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INVESTIGATION OF MATHEMATICAL MODELS FOR HYDRO-VEHICLES WHICH CROSS THE LIQUID SURFACE

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Abstract

In this paper, we have considered the efficiency of using high-performance vehicles interacting with the water surface. Their productivity depends on the seaworthiness, which is determined by hydrodynamic characteristics of vehicles. The increasing speeds of the vehicles lead to the growth of hydrodynamic loads on the structural elements having the contact with the water surface, causing the instability of motion, as well as deformation and destruction of the bottom of boats. To create new hydro-vehicles we have to ensure the accordance of calculated hydrodynamic characteristics to real parameters of apparatus loads at the design stage, that supply sufficient seaworthiness, and therefore, to guarantee their reliability. In this investigation, a new mathematical model for studying hydrodynamic characteristics of boats is developed, which is specialized by experimental results of real tests on the free water. In this research, some numerical results and physical tests are proposed. Some calculations under real technical conditions are provided.

Keywords: Bodies movement, Free surface, Hydrodynamics, