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ROLE OF '*e*' IN ENGINEERING APPLICATIONS

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Abstract

When elements or units are connected in a circuit or system whose individual output takes place at different time slots, then for the circuit or system, the input-output relation will suffer from continuous compounding. The output variable will continuously increase or decrease at a constant rate decided by the circuit or system parameters. Such circuits or system are specified by linear homogenous and non-homogeneous differential equations and the general solutions contain the term (s) e^{mx}. Each term in such differential equation basically specifies the input-output relation of elements in which the outputs are taking place at different time slots. Three examples are taken namely RC circuit, Newton's law of cooling and RLC circuit. The circuit or system equations are linear and hence continuous compounding at constant rate is taking place. When the equations are modified such that there is no definite constant time lag between the terms, then there is no continuous compounding at constant rate. That is, the solution will not contain the term(s) e^{mx}. The general solution of linear differential equation is taken as e^{mx}. The reason is not because the derivatives of first order, second order and so on gives me^{mx}, m^2e^{mx} and so on. Also, not because this pattern leads to a simple general solution of the form $e^{ax} + e^{bx}$. The term(s) e^{mx} is considered as solution, only because, in the circuits or systems which are represented by linear equation, the process of continuous compounding takes place.

Keywords: Complex sinusoids, Continuous compounding in RC circuit, Continuous compounding in RLC circuit, Continuous compounding in cooling process, Euler constant.

1. Introduction

The Euler number 'e' has eminent importance in mathematics, like 0, 1, π and i. All five of these numbers play important roles across mathematics, and these five constants appear in one formulation of Euler's identity.

The Euler number 'e' is not a mere number. It has physical meaning [1]. It represents continuous compounding. Continuous compounding means, continuous addition or subtraction to an initial value at some rate. In order to understand continuous compounding, we will see the compound interest formula. Examples of continuous compounding in engineering applications namely RC circuit, cooling process and RLC circuit are brought out in this paper.

1.1. Formula of compound interest

The well-known compound interest formula is given by,

$$A = P\left\{1 + \frac{r}{n}\right\}^{nt} \tag{1}$$

where A-Final amount, P-Initial principal, r- Interest rate (if % of interest is equal to 100% then, the rate of interest is 100/100=1), n -number of times interest is applied per time period (time period may represent one year or six months or even one day), and t-number of time periods elapsed.

1.2. Definition of 'e'

Let P, Principal =1; r, % of interest =100%; n, number of times interest is applied in one year =1; t, number of years elapsed =1. If we substitute in Eq. (1), then we get, A, the final amount,

$$= 1\left\{1 + \frac{1}{1}\right\}^{1*1} = 2$$

If the interest is applied every month in the year and added to the principal, then n=12. Substituting in Eq. (1),

$$A = 1\left\{1 + \frac{1}{12}\right\}^{12*1} = 2.613$$

If n times the interest is applied per year and added to the principal then, the final amount at the end of the year, $A = 1\left\{1 + \frac{1}{n}\right\}^{n*1}$

From the above, e is defined as [2, 3],

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{n} \tag{2}$$

Which means if compounding is continuous, that is $n \to \infty$, then at the end of the year,

$$A = e \tag{3}$$

1.3. Continuous compounding with constant compounding rate '*r*'

More generally, it is proved that under continuous compounding, that is interest is continuously added to the principal

$$A = Pe^{rt} \tag{4}$$

where, *A*- Final amount; *P*- Principal; *r*- interest rate for a time period; *t*- number of time periods elapsed.

