{28}$$

$$cA_2(n) = \sum_{k=0}^{L} h(k)cA_1(n)(2n+k) \quad \& \quad cD_2(n) = \sum_{k=0}^{L} g(k)cA_1(n)(2n+k) \tag{29}$$

In the receiver side, the reconstruction of $x(n)$ from $cA_1(n)$, $cD_1(n)$, $cA_2(n)$, and $cD_2(n)$ is obtained by:

$$\begin{aligned} x(n) &= cA_0(n) = cA_1(n) + cD_1(n) \\ &= \frac{1}{\sqrt{2}}\sum_k h(n-2k)cA_2(k) + \frac{1}{\sqrt{2}}\sum_k g(n-2k)cD_2(k) + \\ &\quad \frac{1}{\sqrt{2}}\sum_k g(n-2k)cD_1(k) \end{aligned} \tag{30}$$

As the cases of the conventional OFDM, MLT-OFDM and DCT-OFDM. Figure 5 shows the block diagram of the OFDM based on the DWT. The IDWT and the DWT are used instead of the IFFT and FFT respectively [26]. The approximation coefficients were displayed through the wavelet low pass filters and the detail coefficients were illustrated through the wavelet high pass filters. Various wavelet families that meet the principle of orthogonality can be used. Let the wavelet basis function is ϕ(t), the compression factor is l, each subcarrier has been shifted by m, *βlm* are the wavelet coefficients. The inverse discrete wavelet transform (IDWT) outputs ($x(k)$) can be expressed as:

$$x(k) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} 2^{l/2}\beta_l^m \varphi(2^{l/2}k - m) \tag{31}$$

The receiver side, the DWT outputs can be expressed as:

$$\beta_l^m = \sum_{k=0}^{N-1} 2^{l/2}x(n)\varphi(2^{l/2}k - m) \tag{32}$$

To generate N subcarriers of the DWT-OFDM system [27-30], the computational complexity is the order of:

$$O(N) \tag{33}$$

5.2. Wavelet packet transform based system

The decomposition (forward discrete wavelet packet transform (DWPT)) and the reconstruction (inverse discrete wavelet packet transform (IDWPT)) processes can be shown in Fig. 7 using the pair of filters $h(n)$ and $g(n)$ defined above. The pair of filters H and G are represented the up-sampling deconvolution while the pair of filters H-1 and G-1 have represented the down sampling deconvolution [31-33]. These filters are given by the following equations:

$$H(2n) = \sum_{k\in Z} x(k)h(k-2n) \quad \& \quad G(2n) = \sum_{k\in Z} x(k)g(k-2n) \tag{34}$$

$$H^{-1}(n) = \sum_{k\in Z} x(k)h(n-2k) \quad \& \quad G^{-1}(n) = \sum_{k\in Z} x(k)g(n-2k) \tag{35}$$

As the cases of the conventional OFDM, MLT-OFDM, DCT-OFDM, and DWT-OFDM, Figure 5 depicted the WPT based OFDM system, assume $a_{i,j}$ is the constellation of di input data and $\phi_k(n)$ is a discrete function, the output x(n) of IDWPT at the transmitter side is:

$$x(n) = \sum_i \sum_{j=0}^{M-1} a_{i,j}\ \varphi_j(n - iM) \tag{36}$$

At the receiver side, the received signal $y(t)$ is

$$y(n) = x(n) * h(n) + w(n) = \sum_k h(k)x(n-k) + w(n) \tag{37}$$

Under perfect channel estimation and absence of noise, in the output of the DWPT, the $a_{i,j}$ can be estimation by the inner product (‹ . , . ›) of functions as:

$$a_i = \langle x(n-k), \varphi_i(n) \rangle \tag{38}$$

For N subcarriers, The WPT have a computational complexity of [29, 34-36]:

$$O(N \log N) \tag{39}$$

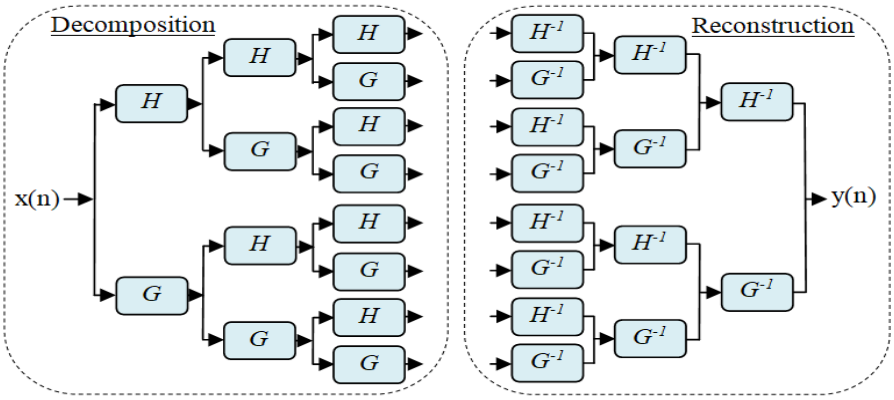

Fig. 7. Decomposition and reconstruction of the DWPT.

5.3. Complex wavelet packet transform based system

Complex wavelet transform (CWT), i.e., complex wavelet packet transform (CWPT) was proposed to solve the lack of shift-invariance problem related to the WPT based systems [37-41]. Where the CWP is the complex version of the WPT. Figure 5 shows the block diagram of the OFDM based on the CWPT, where the IFFI and the FFT those in the conventional OFDM system are replaced by the ICWPT and CWPT respectively in the CWPT based system.

As the cases of the conventional OFDM, MLT-OFDM, DCT-OFDM, DWT-OFDM, and WPT-OFDM, Fig. 5 shows the CWPT based OFDM system. Assume that T is the symbol period, M is the number of subcarriers, $\phi_{l,m}(n)$ is the complex wavelet packet function and d_m are the sequence of transmitted symbols. So, the transmitted signal $x(n)$ at the output of the ICWPT is:

$$x(n) = \sum_{m=i}^{M} \sum_{k=0}^{\infty} d_m(k) \varphi_{l,m}(n-kT) \tag{40}$$

At the receiver side, the received signal $y(n)$ is described by:

$$y(n) = h(n)x(n) + w(n) \tag{41}$$

where $h(n)$ is the channel response and $w(n)$ are the discrete version of continues noise signal $n(t)$, the estimated received data at the output of the CWPT is:

$$\hat{d}_i(n) = \sum_{m=i}^{M} \sum_{k=0}^{\infty} y(n) \varphi^{*}{}_{l,m}(n-kT) \tag{42}$$

The CWPT, have the same order of the FFT and WPT computational complexity, i.e., it's of the form of [42, 43]:

$$O(N) \tag{43}$$

5.4. Dual-tree complex wavelet transform based system

Another version of the CWT, i.e., the dual-tree complex wavelet transform (DTCWT) was also proposed for the OFDM system [44-48]. As shown in Fig. 8, the DTCWT uses two DWTs, the upper part demonstrating the real portion while the lower part demonstrating the imaginary portion [49-53]. For the DTCWT decomposition, the real portion use slow, high pass filter h_0, h_1 respectively, and the imaginary portion uses low and high pass filter g_0 and g_1 respectively. On the other hand, for the DTCWT reconstruction, the real portion uses low and high pass filter $\tilde{h}_0$ and $\tilde{h}_1$ respectively and the imaginary portion uses low and high pass filter $\tilde{g}_0$ and $\tilde{g}_1$ respectively. The sequences of wavelet functions ψ(t) and the scaling functions ϕ (t) are defined using these filters as follows:

$$\psi_h(t) = \sqrt{2}\sum_n h_1(n)\varphi_h(2t-n) \quad \& \quad \varphi_h(t) = \sqrt{2}\sum_n h_0(n)\varphi_h(2t-n) \quad (44)$$

$$\psi_g(t) = \sqrt{2}\sum_n g_1(n)\varphi_g(2t-n) \quad \& \quad \varphi_g(t) = \sqrt{2}\sum_n g_0(n)\varphi_g(2t-n) \quad (45)$$

