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FAULT DETECTION FULL ORDER FILTER APPLY TO DISCRETE TIME-INVARIANT LINEAR SYSTEM

MASOOD AHMAD, ROSMIWATI MOHD-MOKHTAR*

School of Electrical and Electronic Engineering, Universiti Sains Malaysia,
Engineering Campus, Nibong Tebal, 14300 Pulau Pinang, Malaysia
*Corresponding Author: eerosmiwati@usm.my

Abstract

In this paper, a full order fault detection filter (FDF) is designed for a linear discrete time-invariant system subjected to unknown inputs and uncertainty. The problem in fault detection filter design is derived as $H\infty$ optimization based on the linear matrix inequality (LMI) approach. The proposed observer not only consider the unknown inputs but also system uncertainty as to distinguish the present approach from the previous approaches. The performance of the designed FDF is evaluated by detecting different types of sensor faults in the linear model of DC motor through simulations in MATLAB / Simulink. Results show that the proposed robust fault detection filter has successfully detected all the faults irrespective of unknown inputs and system uncertainties.

Keywords: Fault detection, $H\infty$ optimization, Residual, Uncertain systems.

1. Introduction

Reliability, safety, environmental protection, and desired performance are the foremost design specifications of every engineering system. In this regard, several state-of-the-art control systems have been developed by researchers and manufacturers to ensure their optimal performance for attaining the predetermined goals. However, any abnormal behaviour / fault in the sensor, actuator, and system components reduces the overall system performance and, even in some cases, leads to a hazardous situation both in terms of human mortality and financial loss. Therefore, it is imperative need to detect and locate the abnormal behaviour that arises in the system at a very early stage.[1-3].

The objective of the fault detection system is to indicate the occurrence of a fault in the system and its time of occurrence. Fault detection methods are generally divided into two main streams: data-driven methods [4] and model-based methods [5]. Data-driven methods are adopted when system dynamics could not model mathematically due to insufficient knowledge of system behaviour. Contrary, model-based fault detection (FD) methods are preferred when sufficient knowledge of system dynamics is available and could be represented in mathematical equations. Model-based techniques have replaced the hardware redundancy with a system mathematical model/analytical redundancy. In this technique, residual is generated which depicts the change in real system output and system model output [6]. Several model-based methods such as output observer [7], parity relation [8], and parameters estimation methods [9] and references therein, are investigated to generate the good residual, zero in normal operation.

Among model-based fault detection methods, the observer-based FD technique for the dynamic system has received much attention from researchers. The core idea of an observer-based FD system, so-called FDF, is to obtain residual from actual measurement of the system and then compare the evaluated residual with the threshold. An alarm is released if the evaluated residual exceeded the threshold [5]. Good residual tends to either perfectly decouple the unknown input or establish an appropriate agreement between robustness to the unknown inputs and the simultaneously high sensitivity to the fault, which is the prime objective of model-based FD schemes [10]. To obtain a good residual, initial research was devoted to an unknown input decoupling problem [11, 12] followed by the optimization techniques in the 1990s to design an optimal observer using LMI formulation [13, 14]. A unified solution is also developed for multi-objective optimization problems for a linear system in [15] by solving the Riccati equation.

Abundant research has been reported for the observer design for an ideal system model assuming only the unknown inputs, see references [1-3, 6-15]. Most of the reported work, so far, either deal with unknown inputs or uncertainties and limited to continuous-time systems with the prime focus on robust fault detection system design. On the other hand, a perfect mathematical model is never available because of the modelling error, linearization, and component aging issues in the practical system. Owing to these reasons, the above results cannot generate an optimum solution. The uncertainty in the system model produces biasness in residual. This somehow can be misinterpreted as a sensor or actuator fault in the system. Therefore, it is vital to consider these effects in the design of fault detection systems.

In this manuscript, a robust FDF (RFDF) design for a linear discrete-time invariant system with norm-bounded uncertainty and unknown inputs is proposed. The obtained

FDF is synthesized as a robust residual generator to minimize the H_∞ norm of the transfer function from disturbance to residual using LMI formulation in presence of uncertainty and unknown inputs (UIs). Also, in the last stage, the adaptive threshold approach is employed for successful fault detection. The effectiveness of the proposed FDF is demonstrated by detecting the sensor fault in the DC motor model.

