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BINARY GREY WOLF TECHNIQUE FOR OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNIT WITH FULL NETWORK OBSERVABILITY

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Abstract

Phasor measurement unit (PMU) is becoming a vital device for the monitoring, control and protection of modern power systems. The optimal PMU placement (OPP) problem deals with the allocation of a minimum number of PMUs along with its locations for achieving full observability of the network. In this paper, a Binary Grey Wolf Optimization (BGWO) technique is proposed to minimise the PMU numbers along with maximisation of Measurement redundancy (MR) at the buses. To solve the OPP problem, several factors are considered, such as effects due to zero injection buses (ZIBs), single PMU failure and the presence of channel limit. The proposed method is compared to other heuristic methods to demonstrate the superiority and effectiveness of the algorithm. The analysis of the results shows that the proposed technique is equally efficient in terms of solving the problem along with higher observability index as compared to previously developed algorithms.

Keywords: Binary grey wolf optimization (BGWO), Channel limit, Measurement redundancy (MR), Observability, Optimal PMU placement (OPP), Phasor measurement unit (PMU).

1. Introduction

Phasor Measurement Unit (PMU) is a real-time measuring tool equipped with the Global Positioning System (GPS) to provide synchronised measurements of both voltage and current phasors [1]. The blackouts of 14th August 2003 in North America and 30th and 31st July 2012 in India have known examples connected with the lack of real-time measurement data or monitoring of the power network. In response, the placement of PMU over the network could be a promising solution for the security and reliability problem. Due to the high production, installation and maintenance costs and the lack of communication means, it is not possible to install the PMUs at every bus locations in the power network. The PMU placement should be in the range of 20%-30% of the entire buses available for a particular system [2]. Hence the main aim of the OPP problem is to minimise the number of PMUs along with its locations to make the entire electrical network completely observable [3].

Several optimisation techniques have been developed previously to solve this OPP problem. The most common method used for finding a minimum number of PMU is Integer Linear Programming (ILP) which solves the problem with reduced computational time [4, 5]. The exhaustive search (ES) approach is used in [6] for PMU placement with complete observability of the network. In [7] genetic algorithm (GA) is proposed to solve the OPP considering the security issues. In [8] simulated annealing (SA) method is implemented for the solution of the communication constrained OPP problem. An OPP problem with multiple solutions is developed in [9] using the exponential binary particle swarm optimisation (PSO). This algorithm used the non-linear inertia-weight co-efficient for the improvement of the searching capability. In [10] Ant colony optimisation (ACO) algorithm is proposed to find the optimal locations and a minimum number of PMU channels by optimising the control variables. In [11], a binary artificial bee colony (ABC) algorithm is proposed to minimise the PMUs in the presence of ZIBs and conventional flow measurements. The binary version of cat swarm optimisation (CSO) is implemented in [12] for the solution of OPP problem with complete observability of the network. In [13], a modified binary cuckoo algorithm is introduced to optimise the PMU numbers under the normal operating condition, single PMU loss and line loss. A graph theory with the multi-criteria decision-based approach (MCDM) is proposed to minimise the PMUs by maintaining complete observability in [14]. The proposed method is solved considering different possible contingencies. In [15], a teaching-learning based optimisation (TLBO) algorithm is used for OPP problem both under ZIBs and without taking ZIBs. In [16], the binary version of Jaya algorithm is developed to minimise the PMUs along with to obtain maximum redundancy. The firefly algorithm is presented in [17] to find minimum PMU numbers while considering the channel limits and also maximise the measurement redundancy. The GA, PSO, FA are some few methods in which the channel limit of PMUs considered for the placement problem.

The complete observability of the power system network can be analysed by two widely used approaches named as numerical and topological observability. In numerical observability analysis, a system can be fully observable if the number of unknown states and the rank of the associated Jacobian matrix is identical [18]. However, this observability approach involves massive matrix manipulation with high computational time for large scale electrical network. In topological observability analysis, a full ranked spanning tree can be constructed in which each branch can be directly measured. In this paper, a topological observability approach is considered to make the whole network completely observable.

