

DESIGN AND PERFORMANCE ANALYSIS OF ECG DATA COMPRESSION USING CONVOLVED WINDOW- BASED COSINE MODULATED FILTER BANK

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Abstract

Electrocardiogram (ECG) is the graphical record of the electrical activity of the heart muscle recorded by an electrocardiograph. Current medical diagnostic procedures are based on the longer recordings of electrocardiogram signals. Efficient and reliable compression techniques are required to reduce the amount of data storage of long-term electrocardiogram recordings. In this work, convolved fixed window-based optimized near-perfect reconstruction cosine modulated filter bank is employed for electrocardiogram signal compression. ECG signal is decomposed into different subband signals through the analysis filter bank. The subband signals are thresholded, quantized and then encoded to obtain signal compression. The experiment is carried out on electrocardiogram signals obtained from Massachusetts Institute of Technology - Beth Israel Hospital (MIT-BIH) arrhythmia database. Compression ratio, percentage root-mean-square difference and peak signal to noise ratio, are the attributes used for testing the effectiveness of the algorithm. Performance comparison of the unconvolved and convolved window-based filter banks is presented for various ECG signals. The proposed technique also illustrates better performance when compared with other existing algorithms.

Keywords: Cosine modulated filter banks, Compression ratio, ECG compression, Finite impulse response filters, Signal reconstruction.

1. Introduction

Electrocardiogram (ECG) signal is a vital parameter in many clinical diagnoses related to the cardiovascular system of the human body. Huge amount of data is generated by the digitized ECG. An effectual compression technique is necessary for proper transmission/storage. The ECG data compression plays a pivotal part in applications like telemedicine, ambulatory ECG monitoring, wearable healthcare devices and patient databases in hospitals [1-3]. The purpose of the compression technique is to minimize the data space without distorting the signal morphological features during reconstruction. The compression of the ECG signal reduces the bits per sample required to correctly represent the reconstructed ECG at the receiver side. Further, it makes the transmission of data faster at a reduced bandwidth.

During past few decades, many researchers focussed on ambulatory ECG data compression. The research has resulted in different ECG compression algorithms which are broadly classified as lossless and lossy compression methods. In lossless compression exact reconstruction of input signal is obtained. In lossy compression methods some redundant information may be removed for achieving a better compression ratio. These techniques are also classified as direct time-domain techniques, transform-domain techniques, and parameter extraction techniques. The direct data compressors use direct analysis of the actual signal samples. It involves use of predictive or sample interpolating algorithms. This results in redundancy reduction [4]. With the advent of advanced digital signal processing techniques, transform based methods gained momentum over other techniques. The transform based methods include Fourier Transform (FT), Discrete Cosine Transform (DCT) and Wavelet Transform (WT). The parameter extraction methods are based on some long term prediction algorithms.

Recently, techniques such as Compressed Sensing (CS), Principal Component Analysis (PCA), Discrete Wavelet Transform (DWT), and Singular Value Decomposition (SVD) have been extensively explored in ECG compression [5-14]. In [5], multiscale principal component analysis (MSPCA) is reported for multichannel electrocardiogram (MECG) data compression. Kumar et al. [6] presented use of Beta wavelet along with lossless encoding. Compressed sensing (CS) techniques for ECG compression are given in [7, 8]. Polan'ia et al. [7] proposed a CS-based approach involving wavelet decomposition. Craven et al. [8] proposed a compressed sensing scheme combining entropy based coding along with quantization. This scheme provided effective compression by redundancy removal. In CS techniques, the distortion level is not effectively reduced. Padhy et al. [9] proposed a multi-lead ECG (MECG) signal compression through singular value decomposition (SVD). During 2017, number of papers were published on ECG compression using various techniques such as embedded zero tree wavelet (EZW) compression [10], dual encoding technique [11, 12] and temporal modelling using principal component analysis [13]. Peng et al. [10] proposed an improvement over the classic EZW compression algorithm. Nagendra and Raghavarao [11] and Jha and Kolekar [12] employed a DCT based dual encoding based scheme for ECG compression. Kumar et al. [13] presented an improvised method for utilizing beat correlation and principal component analysis (PCA).

Multirate signal processing has also played a vital role in ECG compression in the last few decades. Çetin et al. [14] showed that sub-band coding decomposition based schemes are superior to the transform coding schemes in context of compression ratio. ECG compression based on multirate filter banks was presented

in [2] and [15-17]. There are different types of filter banks available for the numerous and varied applications in signal processing domain. Out of these, cosine modulated filter bank (CMFB) is a popularly used filter bank. It offers better computational efficiency and simple design. The CMFB is implemented by cosine modulation of the prototype low pass filter. It is then optimized to reduce the reconstruction error [18, 19].