The remaining of the paper can be stated as follows: Section 2 discusses works and literature related to the study. Section 3 is dedicated to the preliminaries and problem formulation. Section 4 presents proof of the main theorem proposed for RFDF. Sections 5 demonstrates the residual evaluation and threshold computation. Section 6 illustrates the derived results obtained from the simulation results of the DC motor. Concluding remarks are given in Section 7.

2. Related Works

In real scenarios, a physical system is affected by unknown disturbances in different parts of the system such as process and measurements. The unknown disturbance changes the performance characteristics of the system, even in fault-free cases, and as a result, the behaviour of the real system varies from the nominal model. The discrepancy in system behaviours cause the residual to deviate from zero. In model-based fault detection methods, fault detection is carried out by robust residual generation in which residual is robust against all undesired inputs such as process and measurement disturbance. Ideally, unknown input should perfectly be decoupled and do not influence the residual generation process. Initially, it was proposed to decouple the unknown inputs from the state estimation process using the disturbance distribution matrix. If the states are decoupled from the unknown inputs then residual is, obviously, also independent [16, 17].

In literature, unknown input observers, Eigenstructure assignment approach, and geometric approach has been adopted for disturbance decoupling from residual generation process [18]. However, in most practical scenarios, the realization of the perfectly decoupling of unknown inputs, because of rigorous existing conditions, is not possible [5]. An alternate strategy, widely adopted, is to design an observer to make a suitable compromise between the robustness to unknown inputs and the sensitivity to the faults. This makes the FD problem, a multi-objective design problem. For this purpose, H_∞ norm represents the maximum influence of disturbance on the residual signal and is widely used to improve the robustness of residual against the unknown inputs [19, 20]. The observer-based FD system in [21] is designed to detect the current sensor fault in an induction motor drive. A differential geometric approach is used in this paper to detect and isolate the single and multiple faults, i.e., disconnection, offset, and constant gain faults. The prime objective of all the above results is to meet the robustness criterion with application to linear systems subjected to the unknown disturbance only.

In the last few decades, it has been observed that linear matrix inequality is used for solving the H_∞ optimization problem in robust observer design. To handle the uncertainty, robust observer-based FDF is designed for a linear time-invariant system subjected to disturbances and modelling errors in [22]. The robust FD problem is formulated as an H_∞ model matching problem and the solution of the optimization problem is given in linear matrix inequality form in the said paper. Li et al. [23] extended the same work discussed in [15] for a continuous-time linear uncertain system subjected to polytopic uncertainty utilizing the iterative LMI approach.

Ahmad and Mohd-Mokhtar [24] designed the H- index fault-sensitive FDF for an uncertain system using the LMI framework to detect the sensor fault in the DC motor model. The proposed filter improved the fault sensitivity of the residual rather than attenuating the effect of unknown inputs. Proportional integral observer or PIO is formulated by Farhat and Koenig [25] as a multi-objective optimization problem for the continuous-time uncertain linear system and minimized the H_∞ norm in the LMI framework. The proposed PIO has ensured the minimum effect of disturbance and uncertainties to residual.

According to the best of the authors' knowledge, despite many nice results for robust fault detection of a linear system with disturbance only, there is, still, scarcity of solutions that can be applied to linear discrete-time systems with system uncertainties. Motivated by the above discussion and the indispensable need for robust fault detection for uncertain systems, this contribution proposed a solution for robust fault detection system design. The problem is formulated as an H_∞ optimization problem for a linear discrete time-invariant system with normed bounded system uncertainty and deterministic unknown disturbances.

Notations:

For matrix A, $A^T$and A^{-1} stand for the transpose and inverse of A, respectively. $A > 0$ $(A < 0)$ denotes A is positive (negative) definite. I represents identity matrix and 0 represents zero matrix, both with appropriate dimensions, respectively. $d(k) \in l_2[0, N]$ means $\sqrt{\sum_{k=0}^{N} d^T(k)d(k) < \infty}$ where N is a positive integer. The H_∞ norm of a transfer function $G(s)$ is denoted by $\|G(s)\|_\infty = {}^{sup}_{\theta}\, \sigma_{max}(G(e^{j\theta}))$ where σ_{max} denote the largest singular value of $(\,.\,)$.

3. Problem Formulation

Consider the linear discrete-time uncertain system is represented by

$$x(k+1) = (A + \Delta A)\, x(k) + (B + \Delta B)\, u(k) + E_d\, d(k) + E_f\, f(k)$$

$$y(k) = (C + \Delta C)\, x(k) + (D + \Delta D)\, u(k) + F_d\, d(k) + F_f\, f(k) \tag{1}$$

where $x(k) \in R^n, u(k) \in R^p$ and $y(k) \in R^m$ denote the state vector, control input, and measured output vector respectively. Furthermore, $d(k)$ and $f(k)$ are l_2 norm bounded unknown input, i.e.,$\|d(k)\|_2 \leq \delta_d$, and a fault signal to be detected. $(A, B, C, D, E_d,\ E_f,\ F_d,\ F_f)$ are known matrices with compatible dimensions. $(\Delta A, \Delta B, \Delta C, \Delta D)$ are additive-type, norm bounded uncertainties given as

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} H_1 \sum(t) G_1 & H_1 \sum(t) G_2 \\ H_2 \sum(t) G_1 & H_2 \sum(t) G_2 \end{bmatrix}$$

Where $\sum(t)$ is a time-varying unknown parameter but bounded with a condition that $\sum^T(t)\sum(t) \leq I$. Throughout the paper, 2 assumptions are referred to in which

A1: (C, A) is observable.

A2: $\begin{bmatrix} A - e^{j\theta} I & E_d \\ C & F_d \end{bmatrix}$ has full row rank, while $\theta \in [0,2\pi]$.

Observer-based FDF can be represented by the following equations

$$\begin{cases}\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L\big(y(k) - \hat{y}(k)\big) \\ \hat{y}(k) = C\hat{x}(k) + Du(k) \\ r(k) = y(k) - \hat{y}(k)\end{cases} \quad (2)$$

$\hat{x}(k) \in R^n$ and $\hat{y}(k) \in R^m$ are state estimation vector and estimated output vector, respectively. $r(k)$ is named as a residual signal. The design parameter of FDF is represented by L. The dynamics of said filter is described by a new vector, i.e., state estimation error vector

$$e(k) = x(k) - \hat{x}(k)$$

$$e(k+1) = (A - LC)e(k) + (\Delta A - L\Delta C)x(k) + (\Delta B - L\Delta D)u(k) + (E_d - LF_d)d(k) + \big(E_f - LF_f\big)f(k) \quad (3)$$

$$r(k) = Ce(k) + \Delta Cx(k) + \Delta Du(k) + F_d d(k) + F_f f(k) \quad (4)$$

Due to uncertainty, $(\Delta A, \Delta B, \Delta C, \Delta D)$, and unknown input, $d(k)$, affecting the estimation process, thus, residual is sensitive not only to a fault but unknown input, control input, and state of the system as well. From Eqs. (1) and (3), it is clear that $e(k)$ and $x(k)$ demonstrate the dynamics of an observer and the dynamics of an uncertain system, respectively. Moreover, $d(k)$ and $u(k)$ are inputs of the observer that deteriorate the performance of an observer.

Assuming fault free case ($f(k) = 0$), the dynamics of Eq. (3) is governed by introducing a new vector

$$\bar{x}(k) = \begin{bmatrix} e(k) \\ x(k) \end{bmatrix} \text{ and } \bar{u}(k) = \begin{bmatrix} u(k) \\ d(k) \end{bmatrix}$$

The new system is represented as

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}\bar{u}(k)$$

$$r(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k) \quad (5)$$

where

$$\bar{A} = \begin{bmatrix} A - LC & \Delta A - L\Delta C \\ 0 & A + \Delta A \end{bmatrix} \text{ and } \bar{B} = \begin{bmatrix} \Delta B - L\Delta D & E_d - LF_d \\ B + \Delta B & E_d \end{bmatrix}$$

$$\bar{C} = [C \quad \Delta C] \text{ and } \bar{D} = [\Delta D \quad F_d]$$

To establish a robust FDF design problem as H_∞ optimization problem, the following performance index is considered in this paper [26]:

$$\min_L \|G_{r\bar{u}}\|_\infty \quad (6)$$

According to the authors' best knowledge, only a few results are available in the literature for solving Eq. (6) for the discrete-time uncertain system.

Robust FDF in Eqs. (2)-(4) is articulated as H_∞ minimization problem, in which, to identify the observer gain, L such that matrix $(A - LC)$ is asymptotically stable and the performance index as in Eq. (7) is small.

$$\min_L \sup_{\bar{u}, \Delta} \frac{\|r(k)\|_2}{\|\bar{u}\|_2} \quad (7)$$

Thus, the main objective of work in this paper is to design an observer based FDF in such a way that the obtained residual is as robust as possible to unknown inputs and model uncertainties.

3.1. Residual evaluation and decision logic

After obtaining the good residual, the rest of the task in the model-FD scheme is to evaluate the residual and to compute the threshold. In an ideal case, the residual signal should be zero when there is no fault. However, in practice, process disturbance, measuring noise, and model uncertainty causes the residual to deviate from zero in the fault-free case. To prevent the FD system from generating the false alarm and inferring the existence of a fault, residual evaluation, and threshold settings are used.

For residual evaluation purposes, a norm-based evaluation function is considered in this paper [5].

$$J(k) = \|r(k)\|_2 = \sqrt{\textstyle\sum_{k_1}^{k_2} r^T(k) r(k)} \tag{8}$$

and the threshold is defined as

$$J_{th} = \sup_{\Delta, d \neq 0, f=0} \|r(k)\|_2 \tag{9}$$

Decision logic for fault detection is taken as

$$J \geq J_{th} \text{ ; fault alarm}$$
$$J < J_{th} \text{ ; fault-free} \tag{10}$$

4. Robust Fault Detection Filter

The important property of LMI is that it can deal with uncertainty. Therefore, the LMI approach is used to design the robust FDF in Eq. (2) for an uncertain LTI system.

Theorem:

For a discrete-time uncertain system in Eq. (1) and FDF in Eqs. (3)-(5) considering that assumptions (A1)-(A2) are satisfied. Given $\gamma > 0$, if there exist a scalar $\varepsilon > 0$, matrices $P_1 > 0, P_2 > 0, P_3 > 0$ and X_1, X_2 hold for the following LMI, then FDF is asymptotically stable and H_∞ performance index in Eq. (7) is satisfied.

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & {H_2}^T & {H_3}^T P & 0 \\ 0 & -P & 0 & {C_0}^T & {A_0}^T P & {\underline{G_1}}^T \varepsilon \\ 0 & 0 & -\gamma^2 I & {D_0}^T & {B_0}^T P & {\underline{G_2}}^T \varepsilon \\ H_2 & C_0 & D_0 & -I & 0 & 0 \\ PH_3 & PA_0 & PB_0 & 0 & -P & 0 \\ 0 & \varepsilon \underline{G_1} & \varepsilon \underline{G_2} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0$$

Furthermore, the observer gain matrix can be determined by $L = {P_1}^{-1} X_1$. RFDF can be obtained by repeating the above theorem in MATLAB LMI Toolbox until γ is minimized. The following lemma will be used to prove the above theorem, so it is presented first [5].

Lemma 1:

If there exists an arbitrary positive scalar $\varepsilon > 0$ and a symmetric positive definite matrix P, satisfying $(\varepsilon I - H^T PH)^{-1} > 0$ then

$$(A + H\sum G)^T P(A + H\sum G) \leq A^T PA + A^T PH(\varepsilon I - H^T PH)^{-1} H^T PA + \varepsilon G^T G$$

Proof of Theorem:

Equation (6) can be expended as

$$\|G_{r\bar{u}}\|_\infty = \frac{\|r(k)\|_2}{\|\bar{u}(k)\|_2} < \gamma \quad (11)$$

In this way, the control objective is written as

$$\|r(k)\|_2^2 < \gamma^2 \|\bar{u}(k)\|_2^2 \quad (12)$$

Consider a Lyapunov function $V\ (k)$ as follows.

$$V\ (k) = \bar{x}^T(k) P \bar{x}(k) \quad , \quad V(0) = 0 \quad (13)$$

The system will be stable if

$$V(k+1) - V(k) < 0 \quad (14)$$

Equation (12) can be written as

$$\sum_{k=0}^{\infty} [r^T(k) r(k) - \gamma^2 \bar{u}^T(k) \bar{u}(k)] < -V(\infty) = 0 \quad (15)$$

$$\sum_{k=0}^{\infty} [r^T(k) r(k) - \gamma^2 \bar{u}^T(k) \bar{u}(k) + V(k+1) - V(k)] < 0 \quad (16)$$

To satisfy Eq. (7), the above matrix equation should be $-ve$ definite. Now, using the augmented system Eq. (5), the above relation can be represented as

$$[r^T(k) r(k) - \gamma^2 \bar{u}^T(k) \bar{u}(k) + V(k+1) - V(k)] < 0 \quad (17)$$

$$[\bar{x}^T(k) \quad \bar{u}^T(k)] \left(\begin{bmatrix} \bar{A}^T \\ \bar{B}^T \end{bmatrix} P [\bar{A} \quad \bar{B}] + \begin{bmatrix} \bar{C}^T \\ \bar{D}^T \end{bmatrix} [\bar{C} \quad \bar{D}] + \begin{bmatrix} -P & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \right) \begin{bmatrix} \bar{x}(k) \\ \bar{u}(k) \end{bmatrix} \quad (18)$$

Constant and uncertain matrices are separated as

$$\begin{bmatrix} \bar{C} & \bar{D} \\ \bar{A} & \bar{B} \end{bmatrix} = \begin{bmatrix} C_0 & D_0 \\ A_0 & B_0 \end{bmatrix} + \begin{bmatrix} \Delta\bar{C} & \Delta\bar{D} \\ \Delta\bar{A} & \Delta\bar{B} \end{bmatrix} \quad (19)$$

where

$$\begin{bmatrix} C_0 & D_0 \\ A_0 & B_0 \end{bmatrix} = \begin{bmatrix} C & 0 & 0 & F_d \\ A - LC & 0 & 0 & E_d - LF_d \\ 0 & A & B & E_d \end{bmatrix}$$

$$\begin{bmatrix} \Delta\bar{C} & \Delta\bar{D} \\ \Delta\bar{A} & \Delta\bar{B} \end{bmatrix} = \begin{bmatrix} H_2 \\ H_1 - LH_2 \\ H_1 \end{bmatrix} \sum [0 \quad G_1 \quad G_2 \quad 0]$$

Representing the above matrices as

$$\underline{A}_o = \begin{bmatrix} C_0 & D_0 \\ A_0 & B_0 \end{bmatrix}, \quad A_o = \begin{bmatrix} A - LC & 0 \\ 0 & A \end{bmatrix}, \quad C_o = [C \quad 0], \quad D_o = [0 \quad F_d]$$

$$B_o = \begin{bmatrix} 0 & E_d - LF_d \\ B & E_d \end{bmatrix}, \quad \underline{H} = \begin{bmatrix} H_2 \\ H_3 \end{bmatrix}, \quad H_3 = \begin{bmatrix} H_1 - LH_2 \\ H_1 \end{bmatrix}$$

$\underline{G} = [\underline{G_1} \quad \underline{G_2}]$, $\underline{G_1} = [0 \quad G_1]$, $\underline{G_2} = [G_2 \quad 0]$

By using Lemma 1, it is easy to write Eq. (18) as

$$[\bar{x}^T(k) \quad \bar{u}^T(k)]\left(\left(\underline{A_o} + \underline{H}\sum\underline{G}\right)^T \underline{P}\left(\underline{A_o} + \underline{H}\sum\underline{G}\right) - \overline{\underline{P}}\right)\begin{bmatrix}\bar{x}(k)\\ \bar{u}(k)\end{bmatrix} \tag{20}$$

$$\left(\underline{A_o} + \underline{H}\sum\underline{G}\right)^T \underline{P}\left(\underline{A_o} + \underline{H}\sum\underline{G}\right) - \overline{\underline{P}} \le \underline{A_o}^T\underline{P}\,\underline{A_o} + \underline{A_o}^T\underline{P}\,\underline{H}\left(\varepsilon I - \underline{H}^T\underline{PH}\right)^{-1}\underline{H}^T\underline{P}\,\underline{A_o} + \varepsilon\underline{G}^T\underline{G} - \overline{\underline{P}} \tag{21}$$

where

$\underline{P} = \begin{bmatrix} I & 0\\ 0 & P\end{bmatrix}$ and $\overline{\underline{P}} = \begin{bmatrix} P & 0\\ 0 & \gamma^2 I\end{bmatrix}$

Using Schur complement lemma [26],

If $A_{11} < 0$ then $\begin{bmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} < 0$ iff $A_{22} - A_{21}A_{11}^{-1}A_{12} < 0$

The following relations of Eq. (21) are obtained.

$$\begin{bmatrix} \underline{H}^T\underline{PH} - \varepsilon I & \underline{H}^T\underline{P}\,\underline{A_o}\\ \underline{A_o}^T\underline{P}\,\underline{H} & \underline{A_o}^T\underline{P}\,\underline{A_o} + \varepsilon\underline{G}^T\underline{G} - \overline{\underline{P}}\end{bmatrix} < 0 \tag{22}$$