In this paper, the Binary Gray Wolf Optimization (BGWO) is implemented to find the solution for the OPP problem and to ensure a full observable network with maximum redundancy index. GWO is a newly developed meta-heuristic algorithm based upon the hunting behaviour of grey wolves. In this work, a novel binary version of GWO approach is proposed to solve the OPP problem as the solution is restricted to the binary values (0, 1). The BGWO is very simple and easy in implementation as it can converge to the best optimal solution. The proposed method solved the OPP problem considering the effect due to ZIBs, single PMU failure and the channel limits. The method is validated using four IEEE standard systems such as 14-bus, 30-bus, 57-bus and 118-bus.

Measurement redundancy (MR) is an essential parameter for the secure operation of the power system network. In this work, MR calculation is incorporated with the minimisation of PMU problem for the best placement location. The placement set, which has maximum MR value, is considered as the best optimal location to install PMUs.

This paper is categorised into the following sections. Section II describes the placement problem of PMUs, along with the optimal placement rules for each considered case taken in this work. Section III describes the binary version of the GWO algorithm along with the steps involved for placing PMUs at desired locations to maintain complete observability. The simulation result and discussion are explained in section IV, and Section V explains the conclusion of the proposed work.

2. Problem Formulation

The optimal placement of the PMU problem is defined as to minimise the number of PMUs along with their installation locations in order to get complete observability of the entire system and to maximise the system redundancy. Hence for N number of buses, the objective function $(J(z))$ can be represented as the following Eq. (1) [19, 20]:

$$\begin{aligned} &min\,J(z) = \sum_{i=1}^{N} z_i + w \times (M - A_1 Z)^T\,(M - A_1 Z) \\ &s.t\; F(z) = A_1 \cdot Z \geq b \end{aligned} \qquad (1)$$

where w is the weighting factor and z_i is the placement of PMU of the binary vector Z at i^{th} bus and can be defined as follows (Eq. (2)):

$$z_i = \begin{cases} 1, if\ PMU\ is\ installed\ at\ bus\ i \\ 0, otherwise \end{cases} \qquad (2)$$

$F(z)$ is the observability constraint vector, and A_1 is the binary connectivity matrix which gives the information related to the connection of buses to each other in the power system and can be defined as the following Eqs. (3) and (4):

$$A_{1_{n\times n}}(i,j) = \begin{cases} 1, if\ bus\ i = j \\ 1, if\ bus\ i\ is\ connected\ to\ j \\ 0, otherwise \end{cases} \qquad (3)$$

$b_{n\times 1} = [1\ 1\ 1 \dots\dots\dots\dots]^T$ = unit vector of length N_b (4)

M represents the desired MR value, and it can be defined as follows:

$$M = [M_1\ M_2\ M_3 \dots\ M_N]^T_{1\times N} \qquad (5)$$

The value of A_1Z in the Eq. (1) defines the bus observability count from the OPP solution set. The value of each element of M depends on the sum of the connectivity of the buses and 1, including itself. For example, if one bus is connected to 4 number of buses then the value of M is set to be 6 {4 (connected bus) + 1 (bus itself) + 1 = 6}. Hence the minimisation of the difference between the desired value and actual value gives the maximum value of MR. This MR calculation is another important factor which is added to the objective function of the OPP problem.

A system is said to be fully observable if all buses connected to the entire system can be monitored directly or indirectly with the PMU by measuring the bus voltages and its branch currents. The bus at which a PMU is installed can directly measure the voltage and branch currents associated with it. Hence, the direct measurement involves voltage and branch current values directly from the PMU placed buses, whereas for indirect measurement voltage values of PMU installed neighbouring buses can be calculated by using ohm's law and Kirchhoff’s current law (KCL). By applying both the direct and indirect measurement approach, the entire electrical network can be fully observable with the following rules [20].

i. If the bus voltage and associated branch currents are known, then the other end bus voltage can be calculated.

ii. If the voltages are known for two connected buses, then the branch currents linked these buses can be calculated.

In this paper, the solution for OPP problem is obtained by considering the effect due to ZIBs, single PMU failure and channel limit of PMU which have been discussed below.