The main motivation of the paper is the fact that the convolved windows based filter banks have not been reported in sub-band coding for ECG compression. To fill this research gap, this paper aims at finding the application of convolved windows in effective ECG compression. The objective of the paper is to design an ECG compression scheme for clinically acceptable quality of reconstructed ECG signal.

In this approach of ECG compression, input signal is split into disjoint frequency bands with the help of subband filters. The subband decomposition is said to be uniform if all subbands have the same energy over the entire range of frequency distribution. The frequency spectrum of the ECG signal ranges up to 100 Hz. Most of the spectral energy lies in lower frequency range up to 35 Hz [20]. Therefore most of the higher frequency subband signal samples can be discarded. Remaining significant coefficients must be capable to represent the reconstructed signal with acceptable fidelity. Threshold level determines whether a sample is to be rejected or discarded. The threshold level influences the quality of reconstruction and signal compression.

2. Theoretical Background

Due to the high stability and linear phase property, FIR filters are widely used in multirate signal processing [18]. An N^{th} order $p(n)$ filter with passband edge frequency ω_p , stopband edge frequency ω_s , and cut off frequency $\omega_c = (\omega_p + \omega_s) / 2$, is given by $p(n) = h_{id}(n)w(n)$ where $h_{id}(n)$ is shifted impulse response of the ideal low pass filter and $w(n)$ is the window function [21]. It is given in Eq.(1) [21],

$$h_{id}(n) = \frac{\sin(\omega_c(n-0.5N))}{\pi(n-0.5N)}, \quad n \in Z \tag{1}$$

2.1. Convolved windows

Harris reported that the self-convolution or cross convolution of the classical windows leads to another class of window functions called convolved windows [22]. Reljin et al. [23] proposed windows obtained by multiple convolutions of weighted cosine parent windows in time domain. They have flat top main lobe and low sidelobe values. This helps in reduction of spectral leakage and harmonic interference [24, 25]. So, convolved windows are applied in harmonic analysis and periodic signal parameter estimation [26]. They are also applied for reducing the spectral leakage error [27, 28]. The convolution of a window function with itself results in self convolution windows. A parent window of length N is self-convolved with itself $(C-1)$ times. A C^{th} -order convolved window is thus obtained. Here, C represents the number of parent windows. Let us consider a parent window (of length N), $w_1(n)$, $n = 0, 1, 2, \dots, N-1$. Then the convolved window of length N_c is obtained by $(C-1)$ times convolution of $w_1(n)$. It is defined by Eq. (2) [23].

$$w_c(n) = \underbrace{w_1(n) * w_1(n) * w_1(n) \dots \dots \dots w_1(n)}_{(C-1) \text{ times convolution}} \tag{2}$$

where, $N_c = (C*N)-(C-1)$

The resulting window provides C times greater sidelobe level and sidelobe fall off rate (SLFOR) than the parent window. Here, constant – length parent window type of convolved windows is employed in FIR filter design. The designed low pass filter provides high stopband attenuation and high SLFOR. These attributes of the filter lead to further reduction of aliasing error [29, 30].

2.2. Cosine modulated filter banks

In cosine modulated filter bank (CMFB), the analysis and synthesis filter banks are obtained by cosine modulating a predesigned low pass FIR filter. Therefore, an efficient filter bank can be designed by optimizing the coefficients of a single prototype filter. This approach further reduces the computational complexity and is easy to implement. Analysis filter bank $h_k(n)$ and synthesis filter bank $f_k(n)$ are derived by cosine modulating the prototype FIR filter coefficients $p(n)$ [31, 32]:

$$\left. \begin{aligned} h_k(n) &= 2p(n) \cos\left((2k+1)\frac{\pi}{2M}\left(n - \frac{N-1}{2}\right) + (-1)^k \frac{\pi}{4}\right) \\ f_k(n) &= 2p(n) \cos\left((2k+1)\frac{\pi}{2M}\left(n - \frac{N-1}{2}\right) - (-1)^k \frac{\pi}{4}\right) \end{aligned} \right\} \text{where } 0 \leq n \leq N - 1 \text{ and } 0 \leq k \leq M - 1 \quad (3)$$

Here N is the filter length. M is the number of channels in the filter bank.

2.3. ECG signal reconstruction measures

The quality of signal reconstructed after compression is assessed by different reconstruction error parameters. Let $x[n]$ and $y[n]$, be the original and retrieved signal. The percentage root-mean-square difference is defined as [17], [33-36],

$$PRD = \sqrt{\frac{\sum_{n=1}^N (x[n]-y[n])^2}{\sum_{n=1}^N (x[n])^2}} \times 100 \quad (4)$$

where N is the length of ECG signal.

As the PRD strongly depends on the mean value, it is better to use PRD1. In PRD1 the signal mean value is subtracted from the sample value. The PRD1 is evaluated by the Eq. (5) [36],

$$PRD1 = \sqrt{\frac{\sum_{n=1}^N (x[n]-y[n])^2}{\sum_{n=1}^N (x[n]-\bar{x}[n])^2}} \times 100 \quad (5)$$

where $\bar{x}[n]$ is the signal mean value.

If PRD1 value is between 0 and 9%, the quality of the recovered signal is either ‘very good’, or ‘good’ [34-36]. Hence it is ensured that the PRD1 value, using Eq. (5), must be below 9%. The quality of retrieved signal should also be ascertained subjectively by visual examination.

The compression ratio (CR) is expressed as Eq. 6. [36],

$$CR = \frac{\text{The number of bits in original ECG signal}}{\text{The total number of bits in the compressed data}} \quad (6)$$

Peak signal to noise ratio (PSNR) in decibels (db), is evaluated as per Eq. (7) [36],

$$PSNR(db) = 20 \log_{10} \frac{\max\{x[n]\}}{\sqrt{\frac{1}{N} \sum_{n=1}^N (x[n] - \bar{x}[n])^2}} \quad (7)$$

2.4. Proposed method

In this work convolved window-based cosine modulated filter banks are employed for ECG compression. The complete scheme of ECG compression system is shown in Fig. 1.

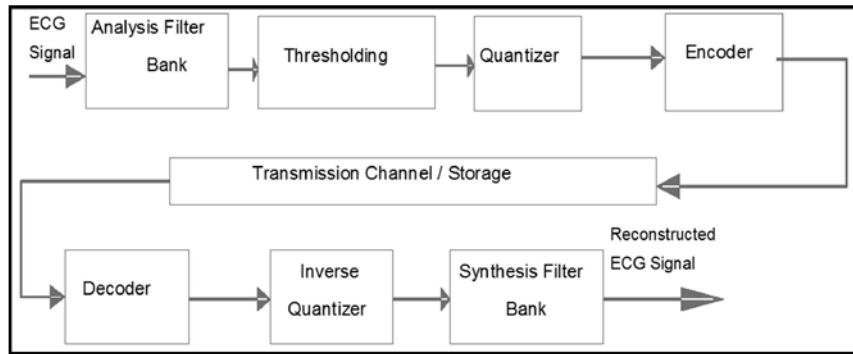


Fig. 1. ECG compression system.

A 32-channel Near-Perfect Reconstruction (NPR) cosine modulated filter bank with analysis and synthesis filters is designed for ECG signal decomposition and reconstruction. Prototype filters for CMFB are designed using three fixed window functions, namely, Hanning Window, Hamming Window and Blackman Window as well as their convolved versions [29, 30]. Fixed window functions such as Blackman window offer low peak sidelobe with relatively narrow transition bandwidth [37]. Convolution of these windows leads to further improvement in peak sidelobe behaviour and transition bandwidth. Passband edge frequency $\omega_p = 0.0156\pi$ and stopband edge frequency $\omega_s = 0.045\pi$, are taken as the filter design parameters [29, 30]. The prototype filters are optimized for reconstruction error using the golden section search method [29, 30]. These optimized prototype filters are then used to generate the filter banks [38].

2.4.1. ECG data compression encoder stage

The ECG sample signals are taken from the MIT-BIH arrhythmia database [39]. The 1024-baseline added to each lead for storage purpose was removed before processing. A block of 1024×32 ECG signal data samples is decomposed through analysis bank of the designed 32 channel NPR CMFB. The resulting signal is in frequency domain with non-uniform information distribution. Since ECG is a low frequency signal (up to 100Hz) most of the information is concentrated in initial low frequency subbands (up to 35 Hz). That means that the ECG signal, without significant loss of quality, can be represented using fewer samples from the lower bands of the analysis filter bank. The coefficients of subband decomposed signal $X_m(n)$ are arranged in the decreasing order of their absolute magnitude. The

threshold level limits are initialised. The *Thr_high* is initialised with the 50% of absolute value of the maximum coefficient value. The *Thr_low* is initialised with zero. The starting threshold value *Thr* is made equal to *Thr_high*. The subband decomposed signal coefficients having value lower than the threshold value *Thr* are discarded, that is, set to zero (insignificant coefficients). The significant coefficients are then Huffman entropy encoded [40] while the subbands with all or most of the coefficients zero are run length encoded (RLE). RLE is a lossless data compression technique. It compresses the data sequence by noting the similar data and its count of occurrence. A significant data sample map is also prepared to keep record of non-zero (significant) data samples in the subbands. The map consists of '0' and '1' values. A '0' indicates that the significant coefficient is zero and a '1' indicates that the significant coefficient is non-zero. The significant data sample map is also RLE encoded and is sent along with the entropy encoded significant samples, subband number and count of total significant coefficients. The total size of the encoded information is significantly less than the original input signal data sample size, thus achieving compression.

2.4.2. ECG data compression decoder stage

At the receiver side, the decoder extracts information from each encoded data block of every subband. The extracted information comprises of the signal length, entropy encoded significant coefficient samples, RLE encoded significant data sample map along with subband number and the count of total significant coefficients for that subband. The received entropy encoded significant samples are Huffman decoded while the significant data sample map is run length decoded. The '0' value in significant data sample map is decoded as '0'. The '1' value in the significant data sample map indicates significant coefficient whose value is obtained from the significant coefficient array. The process is repeated till all significant coefficients are placed at their corresponding locations as indicated by the significant data sample map. The process of decoding repeats to regenerate M-channels subband information at the receiver side.

All M-channel decompressed subbands are then sent to synthesis filter bank for reconstruction of ECG signal. Various ECG measures such as PRD, PRD1, CR and PSNR are computed from the obtained reconstructed ECG signal.

The PRD and PRD1 values are strongly dependent on number of significant coefficients contributing to the reconstruction signal. If the threshold is higher, a smaller number of significant coefficients will be available for signal reconstruction at the synthesis filter bank side. Hence the quality of reconstruction may have poor fidelity with higher values of PRD and PRD1 but has relatively better CR. If threshold is too low, large number of significant coefficients will be passed on to synthesis filter bank. After synthesis, the quality of reconstruction will be of high fidelity with low PRD1 values but relatively poor CR. Hence the threshold values are to be iteratively adjusted such that PRD1 value lies between 8.9 and 9.0% [34-36]. This is known as recursive thresholding. The entire process is repeated for new value of the threshold. This iterative process ends when the PRD1 value is achieved within the range of 8.9 to 9.0%. This method does not require any prior knowledge about the signal. So it can be applied to any recorded waveform.

The complete algorithm of the proposed method is given in the flowchart shown in Fig. 2.

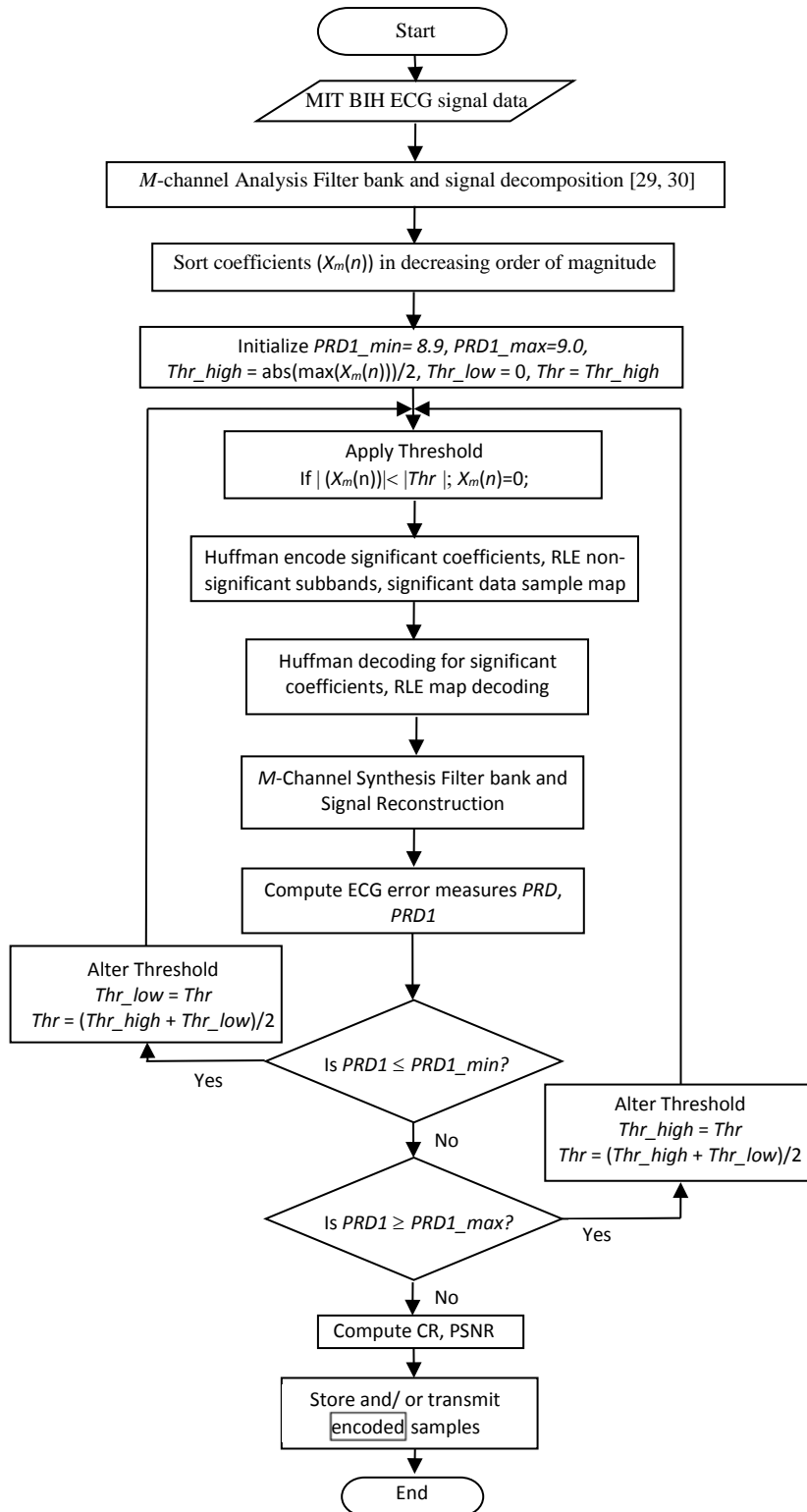


Fig. 2. Flowchart of the proposed ECG compression scheme.

3. Results and Discussions

The proposed work is simulated on the MATLAB v2015 run on Intel (R) Core (TM) I7-6500 processor (2.50 GHz). The experiment is carried out on the ECG signal Record No. 100, 107, 112, 117 and 119 of the MIT-BIH Arrhythmia database. The MIT-BIH database constitutes of 48 two-channel ambulatory ECG recordings of 30 minutes each. The sampling frequency of the recorded ECG signals is 360 Hz with a resolution of 11 bits per sample. Table 1 presents the values of performance parameters (PRD, PRD1, CR and PSNR) obtained for the five sets of ECG records using unconvolved and convolved windows based filter banks (PRD1 value is limited to the range of 8.9 to 9.0%). It is observed from the table that the CR obtained using convolved windows offer a marginal improvement as compared to corresponding unconvolved windows. Best results for compression (CR=11.16) and (PSNR=39.66 dB) are obtained for the ECG Record No.117, for a relatively low PRD value of 2.41. Lower PRD indicates less distortion in the retrieved signal and more resemblance to the original signal.

Table 1. Comparison of the Compression Ratio (CR) Obtained for different ECG Signals using unconvolved and convolved window-based filter bank designs (PRD1 = 8.9 to 9%).

Window	Unconvolved Window				Convolved Window			
	PRD %	PRD %	CR	PSNR dB	PRD %	PRD %	CR	PSNR dB
ECG Record No. 100								
Hanning	4.26	8.97	10.73	37.02	4.25	8.99	10.92	37.06
Hamming	4.26	8.99	10.38	36.98	4.25	8.98	11.16	37.00
Blackman	4.23	8.93	10.78	37.05	4.22	8.93	10.93	37.02
ECG Record No. 107								
Hanning	8.65	8.97	10.86	32.53	8.63	8.95	10.97	32.53
Hamming	8.60	8.91	10.90	32.53	8.65	8.97	10.95	32.49
Blackman	8.63	8.95	10.90	32.49	8.65	8.99	10.92	32.52
ECG Record No. 112								
Hanning	2.86	8.98	10.15	36.09	2.24	8.92	11.11	38.26
Hamming	2.25	8.99	10.38	38.23	2.24	8.95	10.97	38.21
Blackman	2.24	8.96	10.88	38.24	2.23	8.93	11.06	38.25
ECG Record No. 117								
Hanning	2.91	8.97	10.17	38.01	2.41	8.93	11.16	39.66
Hamming	2.40	8.91	11.16	39.58	2.41	8.94	10.96	39.64
Blackman	2.43	8.98	11.02	39.64	2.43	8.99	11.11	39.63
ECG Record No. 119								
Hanning	4.87	8.94	10.54	34.01	4.89	8.97	11.06	34.02
Hamming	4.88	8.95	11.11	34.01	4.87	8.92	10.97	34.02
Blackman	4.90	8.99	10.75	34.05	4.88	8.94	10.75	34.02

The CR and PSNR values are also determined over a wide range of PRD and PRD1 values for all the ECG records under study. Results corresponding to the ECG Record No.117 using convolved Hanning, Hamming, and Blackman windows are presented in Tables 2, 3 and 4 respectively.

Tables 2, 3 and 4 show that the PSNR value decreases with the increase in CR. For lower CR, the PSNR is higher and for higher CR values the PSNR is low. Thus the signal reconstruction quality reduces with the higher level of compression ratio. The analysis shows that sufficient level of compression is achieved for an acceptable range of reconstruction.

Table 2. Performance parameters with convolved Hanning Window (for ECG Record No. 117).

PRD%	PRD1%	CR	PSNR (dB)
2.06	7.60	9.65	41.04
2.35	8.68	10.97	39.89
2.41	8.93	11.16	39.66
2.48	9.16	11.75	39.42
2.58	9.55	12.62	39.05
3.04	11.25	13.66	37.63
4.97	18.36	16.83	33.38
8.93	33.02	21.90	28.28
15.10	55.83	31.61	23.71

Table 3. Performance parameters with convolved Hamming Window (for ECG Record No. 117).

PRD%	PRD1%	CR	PSNR (dB)
2.06	7.62	9.07	41.03
2.39	8.86	10.92	39.72
2.42	8.94	10.96	39.64
2.44	9.02	11.39	39.57
2.47	9.14	12.40	39.46
2.54	9.42	12.56	39.19
2.69	9.96	12.83	38.71
3.18	11.78	14.41	37.25
4.97	18.39	17.70	33.38

Table 4. Performance parameters with convolved Blackman Window (for ECG Record No. 117).

PRD%	PRD1%	CR	PSNR (dB)
2.26	8.36	10.69	40.23
2.42	8.96	11.11	39.63
2.57	9.50	12.23	39.12
2.86	10.58	13.84	38.19
3.47	12.82	15.63	36.52
4.97	18.13	18.19	33.58
5.51	20.35	19.12	32.51
9.11	33.69	27.07	28.13
15.84	58.56	39.24	23.33

The plot between PRD and CR for ECG record No.117 is illustrated in Fig.3. From this figure, it is evident that for the lower range of CR, the performance of all three convolved windows under study are comparable while for higher values of CR, the performance of convolved Blackman window is better than the other two windows. This figure also illustrates the comparison of the given algorithm with the result published by Kumar et al. [13], which is redrawn in blue color. It is clearly observed that the proposed technique provides much better compression ratio for a given PRD values [13]. The value of PRD in [13] is 5.04 when the CR achieved is 14.58 while the proposed technique yields higher CR values of 16.83, 17.70 and 18.19 (for Hanning, Hamming and Blackman window respectively) at comparatively lower PRD value of 4.97.

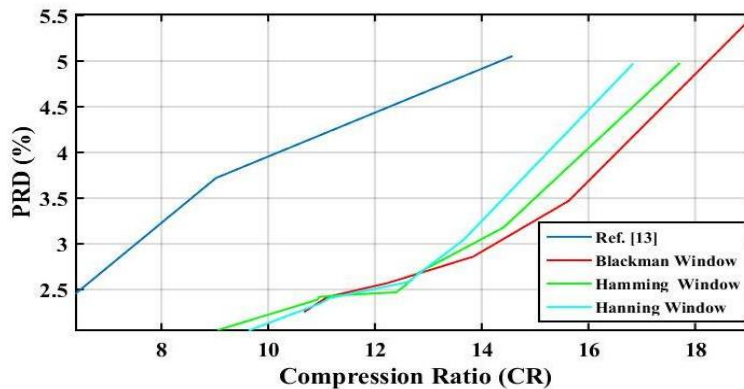
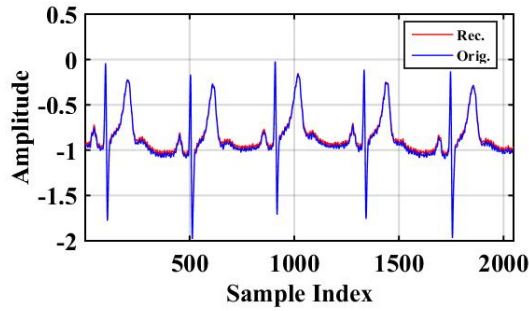


Fig. 3. Relationship between CR and PRD for Record No. 117.

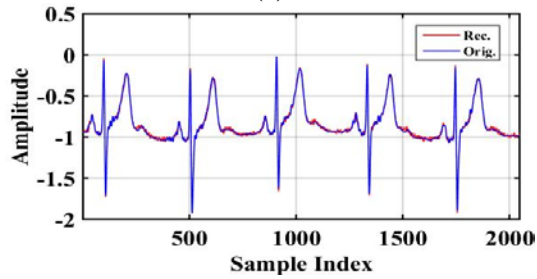
Figure 4(a) shows the original and reconstructed signals for ECG Record No. 117 using unconvolved Hanning window-based filter bank while Fig. 4(b) shows the original and reconstructed signals for ECG Record No. 117 using convolved Hanning window-based filter bank. Similarly, Figs. 5 and 6 show the original and reconstructed signals using unconvolved and convolved Hamming, and unconvolved and convolved Blackman window-based filter bank, respectively.

It is observed from these figures that the QRS complexes and all other vital morphological features of the ECG signal are excellently preserved in the reconstructed ECG signal. Since a large number of high frequency signals have been discarded due to recursive thresholding, the retrieved ECG signal appears to be smooth and free of high frequency noise.

A more diverse comparison of the performance of the proposed technique with other existing techniques is given in Table 5. The summary of performance parameters is given for the MIT-BIH ECG Record No.117 for all techniques referred in [5-8] and [13]. Generally, PRD is used as the performance parameter by the authors; however Craven et al. [8] have used PRD1 as the quality parameter. Therefore, PRD and PRD1, both values are mentioned in the table as per the results available in the corresponding research articles. The highest value of CR obtained through the technique proposed in this work is 11.16 with a relatively low value of PRD and PRD1. For a comparable PRD value, the results obtained are quite better as compared to [5-8] and [13]. Similarly CR is also higher as compared to the CR in [8] for a lower value of PRD1.



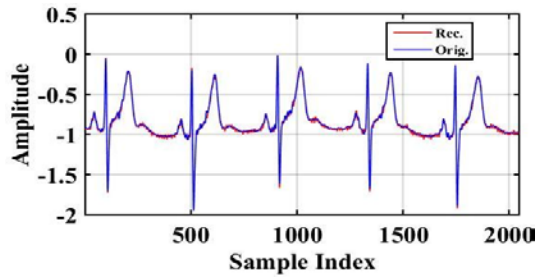
(a)



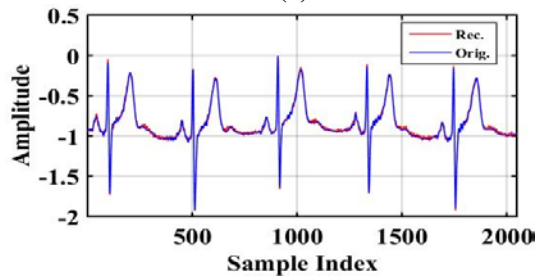
(b)

Fig. 4. ECG Record No.117 using Hanning window CMFB

(a)With unconvolved window (b) with convolved window.



(a)



(b)

Fig. 5. ECG Record No.117 using Hamming window CMFB

(a) With unconvolved window (b) with convolved window.

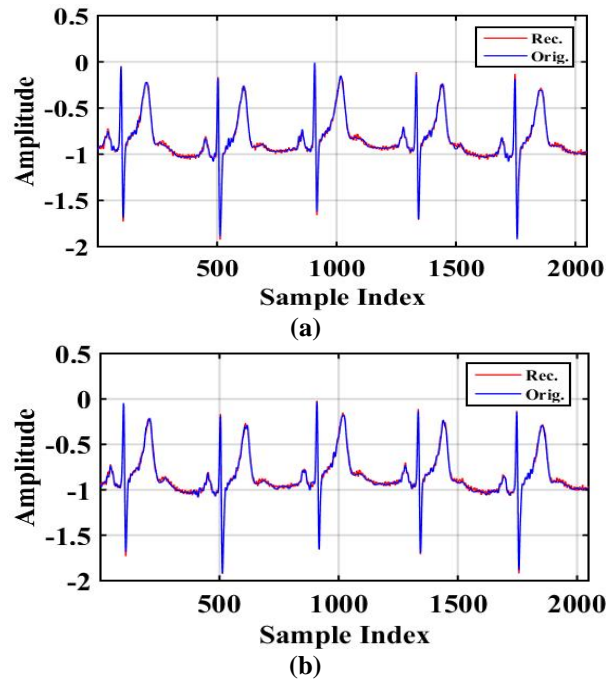


Fig. 6. ECG Record No.117 using Blackman window CMFB with unconvolved window (a) with convolved window.

Table 5. Performance comparison of the proposed method with other existing techniques for ECG record 117.

Algorithm	PRD %	PRD1 %	CR	Technique used
Sharma et al. [5]	2.09	-	5.98	Principal Component Analysis (PCA)
Kumar et al. [6]	2.5	-	5.95	Beta wavelet
Polan'ia et al.[7]	2.11	-	6.8	Compressed Sensing
Craven et al. [8]	-	9.0	10.88	Compressed Sensing
Kumar et al. [13]	3.72	-	9.02	Principal Component Analysis (PCA)
Proposed Algorithm	2.41	8.93	11.16	NPR CMFB (Convolved window-based)

The proposed method is also applied on ECG Record No.112 to compare the results published in [13] and it has been observed that the proposed method provides better compression with lower value of PRD. Based on Kumar et al. [13], the CR is 8.7 at a PRD value of 2.48 while in the proposed technique its value is 11.11 at 2.24 PRD.

4. Conclusions

In this paper, ECG data compression using fixed and the convolved windows based filter bank design has been presented. The result showed that the data compression of convolved windows is marginally superior to the unconvolved windows. The proposed scheme was evaluated for five different sets of ECG signals from MIT-BIH

ECG Arrhythmia database. The set of signals are of variety, having different rhythms, wave morphologies and abnormal heartbeats. Experimental results are evaluated on the basis of PRD, PRD1, PSNR and CR. Following are the observations made on the basis of investigations:

- The CR obtained using convolved windows offer marginal improvement as compared to corresponding unconvolved windows. It also gave better CR values when compared to other existing techniques.
- PSNR values obtained showed that the proposed method is also capable of achieving good quality of reconstruction.
- The traces indicate an excellent preservation of all vital morphological features on reconstruction of compressed ECG signal data.

A good degree of compression ratio with a good quality of reconstruction of ECG signal is desirable for the data storage and signal transmission. The proposed approach has demonstrated that it can be used in such situations where volumes of ECG data are to be stored/transmitted with simultaneous preservice of ECG signal morphologies which are of vital clinical importance. Such schemes will find place in systems like ECG Holter (ambulatory ECG), telecardiography and Internet of Things (IoT)/cloud based healthcare monitoring devices which is the trending future of the healthcare monitoring.

Nomenclatures

C	Order of convolution
$fk(n)$	Synthesis filter bank
$h_k(n)$	Analysis filter bank
$h_{id}(n)$	Impulse response of the ideal low pass filter
N_c	Length of convolved window
$p(n)$	Filter response
PRD	Percentage root mean square difference
$PRD1$	Percentage root mean square difference (sample value-mean value)
ω_c	Cut-off frequency
ω_p	Pass band frequency
$w(n)$	Window function
$w_l(n)$	Parent window function
$w_c(n)$	Convolved window function

Abbreviations

CMFB	Cosine Modulated Filter Bank
CR	Compression Ratio
CS	Compressed Sensing
ECG	Electro Cardio Gram
EZW	Embedded Zero Tree Wavelet
FIR	Finite Impulse Response
MECG	Multi-Channel Electro Cardio Gram
MIT-	Massachusetts Institute of Technology - Beth Israel Hospital
BIH	
MSPCA	Multi Scale Principal Component Analysis
NPR	Near Perfect Reconstruction

PCA	Principal Component Analysis
PRD	Percentage Root Mean Square Difference
PSNR	Peak Signal to Noise Ratio
RLE	Run Length Encoded
SLFOR	Side Lobe Fall-Off Rate

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