Expending Eq. (22) will give

$$\begin{bmatrix} {H_2}^T H_2 + {H_3}^T P H_3 - \varepsilon I & {H_2}^T C_0 + {H_3}^T P A_0 & {H_2}^T D_0 + {H_3}^T P B_0\\ {C_0}^T H_2 + {A_0}^T P H_3 & {C_0}^T C_0 + {A_0}^T P A_0 + \varepsilon\underline{G_1}^T\underline{G_1} - P & {C_0}^T D_0 + {A_0}^T P B_0 + \varepsilon\underline{G_1}^T\underline{G_2}\\ {D_0}^T H_2 + {B_0}^T P H_3 & {D_0}^T C_0 + {B_0}^T P A_0 + \varepsilon\underline{G_2}^T\underline{G_1} & {D_0}^T D_0 + {B_0}^T P B_0 + \varepsilon\underline{G_2}^T\underline{G_2} - \gamma^2 I\end{bmatrix} < 0 \tag{23}$$

The above matrix inequality (MI) includes non-linear terms. Rewrite the above MI into

$$\begin{bmatrix} {H_2}^T & {H_3}^T & 0\\ {C_0}^T & {A_0}^T & \underline{G_1}^T\\ {D_0}^T & {B_0}^T & \underline{G_2}^T\end{bmatrix}\begin{bmatrix} I & 0 & 0\\ 0 & P & 0\\ 0 & 0 & \varepsilon I\end{bmatrix}\begin{bmatrix} H_2 & C_0 & D_0\\ H_3 & A_0 & B_0\\ 0 & \underline{G_1} & \underline{G_2}\end{bmatrix} - \begin{bmatrix} \varepsilon I & 0 & 0\\ 0 & P & 0\\ 0 & 0 & \gamma^2 I\end{bmatrix} < 0 \tag{24}$$

Again, apply the Schur complement lemma given below.

If $A_{22} < 0$ then $\begin{bmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} < 0$ iff $A_{11} - A_{12}A_{22}^{-1}A_{21} < 0$

Equation (24) can be represented as

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & {H_2}^T & {H_3}^T & 0\\ 0 & -P & 0 & {C_0}^T & {A_0}^T & \underline{G_1}^T\\ 0 & 0 & -\gamma^2 I & {D_0}^T & {B_0}^T & \underline{G_2}^T\\ H_2 & C_0 & D_0 & -I & 0 & 0\\ H_3 & A_0 & B_0 & 0 & -P^{-1} & 0\\ 0 & \underline{G_1} & \underline{G_2} & 0 & 0 & -\varepsilon^{-1} I\end{bmatrix} < 0 \tag{25}$$

By doing matrix equivalent transformation, the above nonlinear inequality turns to linear inequality form as

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & {H_2}^T & {H_3}^T P & 0 \\ 0 & -P & 0 & {C_0}^T & {A_0}^T P & {\underline{G}_1}^T \varepsilon \\ 0 & 0 & -\gamma^2 I & {D_0}^T & {B_0}^T P & {\underline{G}_2}^T \varepsilon \\ H_2 & C_0 & D_0 & -I & 0 & 0 \\ PH_3 & PA_0 & PB_0 & 0 & -P & 0 \\ 0 & \varepsilon \underline{G}_1 & \varepsilon \underline{G}_2 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (26)$$

where

$P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$ and $X_1 = LP_1$

By inserting the $H_3, A_0, B_0, C_0, D_0, \underline{G}_1, \underline{G}_2$ and P matrices in the above LMI which is the desired form mentioned in the above theorem. Up to here, the proof of the theorem is completed.

Solving the desired LMI for scalar $\varepsilon > 0$, matrices $P_1 > 0, P_2 > 0, P_3 > 0$ and $X_1,\ X_2$, robust FDF gain can be computed as

$$L = {P_1}^{-1} X_1$$

All other observer matrices in Eq. (2) are known so the RFDF design completes.

5. Computation of Evaluation Function and Adaptive Threshold

Generated residual from RFDF can be represented as [5]

$$\|r(k)\|_2^2 = \left\|r_d(k) + r_u(k) + r_f(k)\right\|_2^2 \quad (27)$$

In fault free case: $\left\|r_f(k)\right\|_2^2 = 0$, so residual evaluation function becomes

$$\|r_d(k) + r_u(k)\|_2^2 \le \|r_d(k)\|_2^2 + \|r_u(k)\|_2^2 \le \delta_d + \delta_u = J_{th} \quad (28)$$

where

$\delta_d = J_{th,d} = \sup[d^T(k)d(k)]$; can be calculated off-line.

$\delta_u = J_{th,u} = \sup \left\|r_{u|f=0,d=0}(k)\right\|_2^2$; can be computed online as

$J_{th,u} = \gamma^2 \|u(k)\|_2^2$.

γ can be determined by using the above theorem. Hence, the final expression for J_{th} is

$$J_{th} = \delta_d + \gamma^2 \|u(k)\|_2^2 \quad (29)$$

It is clear from Eq. (29) that threshold belongs to 2 parts. First part is constant value δ_d and the second part of the threshold depends on system input which is available online. Hence, the threshold for fault detection in an uncertain system is an adaptive type that varies as the system input varies.

6. Simulation Results

The FDF designed in the previous section is applied to a linear dynamic model of DC motor aiming to detect the sensor fault. 2 states are considered in this paper, armature current and shaft angular velocity. Input to the motor is armature voltage,

$u(k)$. The linear model of DC motor with unknown input and uncertainty is represented by the following state-space equation:

$$x(k+1) = (A + \Delta A)\, x(k) + (B + \Delta B)\, u(k) + E_d\, d(k) + E_f\, k(k)$$

$$y(k) = (C + \Delta C)\, x(k) + (D + \Delta D)\, u(k) + F_d\, d(k) + F_f\, f(k) \qquad (30)$$

where

$$x(k) = \begin{bmatrix} w(k) \\ i_a(k) \end{bmatrix}, A = \begin{bmatrix} 0.7408 & 0.0085 \\ -0.0002 & -0.9718 \end{bmatrix}, B = \begin{bmatrix} 0.0001 \\ 0.0282 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = 0, E_d = \begin{bmatrix} -0.864 & 0 \\ 0 & 0 \end{bmatrix}, E_f = B, F_d = F_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Uncertainty in system matrices can be represented $H_1 = H_2 = [0.01 \quad 0.01]^T, G_1 = [0.1 \quad 0.1], G_2 = [0.1]$ and $\sum = 0.9597$ is taken randomly for our purpose. Load torque variation is considered as unknown input to DC motor model, $d(k) \in [-0.01, 0.01]$. To verify the proposed RFDF, the entire DC motor model and observer is implemented in MATLAB Simulink. Different kinds of sensor faults (sensor offset fault, sensor incipient fault and sensor stuck fault) are injected on the DC motor to investigate the performance of RFDF. Step input of 0.7V is applied and it is worth mentioning here that assumptions (A1 and A2) are satisfied.

From several repeatable applications, γ is reduced to 1.01 which is, in fact, the feasibility limit of the proposed LMI. Suppose the upper bound of unknown input is $\delta_d = 0.0001$, then the threshold can be determined by using Eq. (29).

Figure 1 depicts the residual signal in the fault-free case. In the fault-free case, the residual is not exactly zero due to the presence of disturbance and uncertainty in DC motor parameters.

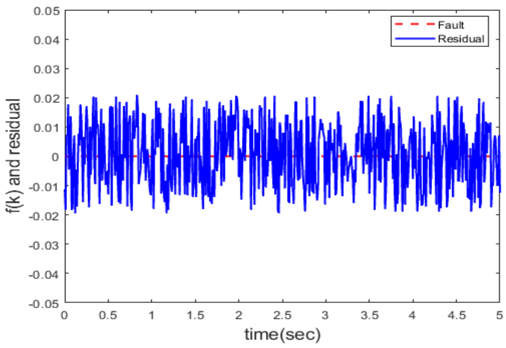
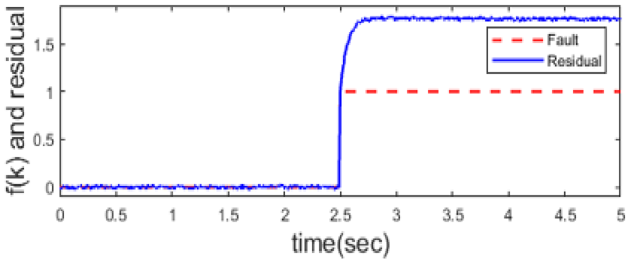