2.1. Effect due to zero injection buses (ZIBs)

The bus that has no generation or load connected to it is known as the zero injection bus (ZIB). Therefore, according to the Kirchhoff's Current Law (KCL), the sum of the flows in the branch currents linked with the ZIB is zero. The main purpose of considering ZIB is to minimise the PMUs further. There are some rules to satisfy the observability criteria in case of ZIBs, as mentioned below [20].

i. An observable ZIB can observe any unobserved bus which is connected to it by using KCL at ZIB.

ii. If all the observable buses are connected to unobserved ZIBs, then that ZIB can be made observable by using KCL.

2.2. Effect due to single PMU failure

Proper monitoring and controlling of power system network should be unaffected during any contingencies. The loss of any single PMU can have the effects of interruption of the entire electrical network. To avoid such type of circumstances, each bus should be observed twice to make the whole network completely observable [20]. Hence the modified unit vector defined as following Eq. (6):

$$b_{n\times 1} = [2\ 2\ 2\ \dots\dots\dots\dots]^T \tag{6}$$

2.3. Effect due to channel limits

The PMUs are usually designed with a fixed number of channels by different manufacturers. Hence the PMUs are limited for a specified number of branch currents and bus voltages to be monitored at one time. Therefore, for a different set of channel limits, the PMU numbers are also varying. The number of PMUs goes on decreasing with the increasing of channel limits. In this paper, channel limits are considered as one of the important contingency condition to solve the OPP problem. In this case, possible combinations are required to be calculated by using the following formula Eq. (7) for a fixed number of PMU channels [21]:

$$BR_i = \begin{cases} BC_i, & for\ L \le BI_i \\ 1, & for\ L > BI_i \end{cases} \tag{7}$$

where BR_i represents the total branch connections for bus i, L represents the Channel limits, BI_i represents the number of neighbouring buses with bus i, BC_i represents the possible combinations of L out of BI_i and can be defined as follows (Eq. (8)):

$$BC_i = \frac{BI_i!}{\left(BI_i-(L-1)\right)!(L-1)!} \tag{8}$$

2.4. Measurement redundancy (MR)

Measurement Redundancy (MR) is one of the important parameter for secure monitoring of the electrical network in solving the OPP problem [22]. In order to calculate the MR, the bus observability index (BOI) and the complete system observability index (CSOI) are required. BOI is defined as the number of times a bus is observed by the placement sets of PMUs whereas the CSOI is the summation of all the BOIs at each bus location. The set which has maximum CSOI is the best location for PMU placement. In this paper, the redundancy concept is introduced to find the best optimal location for a secure network. The BOI and CSOI can be represented as follows (Eq. (9) & (10)):

$$\boldsymbol{BOI_i = A_1 \times Z} \tag{9}$$

$$CSOI = \sum_{i=1}^{N} BOI_i \tag{10}$$

3. Binary Grey Wolf Optimization (BGWO)

3.1. Grey wolf optimization algorithm (GWO)

Grey wolf optimisation algorithm (GWO) is the most potent newly developed meta-heuristic optimisation method proposed by S. Mirjalili et al. [23] in 2014. This algorithm mainly deals with the non-convex optimisation problem. This algorithm is mainly developed by observing the hunting and searching characteristics of Grey Wolf, which belongs to 'Canidae Family'. The average group size of the grey wolf is 5 to 12 members. In GWO, the population is categorised into four major groups of wolves such as alpha wolf (α), beta wolf (β), delta wolf (δ) and omega wolf (ω). Alpha wolf is the best leader among all the group members, which has the responsibility to make the decisions, hunting and other activities. Beta wolf is the second-best leader who helps in supervising and support the alpha wolf. The third leader of the group is the Delta wolf who plays the role of scapegoat but dominate over omega wolves. According to [24], the hunting process of grey wolf includes the following steps:

i. The first step is to track, chase and approach towards the prey.
ii. Secondly Pursuing, encircling the pray
iii. Finally harassing the prey until it stops moving and attacks towards the prey.