Applying the concept to engineering, in a system, if a variable of initial value *P(0)* is continuously compounded by a process at the rate of '*r*' per second, then after '*t*' seconds the value of variable will be,

$$P(t) = P(0)e^{rt} \tag{5}$$

Let *P(0)* is compounded at every pico-second, that is $n = 10^{12}$. Let r = 20%, which means $r = 0.2$.The value of *P(t)* after one second

$$= p(0)\left\{1 + \frac{1}{10^{12}}\right\}^{(10^{12}*0.2*1)}$$

1.4. Continuous compounding at imaginary rate

Comparing with $A = e^{rt}$withe^{jt}, the process rate is '*j*' per second. The process rate is$1\angle 90°$ per second, that is, the process rate is 100% per second but in perpendicular direction. Continuous compounding takes place at the compounding period (say the compounding period is one pico- second) and increase the initial vector at the rate of 100% per second. After one pico- second the increase in magnitude will be

$$= 1 * \left\{1 + \frac{1}{10^{12}}\right\}^{(10^{12}*1*10^{-12})} - 1 = 10^{-12}$$

But the increase is in the perpendicular direction. Continuous compounding in the perpendicular direction does not increase the length but will rotate the initial vector [4].

1.5. Continuous compounding at complex rate

$$A = e^{a+ib} = e\hat{}a.e\hat{}ib \tag{6}$$

In the term e^{a}, the rate is real. It specifies that there will be continuous compounding of length of the initial unity vector, at the rate of 100% per second, for '*a*' seconds.

Let after the real part is continuously compounded for '*a*' second, imaginary compounding at the rate of 100% takes place for '*b*' seconds. It rotates the vector by an angle of '*b*' radians [4].

1.6. Complex sinusoids

Continuous compounding at imaginary rate results in circular motion when viewed in two dimensions. In three dimensions, when time is taken in the *z* plane, a single circular motion will generate helical motion and is said to be complex sinusoid [5]. The projection on X-plane that is real part vs. time, is a cosine waveform. The projection on Y-plane that is imaginary part vs. time is a sine waveform.

1.7. Application of 'e' in engineering – examples

In all these examples, the process of cumulative compounding is not brought out explicitly. But the input-output relationship involves the term 'e'. These examples are selected to show that the process of continuous compounding takes place in all the domains of engineering.

Exponential (Cumulative Compounding) smoothing model [6] is one of the main forecasting models of power system. An algorithm of Newton-Raphson method is used to analyse the parameters of the complex exponential signal [7]. The stability analysis for neural networks with time-varying delay is studied [8]. By using functional differential equations, results about exponential stability criteria, which are delay-dependent, are given.

The exponential window function having two independent parameters is used for the design of prototype FIR filter of the filter banks [9]. The simulation results show that the filter bank designed by the proposed method has better design performance in terms of the amplitude and aliasing errors. Omran et al. [10] dealt with comparing different estimators of the two parameters of general failure exponential model as well as estimating reliability function.

In order to describe the failure and life rule of avionics more truly and to improve the accuracy and efficiency of reliability analysis, Yang [11] proposed a reliability data analysis method based on exponential distribution. Based on the district – based domestic solid waste data of Istanbul Metropolitan Municipality between 2004 – 2019, Akgul et al. [12] made the estimation of the domestic waste amounts for 2020 by exponential smoothing.

2. Description of Proposal

2.1. Physical meaning of e^{mx} in the solution of differential equations

Consider the equation,

$$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy=0 \tag{7}$$

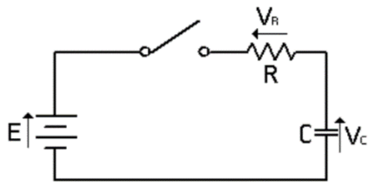
where p and q are constants. It is a linear homogeneous second order differential equation. The general solution of the equation contains the terms e^{mx}, where m is a constant.

It means that in all the systems which is described by the above equation, continuous compounding takes place. The general solution of linear differential equation is taken as e^{mx}. The reason is not because the derivatives of first order, second order and so on gives me^{mx}, m^2e^{mx} and so on. Also, not because this pattern leads to a simple general solution of the form$e^{ax} + e^{bx}$. The term(s) e^{mx} is considered as solution, only because, in the circuits or system which are represented by linear equation, the process of continuous compounding takes place.

2.1.1. Explanation of continuous compounding in RC charging circuit

Let a dc voltage source is applied to an RC circuit as shown in Fig. 1. The equation of the circuit [13] is given by the Eq. (8),

$$iR + \frac{1}{c}\int idt = E \tag{8}$$



The input to resistor is current 'i' and the output is the voltage across the resistor, $V_R = iR$. Similarly, the input to the capacitor is current and the voltage across the capacitor, $V_C = \frac{1}{c}\int i dt$. Each term basically represents the input output relation of the components, but the outputs occur at different time slots. Differentiating Eq. (8),

$$R\frac{di}{dt} + \frac{i}{c} = 0 \tag{9}$$

The first term lag by a time interval of Δt, with respect to second term, which results in continuous compounding. The solution of Eq. (9) is given by,

$$i(t) = \frac{E}{R} e^{\frac{-t}{RC}} \tag{10}$$

As expected, *i(t)* is continuously compounded with the rate of compounding equal to $\frac{-1}{RC}$.

When the switch is closed the capacitor cannot be charged instantaneously to E. Due to reactive component that is due to capacitor, there will be a time delay. Due to this time delay, continuous compounding takes place, which decides the charging current as detailed in the 'signal flow', mentioned below.

Signal flow during continuous compounding in RC circuit

Switch closed -- maximum charging current *Im* flows because capacitor acts as a short-- capacitor is getting charged and acts as a battery with small voltage -- sets a current in opposite direction -- charging current reduces -- reduced charging current charges the capacitor -- capacitor voltage increases which further reduces the charging current -- the compounding continues till the capacitor is fully charged to the supply voltage E -- charging current reduces to zero.

2.1.2. Explanation of continuous compounding in Newton's law of cooling

Newton's law of cooling [14] is given by Eq. (11),

$$dT/dt = -k\,(T - T_S) \tag{11}$$

where, T = temperature at time t and T_s = temperature of the surrounding, k = Positive constant that depends on the area and nature of the surface of the body under consideration.

The term on the left side lag by a time interval of Δt, with respect to the term on the right side, which results in continuous compounding.

The solution of Eq. (11) is given by,

$$T(t) = T_s + (T_o - T_s)\, e^{-kt} \quad (12)$$

where T_o is the initial temperature of the hot body.

As expected, $T(t)$ is continuously compounded with the rate of compounding equal to $-k$.

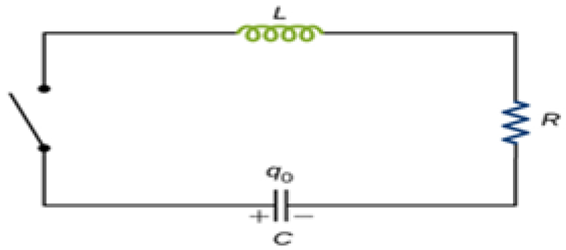
Signal flow during continuous compounding in cooling

The signal flow in continuous compounding is detailed below.

Temperature difference between body and surroundings is maximum – maximum radiation takes place – body temperature falls a bit –the difference in temperature $(T - T_S)$ falls a bit– radiation falls a bit – continuous compounding continues till the body temperature falls to the surrounding temperature.

2.1.3. Explanation of continuous compounding in RLC circuit

Consider the RLC circuit which is shown in Fig. 2



The equation governing the RLC circuit [15], is given by Eq. (13),

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = 0 \quad (13)$$

The input to resistor is current 'i' and the output is the voltage across the resistor, $V_R = iR$. Similarly, the input to the inductor is i and the output is the voltage across the inductor, $V_L = L\frac{di}{dt}$ and the input to the capacitor is current and the voltage across the capacitor, $V_C = \frac{1}{C}\int idt$. Each term basically represents the input - output relation of the components but the outputs occur at different time slots. Differentiating Eq. (13),

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = 0 \quad (14)$$

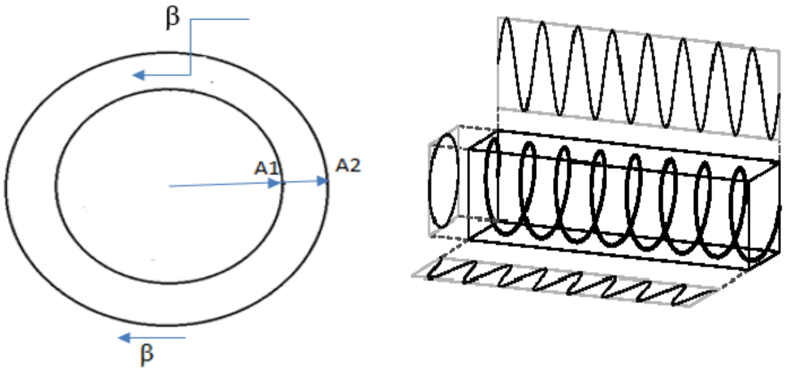
In Eq. (14), the first term lag by a time interval of $2\Delta t$, with respect to third term, which results in continuous compounding. The second term lag by a time interval of Δt, with respect to third term which also will result in continuous compounding. The rates of continuous compounding will be real and different if the roots of the auxiliary equation are real and different. The rate of continuous compounding will be real and same if the roots are real and same. The rates of continuous compounding will be complex conjugates if the roots of the auxiliary equation are complex conjugates.

When the roots are complex conjugates, the general solution will be,

$$i = A1e^{(-\alpha+j\beta)t} + A2e^{(-\alpha-j\beta)t} \quad (15)$$

$$= e^{-\alpha t}\left(A1e^{j\beta t} + A2e^{-j\beta t}\right)$$

The terms in the bracket, generates two complex sinusoids of amplitude A1 and A2 and rotates with the angular velocity of β for a time period of t. In two dimensions, the complex sinusoids are shown as circular motion in Fig. 3(a). In three dimensions, the complex sinusoids will generate helical motion with respect to time as shown in Fig. 3(b). However, the amplitudes will be attenuated due to continuous compounding by the real term -α.



(a) Two dimensions **(b). Three dimensions.**

Fig. 3. Complex Sinusoids

If we substitute, $\alpha = \frac{-R}{2L}$, due to initial conditions (when $t = 0, v_0 = \frac{q_0}{c}$) the real part of the complex sinusoids (cosine terms) vanishes and the solution of the equation is given by,

$$i(t) = \frac{V_0}{\beta L} e^{\frac{-R}{2L}t} (-j\sin(\beta t)) \quad (16)$$

where, V_0 is the initial voltage across the capacitor, $\frac{-R}{2L}$ is the rate of continuous compounding, β is the under damped frequency of oscillation.

Switch closed-- capacitor starts discharging -- sets up changing magnetic field in the inductor -- changing magnetic field develops a voltage across the inductor -- sets up opposite current -- reduces the original current -- under the action of continuous compounding the net current at every instant, sets up increasing magnetic field in the inductor, reducing electric field in the capacitor.

At some point the capacitor is completely discharged --all the energy is now stored as magnetic field in the inductor, excluding the dissipation of energy in the resistor -- again, the energy is transferred from inductor to capacitor under continuous compounding.

3. Results and Discussion

The results are obtained using MATLAB and discussed to prove that the process of continuous compounding is possible only in linear equations.

3.1. Continuous compounding in RC circuit

Basically, each term in the Eq. (8), represents the input-output relation of each element in the circuit. The basic equation multiplied by constant or differentiating the equation or integrating the equation, the time lag of the output of components is not going to be altered. If the output of a component can be arrived only after a time interval of Δt there will be definite continuous compounding at constant rate. In Eq. (9), the first term clearly lags behind the second term by Δt. Hence in Fig. 4, continuous compounding at constant rate takes place.

3.1.1. M File for solution and plotting of RC circuit equation

The M File for solving Eq. (9), with R = 1 KΩ, C = 1 μf , E = 10 V and $i(0) = 0.01$ (10 V/1000 Ω) is detailed below.

```
% M File for simulation of RC circuit
symsi(t)% create symbolic variable i(t)
Di = diff(i);% Di = di/dt
ode = 1000*Di + 1e6*i == 0;%ode  = 1000 di/dt + 10^6 i = 0
iSol = dsolve(ode, i(0) == 0.01); % solve the differential
%equation with initial condition i(0) = 0.01
ezplot(iSol,[0,7e-3])% plot the variable iSol=i(t)
%for t=0 to 7*10^-3
```

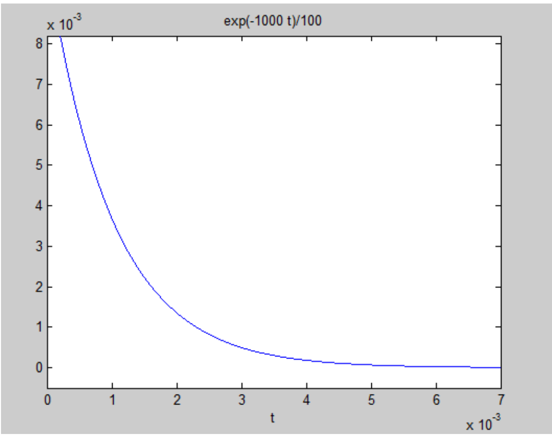


Fig. 4. Continuous compounding in RC circuit.

The solution of Eq. (9) as obtained from MATLAB is $(t) = \frac{1}{100}e^{-1000t}$. The result is as per our expectation that continuous compounding has taken place with rate of -1000 amp/s.

3.1.2. M File for solution and plotting of imaginary RC circuit equation

Let us consider an imaginary capacitor, in which the voltage across the capacitor is, $\frac{1}{c}\int itdt$.Then the equation of RC circuit is a non-linear equation,

$$iR + \frac{1}{c}\int itdt = E \tag{17}$$

Differentiating the Eq. (17),

$$R\frac{di}{dt} + \frac{it}{c} = 0 \tag{18}$$

We cannot derive the plot of term1(t Vs Rdi/dt), from the plot of term 2 (t Vs it/c). This is possible only when the term 2 lags behind term1 by finite time interval. Hence, in a system which is represented by a non-linear equation continuous compounding at constant rate cannot take place.

% M File for simulation of imaginary RC circuit

```
symsi(t)% create symbolic variable i(t)
Di = diff(i);% Di = di/dt
ode = 1000*Di + 1e6*i*t == 0;%ode = 1000di/dt + 10^6it = 0
iSol = dsolve(ode, i(0) == 0.01); % solve thedifferential
%equation with initial condition i(0) = 0.01
ezplot(iSol,[0,7e-3])% plot the variable i(t)
%for t=0 to 7*10^-3
```

Figure 5 shows the input-output relation of the circuit, when the second term of the Eq. (18) is multiplied by *t*. The first term is not lagging by Δ*t* with respect to second term. Hence, there is no continuous compounding at a constant rate. However, in this particular case, continuous compounding at variable rate, proportional to time has taken place [16]. The output as given by the MATLAB solution is,

$$i(t) = \frac{1}{100}e^{-500t^2} \tag{19}$$

The rate of variable continuous compounding is -500 *t*.

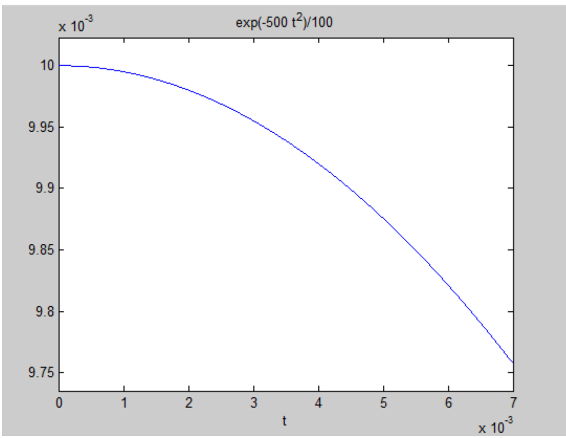


Fig. 5. Input-Output relation of RC circuit with imaginary time dependant capacitor.