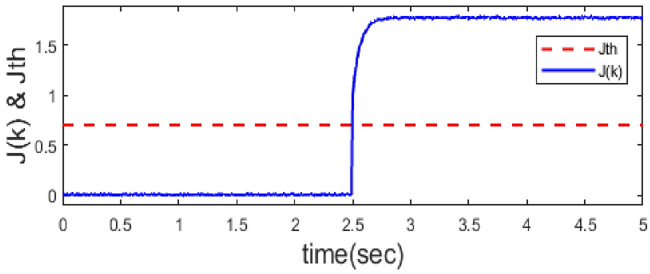
Fig. 1. Residual in a case on null fault.

Abrupt sensor fault is modelled as a step function, and this is applied to the sensor fault input at 2.5 seconds. The residual response to sensor fault is illustrated in Fig. 2(a) and the residual evaluation function, as well as the threshold, is shown in Fig. 2(b). In this case, it is noticed that the magnitude of the evaluation function has increased above the threshold value as the fault occurs in the sensor, which indicates the fault in the speed sensor.

The characteristics of incipient fault change very slowly with time, and they are very difficult to detect. For our purpose, the incipient fault is modelled as a ramp function and is injected into the current sensor at 2 seconds. Fault detection information can be retrieved from Fig. 3.(a) and (b) for incipient fault with a detection time of 0.6 seconds. The sensor stuck fault is simulated by making step function negative at the desired time of the simulation. Figure 4(a) shows the sensor fault and residual response to the fault. Figure 4(b) shows that the residual evaluation function crosses the threshold at the time of fault occurrence. The simulation results show that the proposed RFDF has successfully detected all the faults irrespective of unknown inputs and system uncertainties.



(a) Residual.



(b) Evaluation function and threshold.

Fig. 2. Abrupt fault: (a) Residual and (b) Evaluation function and threshold.

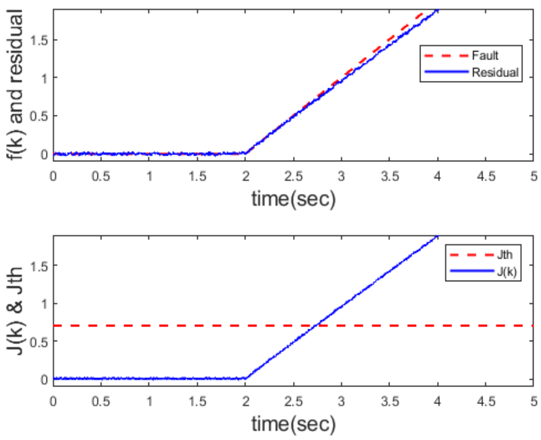


Fig. 3. Incipient fault: (a) Residual and (b) Evaluation function and threshold.

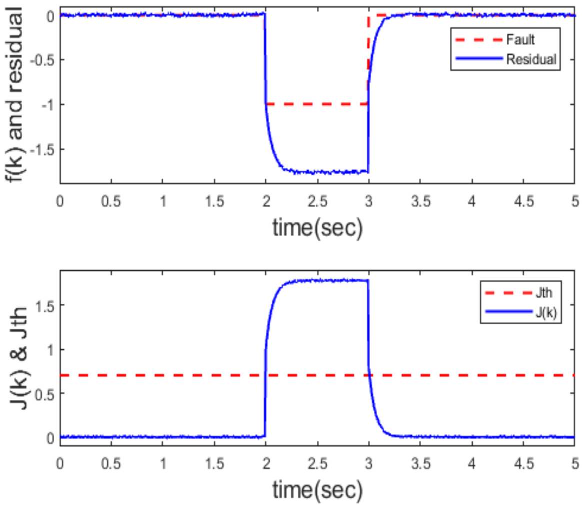


Fig. 4. Failure fault: (a) Residual and (b) Evaluation function and threshold.

7. Conclusion

The detection of premature faults is vital in overcoming future damage to the machine, degradation of performance, and even threatening human health. Full order robust fault detection filter design problem is assessed for discrete time-invariant linear system with both unknown inputs and modelling errors. RFDF is formulated as H_∞ optimization problem in the LMI framework. The application of the proposed observer based FDF has been demonstrated on the electromechanical system (DC motor). Various types of sensor faults are successfully detected using the designed RFDF which verifies the effectiveness of FDF.

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