Mathematically, the alpha wolf is considered as the fittest solution. Consequently, the beta wolf and the delta wolf are considered as second and third-best solutions. The remaining solutions are assumed as omega wolf. In GWO algorithm, the hunting process starts with the alpha (α), beta (β) and delta (δ) wolves, while the omega (ω) wolves follow these leaders. The encircling behaviour of wolves for hunting can be expressed as [23] (Eqs. (11) and (12)):

$$D = \left|C\left(X_P\ (t)\right) - X\ (t)\right| \tag{11}$$

$$X(t+1) = \{X_P(t) - A \cdot D\} \tag{12}$$

where t presents the current iteration; X_P presents the position of the desired prey; X presents the position of the grey wolf, A & C are the co-efficient vectors and represented below in Eqs. (13) and (14):

$$A = 2a \cdot r_1 - a \tag{13}$$

$$C = 2 \cdot r_2 \tag{14}$$

where 'a' is the encircling co-efficient linearly decreased from 2 to 0, according to the following Eq. (15).

$$a = 2 - 2\left(\frac{t}{T_{max}}\right) \tag{15}$$

Mathematically, the new best position of the Wolf can be updated by considering the average position of the three wolves (alpha, beta and delta wolf) by using the following Eq. (16):

$$X(t+1) = \left(\frac{X_1 + X_2 + X_3}{3}\right) \tag{16}$$

where X_1, X_2, and X_3 can be expressed as Eqs. (17), (18) and (19):

$$X_1 = |X_\alpha - A_1(D_\alpha)| \tag{17}$$

$$X_2 = |X_\beta - A_2(D_\beta)| \tag{18}$$

$$X_3 = |X_\delta - A_3(D_\delta)| \tag{19}$$

The position of alpha (D_α), beta (D_β), and delta (D_δ) can be updated based on the Eqs. (20), (21) and (22), respectively, as follows:

$$D_\alpha = |C_1 \cdot X_\alpha - X| \tag{20}$$

$$D_\beta = |C_2 \cdot X_\beta - X| \tag{21}$$

$$D_\delta = |C_3 \cdot X_\delta - X| \tag{22}$$

where C_1, C_2, and $C3$ can be calculated using the Eq. (14).

3.2. Binary grey wolf optimization algorithm

Binary Grey Wolf Optimization (BGWO) technique is the binary version of the GWO technique, where the updated position of the grey wolf is represented in binary form as shown in the following Eq. (23):

$$X_d(t+1) = \begin{cases} 1, if\ SF \geq r_3 \\ 0, otherwise \end{cases} \tag{23}$$

where d represents the searching space dimension and SF is the sigmoid function of x and can be calculated as Eq. (24):

$$SF(x) = \frac{1}{1 + exp\ (-10\ (x - 0.5)\ )} \tag{24}$$

where, $x = \frac{X_{1,d}+X_{2,d}+X_{3,d}}{3}$

The flowchart of BGWO for OPP problem is presented in Fig. 1. The complete discussion about the steps of the proposed approach is given below:

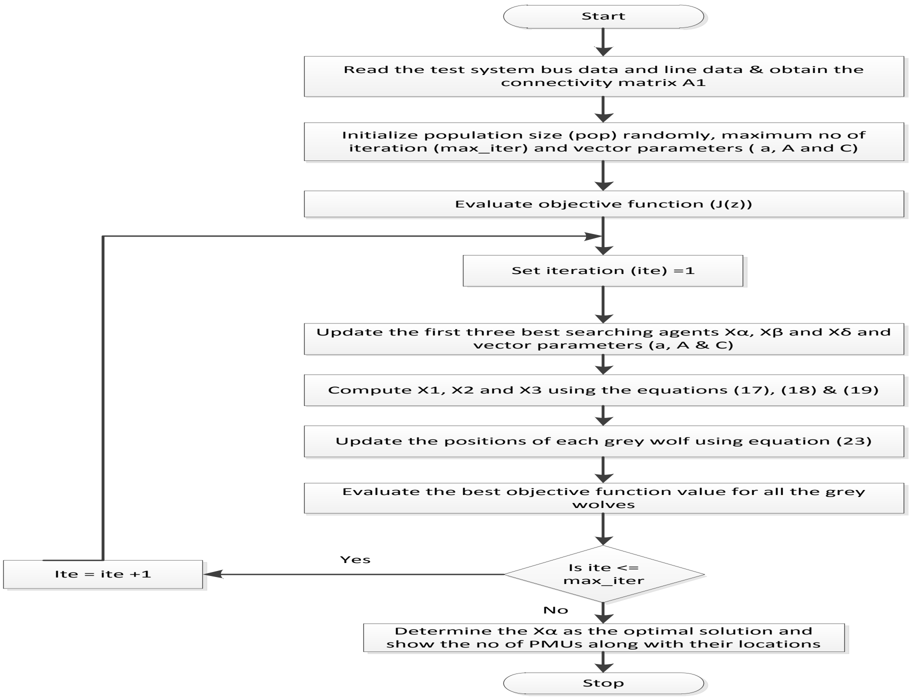

Fig. 1. Flowchart of BGWO for OPP problem.

1. Initialise the population size of grey wolves randomly, the maximum number of iterations (max_iter) and parameters *a*, *A* & *C*.
2. Randomly initialise the position for PMUs in the system.
3. Calculate the objective function value described for each X_α solution in the population.
4. Update the first three best searching agents X_α, X_β & X_δ.
5. Compute X₁, X_2 and X_3 using Eqs. (17), (18) and (19).
6. Update the position of each Wolf by using the Eq. (23).
7. Repeat the procedure from step 3 to 6 up to maximum iteration. Finally, X_α is selected as the optimal solution to the problem.

4. Results and Discussion

The performance of the proposed BGWO technique to solve the OPP problem is tested under four cases- 1) Normal condition, 2) With Zero Injection Bus, 3) Single PMU failure, 4) channel limit for normal operating condition and effects due to ZIBs on IEEE 14, 30, 57 and 118 bus systems using MATLAB software. The results obtained from the proposed algorithm are verified by comparing with previously developed algorithms such as Firefly algorithm [17], BPSO [20, 22], ILP [21], gravitational search [25], Flower pollination [26], Binary integer [28] and GA [27, 29]. As the proposed approach gives the optimal results with multiple placement locations, therefore it is necessary to find the best location such that the measurement redundancy will be more. In this paper, both direct and indirect measurements have been considered for complete observability of the test systems. Here Table 1 gives information about the total number of ZIBs along with their locations and a total number of transmission lines for different IEEE test bus systems.

Table 1. Test system specifications.

IEEE bus systems	Number of ZIBs	Transmission Lines	Location of ZIBs
14-bus	1	20	7
30-bus	6	41	6, 9, 22, 25, 27, 28
57-bus	15	80	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48
118-bus	10	186	5, 9, 30, 37, 38, 63, 64, 68, 71, 81

4.1. Case 1: Normal operation

During normal operation, ZIBs are not taken into consideration, and the entire network is in working mode. The minimum numbers required for placement and the CSOI for the corresponding location obtained from BGWO are presented in Table 2. The optimal PMUs numbers along with their redundancy index obtained from BGWO are compared with Firefly algorithm [17], BPSO [22], gravitational search [25], Flower pollination [26] and GA [27]. The proposed technique gives the same number of PMUs with various possible locations in comparison to other methods as given in Table 3. It can be observed that the CSOI results each bus location for four bus systems are maximum as compared to other methods. The convergence graph for the entire four test systems is shown in Fig. 2 and the optimal PMU allocation for an IEEE-14 bus system during normal operation is shown in Fig. 3.

Table 2. Results of OPP problem at normal operation.

IEEE bus systems	PMU Locations	CSOI	Optimal no of PMUs
14-bus	**2, 6, 7, 9**	**19**	4
	2, 6, 8, 9	17	
	2, 7, 11, 13	16	
	2, 7, 10, 13	16	
30-bus	**2, 4, 6, 9, 10, 12, 15, 20, 25, 27**	**52**	10
	2, 4, 6, 10, 12, 18, 24, 25, 27	51	
	2, 4, 6, 10, 12, 15, 18, 25, 29	50	
	2, 4, 6, 10, 11, 12, 15, 19, 25, 29	48	
57-bus	**1, 4, 6, 9, 15, 20, 24, 25, 28, 32, 36, 38, 39, 41, 46, 51, 53**	**72**	17
	1, 4, 9, 13, 20, 23, 27, 29, 30, 32, 36, 41, 44, 47, 50, 53, 57	68	
118-bus	**3, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114**	**164**	32
	1, 5, 10, 12, 15, 17, 20, 23, 26, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 91, 94, 102, 105, 110, 114	160	

Table 3. Comparative results at normal operating condition.

IEEE Bus systems & CSOI	Proposed method	Gravitational search [25]	GA [27]	Flower pollination [26]	Firefly [17]	BPSO [22]
IEEE 14	4	4	4	4	4	4
CSOI	19	19	19	19	19	19
IEEE 30	10	10	10	10	10	10
CSOI	52	52	42	52	50	52
IEEE 57	17	-	17	17	17	17
CSOI	72	-	70	72	72	71
IEEE 118	32	32	-	32	32	-
CSOI	164	164	-	160	164	-

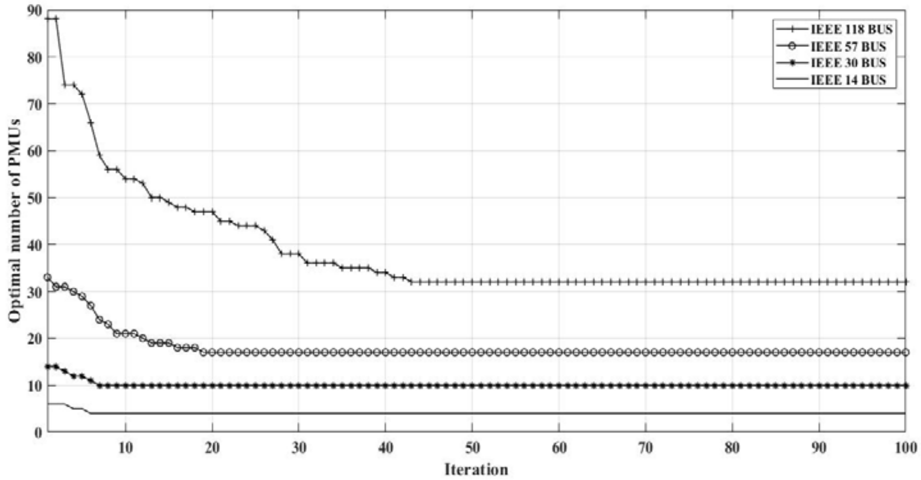

Fig. 2. Convergence graph for IEEE 14, 30, 57 & 118 bus systems under normal operation.

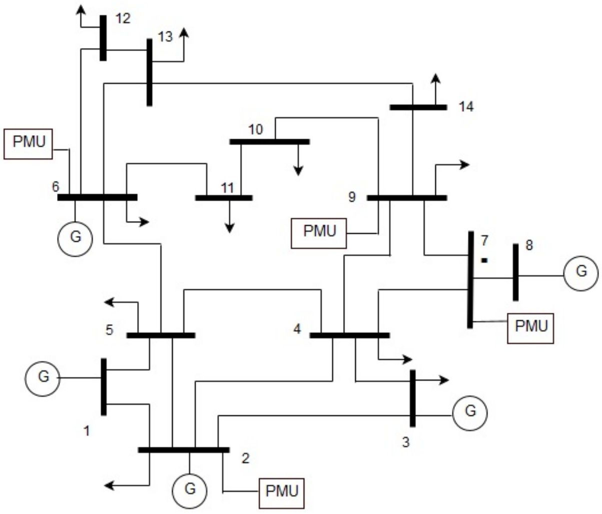

Fig. 3. Optimally allocated PMUs for IEEE 14 bus system.

4.2. Case 2: ZIB consideration

The characteristics of zero injection buses are taken into consideration for further reduction of PMUs. Table 4 shows the results for BGWO while considering ZIBs. The obtained results along with their CSOI are compared with Firefly algorithm [17], BPSO [22], GA [27] and gravitational search [25] method, as shown in Table 5. It can be observed from the results that the CSOI value for IEEE 57 and 118 bus system is more as compared to other methods. However, the best result could not be obtained by BPSO [22] for IEEE 57 and 118 bus system. Therefore, the proposed technique is capable of solving the OPP problem with different possible PMU locations as compared to others. The convergence graph for all the four test systems is shown in Fig. 4.

Table 4. Results of OPP problem with the consideration of ZIB.

IEEE bus systems	**PMU Locations**	**CSOI**	**Optimal no of PMUs**
14-bus	**2, 6, 9**	**16**	3
30-bus	**2, 4, 10, 12, 15, 18, 27**	**41**	7
	2, 4, 10, 12, 15, 19, 27	**41**	
	2, 4, 10, 12, 19, 24, 27	39	
57-bus	**1, 3, 13, 19, 25, 29, 32, 38, 41, 51, 54**	**62**	11
	1, 6, 13, 20, 25, 29, 32, 38, 41, 51, 54	61	
	1, 3, 13, 19, 25, 29, 32, 38, 42, 51, 54	61	
118-bus	**3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110**	**157**	28
	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	156	

Table 5. Comparative results with the consideration of ZIBs.

IEEE Bus systems & CSOI	**Proposed method**	**Gravitational search [25]**	**GA [27]**	**Firefly [17]**	**BPSO [22]**
IEEE 14	**3**	3	3	3	3
CSOI	**16**	15	15	16	16
IEEE 30	**7**	**7**	**7**	**7**	**7**
CSOI	**41**	**36**	**32**	**41**	**34**
IEEE 57	**11**	-	11	11	13
CSOI	**62**	-	57	61	64
IEEE 118	**28**	**28**	**-**	**28**	**29**
CSOI	**157**	**147**	**-**	**157**	**155**

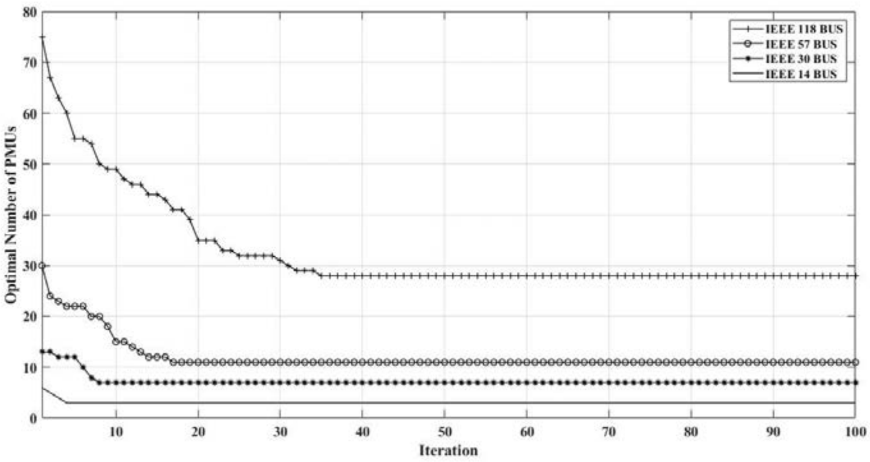

Fig. 4. Convergence graph for IEEE 14, 30, 57 & 118 bus systems under zero injection bus condition.

4.3. Case 3: Single PMU failure

Table 6 presents the solution for OPP in case of a single loss of PMU. PMU failure is one of the most severe contingencies which affect the system observability. So, to avoid such issues, every bus needed to be observed at least twice independently. Hence PMU numbers increased with the increase of bus size. Maximum results are obtained in this case as compared to normal operating case and ZIB consideration. The measurement redundancy is also calculated for the better optimal location from the multiple placement sets.

Table 6. Results of OPP problem with single loss of PMU.

IEEE bus systems	PMU Locations	CSOI	Optimal no of PMUs
14-bus	**2, 4, 5, 6, 7, 8, 9, 11, 13**	**39**	9
	2, 4, 5, 6, 7, 8, 9, 10, 13	**39**	
	2, 3, 5, 6, 7, 8, 9, 11, 13	36	
30-bus	**1, 2, 4, 6, 7, 9, 10, 11, 12, 13, 15, 17, 18, 20, 22, 24, 2526, 27, 28, 30**	**85**	21
	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 22, 23, 25, 26, 27, 29	83	
57-bus	**1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 41, 45, 46, 47, 50, 51, 53, 54, 56, 57**	**130**	33
	1, 2, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 41, 44, 46, 47, 50, 51, 53, 54, 56, 57	129	
118-bus	**2, 3, 5, 6, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 35, 37, 40, 42, 43, 45, 46, 49, 51, 52, 54, 56, 57, 59, 61, 62, 64, 66, 68, 70, 71, 73, 75, 76, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 116, 117**	**309**	68
	2, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 26, 27, 28, 30, 31, 32, 34, 36, 37, 40, 41, 43, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 64, 65, 66, 68, 70, 71, 72, 73, 75, 77, 79, 80, 84, 85, 86, 87, 89, 90, 92, 94, 96, 100, 102, 105, 106, 108, 110, 111, 112, 115, 116, 117, 118	305	