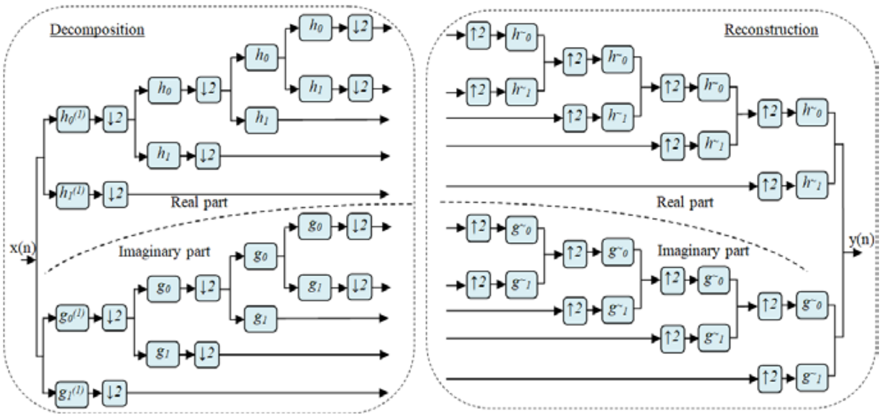

Fig. 8. Decomposition and reconstruction of the DTCWT.

As the cases of the conventional OFDM, MLT-OFDM, DCT-OFDM, DWT-OFDM, WPT-OFDM, and CWPT-OFDM, Fig. 5 shows the DTCWT based OFDM system. At the transmitter side, the transmitted signal *x(t)* is the output of the IDTCWT and the real part of the transmitted signal *x(t)* can be written as [54-58]:

$$R_e[x(t)] = \sum_k x_{ar_{1,k}} R_e[\varphi_{1,k}(t)] + \sum_k \sum_{j=2}^{N/2} x_{dr_{j,k}} R_e[\psi_{j,k}(t)] \quad (46)$$

The imaginary part of the transmitted signal *x(t)* can be written as:

$$I_m[x(t)] = \sum_k x_{ai_{N/2+1,k}} I_m[\varphi_{1,k}(t)] + \sum_k \sum_{j=N/2+2}^{N} x_{di_{j,k}} I_m[\psi_{j,k}(t)] \quad (47)$$

This gives that the total transmitted signal *x(t)* is:

$$x(t) = \sum_k x_{a_{j_0,k}} \varphi_{j_0,k}(t) + \sum_k \sum_j x_{d_{j,k}} \psi_{j,k}(t) \quad (48)$$

At the receiver side, the received signal *y(t)* is

$$y(t) = x(t) * h(t,\tau) = \int_{-\infty}^{\infty} h(\tau) * x(t-\tau) + w(t) \quad (49)$$

y(n) is the discrete version of the continuously received signal *y(t)* is given by:

$$y(n) = x(n) * h'(n) = \sum_k h'(k)x(n-k) + w(n) \quad (50)$$

Assume that the channel *h* is complex and can be written as:

$$H_h = H_{h_R} + jH_{h_I} \quad \& \quad H_g = H_{g_R} + jH_{g_I} \tag{51}$$

Now the received signal *y(k)* will be:

$$y(k) = \frac{1}{2}\left[\left(H_{h_R}x_R - H_{g_I}x_I\right) + j\left(H_{h_I}x_R - H_{g_R}x_I\right)\right] \tag{52}$$

Under perfect channel estimation, the output of the DTCWT can be written as:

$$a_k = 2\left[\frac{\left(H^T{}_{h_R}y_R + H^T{}_{h_I}y_I\right)}{\left(H^T{}_{h_R}H_{h_R} + H^T{}_{h_I}H_{h_I}\right)} + j\frac{\left(H^T{}_{g_R}y_I + H^T{}_{g_I}y_R\right)}{\left(H^T{}_{g_R}H_{g_R} + H^T{}_{g_I}H_{g_I}\right)}\right] \tag{53}$$

Since the DTCWT use two DWT (upper and lower parts), the computational complexity for DTCWT is [42, 43]:

$$O(2N) \tag{54}$$

The complexity of the DTCWT is in less order comparing to the complexity of FFT and WPT.

5.5. Curvelet transform based system

Although the DTCWT has solved some problems related to the real version of the wavelet transform, it still suffers from other problems like lack of directions and the redundancy. As a result, the curvelet transform (CvT) [59] was utilized in the OFDM system [60], mainly the fast version of the CvT has been used, i.e., the fast discrete curvelet transform via Unequispaced fast Fourier transform (FDCT-USFFT) and the fast discrete curvelet transform via wrapping (FDCT-Wrapping) [61].

Let W denote the forward CvT and $\widetilde{W}$ denote the inverse CvT. For the input signal *u(t)* the forward and the inverse transformation can be written as:

$$U(t) = W\{u(t)\} \quad \& \quad u(t) = \widetilde{W}\{U(t)\} \tag{55}$$

The forward and the inverse transformation of the FDCT via frequency wrapping based on the FFT is shown in Fig. 9. For the forward transformation, the input data are converted into FD through the FFT, then the FD curvelet coefficients are obtained at the window function output and then converted to the TD using the IFFT. The inverse transformation is the reverse processing of the forward transformation.

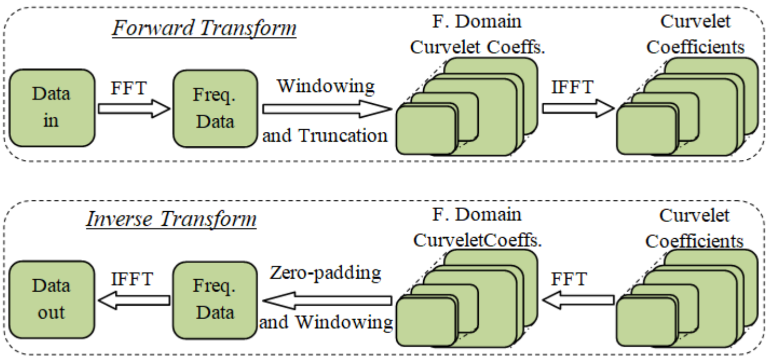

Fig. 9. Forward and Inverse FDCT via frequency wrapping.

As the cases of the above systems, Fig. 5 depicted the CvT based OFDM system. Let W is the forward CvT for the FDCT-USFFT or the FDCT-wrapping and $\widetilde{W}$ is the inverse of the transformation. Now the transmitted signal *x(n)* at the output of the inverse transform of the FDCT-USFFT or the FDCT-wrapping is [61]:

$$x(n) \; = \; \sum_{i=0}^{N} \widetilde{W}_i \; d_i(n) \tag{56}$$

At the receiver side, the received signal *y(n)* is described by:

$$y(n) \; = \; h \; (n) x(n) + w(n) \tag{57}$$

where *h(n)* is the channel response and *w(n)* is the discrete version of continues noise signal *n(t)*, the estimated received data a_i at the output of the forward transform of the FDCT-USFFT or the FDCT-wrapping is:

$$a_i(n) \;\; = \; \sum_{i=0}^{N} W_i\{y(n)\} \;\; = \;\; \sum_{i=0}^{N} W_i\{h(n)x(n)\} \; + \; \sum_{i=0}^{N} W_i\{w(n)\} \tag{58}$$

Assuming perfect channel estimation and absence of noise, in the output of the forward transform of the FDCT-USFFT or the FDCT-wrapping, the final output can be given by:

$$a_i(n) \;\; = \; \hat{d}_i(n) \; = \; \sum_{i=0}^{N} W_i \widetilde{W}_i\{d_i(n)\} \tag{59}$$

The computational complexity of both FDCT algorithms for computing N subcarriers evaluation of the Taylor polynomial is [61-64]:

$$O(N \log N) \tag{60}$$

Thus, it can be deduced that the complexity of the FDCT is of the same order compared by the complexity of Fourier.

6. Advantages/disadvantages of various transforms used in OFDM system

The OFDM system based on the FFT

It possesses some attractive properties such as high data capacity and high spectral efficiency because of overlapping spectra and simple implementation by FFT. On the other side, few drawbacks of the OFDM system based on the FFT are listed as follows: highly sensitive to timing and frequency offsets, high PAPR and high computational complexity ($O(N\log N)$) compared to the other transformation based system.

The OFDM system based on the MLT

They owns all the attractive properties of the OFDM system based on the FFT and outperform the OFDM system based on FFT in term of BER and ICI. Nevertheless, it has the same order of computational complexity ($O(N\log N)$) like the OFDM system based on the FFT.

The OFDM system based on the DCT

They holds all the attractive properties of the OFDM system based on FFT. In addition to that, the OFDM system based on the DCT gives better PAPR results and lower computational complexity ($O(N/2 \; \log N)$) compared to the above two type transformation based systems.

The OFDM system based on the DWT

It preserves all the attractive properties of the OFDM system based on the FFT. Moreover, the OFDM system based on DWT is more bandwidth efficient than an

FFT based OFDM system, no need CP, less sensitive to frequency offset, gives noticeable BER improvements and lower computational complexity ($O(N)$) compared to all the above type transformation based systems.

The OFDM system based on the WPT

It enjoys all the attractive properties of the OFDM system based on the FFT. Furthermore, better combat narrowband interference, inherently more robust with respect to ICI, no need CP compared to the above systems. But the OFDM system based on the WPT has some drawback like lack of shift invariance and high order of computational complexity ($O(N\log N)$) like an FFT and MLT based OFDM systems.

The OFDM system based on the CWPT

It keeps all the attractive properties of an FFT and WPT based OFDM systems. Additionally, an CWPT based OFDM system has a lower computational complexity ($O(N)$) like the OFDM system based on the DWT.

The OFDM system based on the DTCWT

It the presence of nonlinear power amplifier is better, and its BER performance is also better compared to the above types of transformations-based systems. But its shortcoming is that it has a higher computational complexity ($O(2N)$) compared to the DWT and the CWPT based OFDM systems.

The OFDM system based on the CvT

For the FDCT-USFFT and the FDCT-Wrapping recalls most of the gorgeous properties of the above OFDM systems. Besides, it has a better spectral efficiency and better PSD results. But the limitation of the OFDM system based on the CvT is that it has a higher computational complexity ($O(N\log N)$) like an the FFT, the MLT and the WPT based OFDM systems. Computational complexity of various transforms used in the OFDM is shown in Table 1.

Table 1. Computational complexity of various transforms used in the OFDM.

Name of Transformation	Computational complexity
Fast Fourier Transform (FFT)	$O(N \log N)$
Modulated Lapped Transform (MLT)	$O(N \log N)$
Discrete Cosine Transform (DCT)	$O(N/2 \log N)$
Discrete Wavelet Transform (DWT)	$O(N)$
Discrete Wavelet Packet Transform (DWPT)	$O(N \log N)$
Complex Wavelet Packet Transform (CWPT)	$O(N)$
Dual Tree Complex Wavelet Transform (DTCWT)	$O(2N)$
Fast Discrete Curvelet Transform via Unequispaced Fast Fourier Transform (FDCT-USFFT)	$O(N \log N)$
Fast Discrete Curvelet Transform via Wrapping (FDCT-Wrapping)	$O(N \log N)$

7. Conclusions

OFDM system considers as a promising and potential system for the communication systems, currently is adopted in various aspects of communication systems ranging from the wired communication system to the wireless communication system. The aim of this paper has been providing an overview of all the transformation techniques used in the OFDM system starting from the FT then, the LT and the CT then, the WT and ending to the CvT. And at the same time addressing the computational complexity of each type of the used transform. The results show that the FFT, MLT, WPT, and the CvT give the same order of the computational complexity of $O(N \log N)$, while the DCT gives the computational complexity of $O(N/2 \log N)$. on the other hand, the DWT, CWPT, and the DTCWT give the lowest order of the computational complexity which is given by $O(N)$.

To face the future challenges of the OFDM system in the field of communication systems, the door is still open for other new transformation techniques to enter to generate a new OFDM system capable of meeting the future requirements of the next generation of wireless communications.

Abbreviations

CP	Cyclic prefix
CT	Cosine transform
CWPT	Complex wavelet packet transform
CWPT-OFDM	Complex wavelet Packet transform based OFDM system
CvT	Curvelet transform
DCT	Discrete cosine transform
DFT	Discrete Fourier transform
DSL	Digital subscriber line
DWT	Discrete wavelet transform
DTCWT	Dual-tree complex wavelet transform
DCT-OFDM	Discrete cosine transform based OFDM system
DWT-OFDM	Discrete wavelet transform based OFDM system
FD	Frequency domain
FDCT	Fast discrete curvelet transform
FDCT-USFFT	Fast discrete curvelet transform via Unequispaced fast Fourier transform
FT	Fourier transform
FFT	Fast Fourier transform
ICI	Inter-channel interference, Inter-carrier interference
ISI	Inter symbol interference
IDCT	Inverse discrete cosine transform
IDWT	Inverse discrete wavelet transform
IFFT	Inverse Fast Fourier transform
IMLT	Inverse modulated lapped transform
LT	Lapped transform
MCM	Multicarrier modulation
MLT-OFDM	Modulated lapped transform based OFDM system
MTM	multi-tone modulation
OFDM	Orthogonal frequency division multiplexing
TD	Time-domain
USFFT	Unequispaced fast Fourier transform
MLT	Modulated lapped transform
WT	Wavelet transform
WPT	Wavelet packet transform
WPT-OFDM	Wavelet Packet transform based OFDM system

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