3.2. Continuous compounding in cooling process

M File for solution and plotting of Newton's cooling equation

The M File for solving Eq. (11), with T_S = 34°C and k = 0.1 is given below

```
% M File for simulation of cooling process
Syms T(t)    % create symbolic variable T(t)
DT = diff(T);     % DT = dT/dt
TS = 34
k = 0.1
ode = DT+k*(T - TS)==0;
TSol = dsolve(ode, T(0) == 1000) % solve the
% differentialequation with initial condition when t = 0, T = 1000
ezplot(TSol,[0,100])% T vs t for t ranging from 0 to 100
```

The simulated output of the cooling process is shown in Fig. 6.

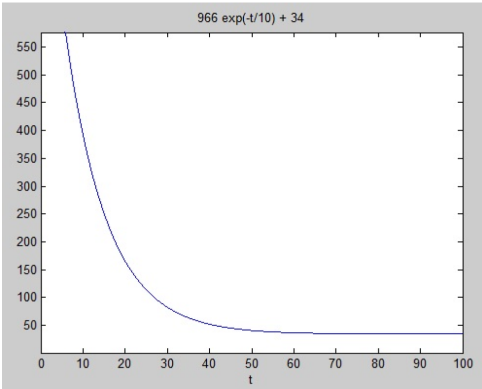


Fig. 6. Hot body temperature with respect to time.

3.3. Continuous compounding in RLC circuit

3.3.1. M File for solution and plotting of RLC circuit equation

The M File for solving Eq. (14), with R = 2 Ω, L= 1 H, C = 1/5 f and $i(0) = 0, \frac{di}{dt}(0) = 10$ is detailed below.

```
% M File for simulation of RLC circuit
symsi(t)% create symbolic variable i(t)
Di = diff(i);% Di = di/dt
D2i = diff(i,2);% D2i = d^2i/dt^2
ode = D2i + 2*Di + 5*i == 0;% ode = d^2i/dt^2 + 2 di/dt + 5i = 0
iSol = dsolve(ode, i(0) == 0, Di(0) == 10)% solve the
%differentialequation with initial conditioni = 0 , di/dt = 10
ezplot(iSol,[0,10])% plot the variable i(t) for t=0 to 10
```

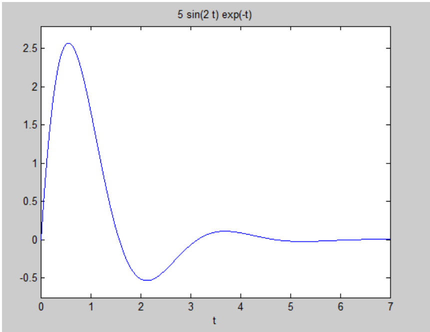


Fig. 7. Continuous compounding in RLC circuit.

3.3.2. M File for solution and plotting of imaginary RLC circuit equation

Consider an imaginary capacitor as defined in Section (3.1.2). The M file to plot *i*(*t*) is given below.

```
% M File for simulation of imaginary RLC circuit
symsi(t)% create symbolic variable i(t)
Di = diff(i);% Di = di/dt
D2i = diff(i,2);% D2i = d^2i/dt^2
ode = D2i + 2*Di + 5*i*t == 0;% ode = d^2i/dt^2 + 2 di/dt + 5it = 0
iSol = dsolve(ode, i(0) == 0, Di(0) == 10))% solve the
%differentialequation with initial conditioni = 0 , di/dt = 10
ezplot(iSol,[0,10])% plot the variable i(t)for t=0 to 10
```

As seen from Fig. 8, there is no continuous compounding in the RLC circuit with time dependant imaginary capacitor. The first few terms, out of number of terms of the MATLAB solution is given in Eq. (20). There is no exponential term in the solution which means that there is no continuous compounding.

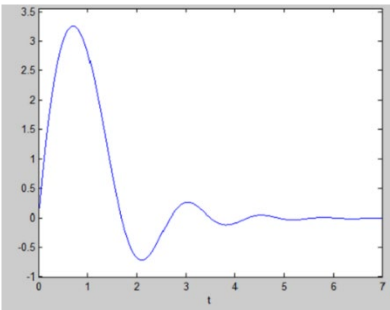


Fig. 8. Input-Output relation of RLC circuit with imaginary time dependant capacitor.