4.4. Case 4: Channel limits under the normal case and ZIB effects

The comparative results of the proposed method with the previous works are presented in Tables 7 and 8 considering channel limits for normal case and for zero bus effects respectively. PMUs are designed with a limited number of channels according to the manufacturers, and their cost also varies depending upon the channels. A PMU which has maximum channel limits can observe the maximum number of buses so that minimum PMUs can be obtained to maintain full observability of the system. The obtained results are the same for all the compared techniques for a normal case, but while considering ZIBs, BGWO gives a better result than ILP [21]. However, the results are consistent over the existing method, and it is proved that the proposed technique is ample enough for solving the optimal placement problem for considering the PMU's channel limit.

Table 7. Comparative results with the consideration of Channel limit for normal operating condition.

IEEE bus systems	Channel Limit (L)	Number of PMUs			
		Proposed method	BPSO [20]	ILP [21]	Binary Integer [28]
14	2	7	7	7	7
	3	5	5	5	5
	4	4	4	4	4
	5	4	4	4	4
30	2	15	15	15	15
	3	11	11	11	11
	4	10	10	10	10
	5	10	10	10	10
57	2	29	29	29	29
	3	19	19	19	19
	4	17	17	17	17
	5	17	17	17	17

Table 8. Comparative results with the consideration of Channel limit and ZIBs.

IEEE bus systems	Channel Limit (L)	Number of PMUs			
		Proposed method	BPSO [20]	ILP [21]	GA [29]
14	2	7	7	7	-
	3	5	5	5	-
	4	4	4	4	-
	5	3	3	3	-
30	2	13	13	13	12
	3	8	8	9	8
	4	7	7	7	7
	5	7	7	7	7
57	2	21	21	21	21
	3	14	14	14	14
	4	12	12	12	13
	5	11	11	11	12

5. Conclusions

This paper proposed the BGWO technique to solve the optimal placement problem and to improve the measurement redundancy. The observability index is also one of the important parameter as it is based on the best optimal location. As the proposed method gives multiple placement sets, it is necessary to find the CSOI for each set for the best PMU location. Four different IEEE test bus systems are considered in this work to solve the OPP problem under normal operation, ZIB effect and Channel limit. The results obtained from the proposed work meet all the observability criteria of the entire system, along with a reduced number of PMUs and maximum redundancy than the previously developed algorithms. The proposed BGWO method is simple and easy to implement to solve any optimisation problem. By applying this method to different standard test systems in different contingency conditions, it demonstrates the comparative ability to achieve an optimal solution with higher CSOI to the existing methods.

Nomenclatures

A & C	Coefficient vectors
A_1	Binary connectivity matrix
a	Encircling coefficient
BOI_i	Bus Observability Index for ith bus
b	Unit vector
d	Dimensional searching space
L	Channel limit
M	Measurement Redundancy vector
r_1, r_2, r_3	random vectors distributed between [0, 1]
T_{max}	Maximum iteration
w	Weighting factor (taken as 0.01)
X_α	The position vector of the alpha wolf
X_β	The position vector of the beta wolf
X_δ	The position vector of delta wolf

Greek Symbols

α	Alpha wolves in the grey wolf family
β	Beta wolves in grey wolf family
δ	Delta wolves in the grey wolf family
ω	Omega wolves in the grey wolf family

Abbreviations

ABC	Artificial Bee Colony
ACO	Ant Colony Optimization
BGWO	Binary Grey Wolf Optimization
BPSO	Binary Particle Swarm Optimisation
CSO	Cat swarm optimisation
CSOI	Complete System Observability Index
ES	Exhaustive Search
GWO	Grey Wolf Optimization
MCDM	Multi-criteria decision making
SA	Simulated Annealing
SF	Sigmoid function

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