3.3.3. Comparison of considered applications

In all the above applications, the compounding is negative. The negative compounding takes at different rates. In the applications where the compounding is positive, the system will be saturated at some point and will become non-linear. The comparison of the considered applications is shown in Table 1.

Table 1. Comparison of considered applications.

S. No.	Application	Sign of Compounding	Rate of Compounding
1.	RC Circuit	Negative	$-\frac{1}{RC}$
2.	Cooling Process	Negative	-k
3.	RLC Circuit	Negative	$-\frac{R}{2L}$

4. Non-Linear System – Pendulum–Non-Existence of Continuous Compounding

In the case of systems which are described by non-linear differential equations, the process of continuous compounding cannot exist.

An example of non-linear problem is the dynamics of pendulum under the influence of gravity. The non-linear equation describing the system is given by

$$\frac{d^2\theta}{dt^2} + \sin\theta \quad = 0 \tag{21}$$

The first term is not lagging by Δt or by $2\Delta t$ with reference to the second term. Hence, continuous compounding cannot take place in non-linear systems.

M File for the solution and plotting of pendulum equation

symstheta(t)% create symbolic variable theta(t)
Dtheta = diff(theta);% $D\theta = \frac{d\theta}{dt}$
D2theta = diff(theta,2);% $D2\theta = \frac{d^2\theta}{dt^2}$
ode = D2theta + 2*Dtheta + 5*theta*t == 0;
% ode $=\frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt} + 5\theta t = 0$
thetaSol = dsolve(ode, theta(0) == 0, Dtheta(0) == 10)
% solve thedifferentialequation with initial 4623ondition = 0 , $\frac{d\theta}{dt} = 10$
ezplot(thetaSol,[0,10])
% plot the variable θfor t=0 to 10

There is no exponential term in the solution, written at the top of Fig. 9. Hence there is no continuous compounding in the case of pendulum system which can be seen from Fig. 9, i.e., if we join the peaks of the curve in Fig. 9, exponential decay cannot be obtained.

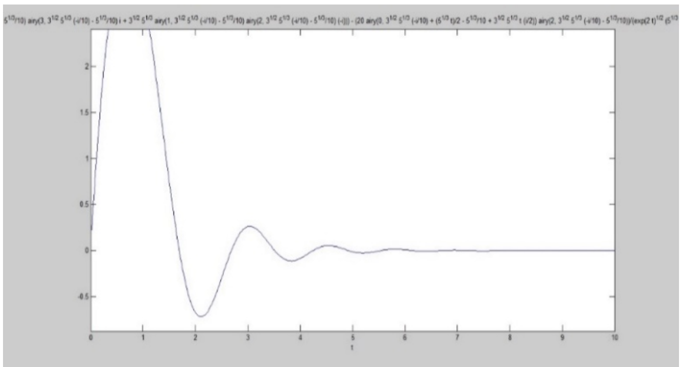


Fig. 9. Time and angle relation in pendulum equation.

5. Conclusion

When a circuit or system is expressed by a linear differential equation, then the circuit or system will suffer by continuous compounding, since each term basically lags behind the other terms. The reason is that when different units are connected in a system, the individual unit's output takes place at different time slots. In most of the applications the actual process of continuous compounding between the units are not identified and normally satisfied with the mathematical model of the system.

In the future, it is proposed to take up the systems which have solutions containing '*e*' and establish the physical continuous compounding process taking place in the systems.

Nomenclatures

E	DC source voltage, V
i	Current, Amp
I_m	Maximum charging current, Amp
K	Positive constant
n	Number of times interest is applied per time period
P	Initial principal
r	Interest rate
t	Number of time periods elapsed
T	Temperature at time t
T_S	Temperature of the Surrounding
V_0	Initial voltage, V
V_C	Voltage across capacitor, V
V_L	Voltage across inductor, V
V_R	Voltage across resistor, V

Greek Symbols

α	Rate of continuous compounding, amp/s
β	Under damped frequency of oscillation, Hz
θ	Angle of deflection, deg.

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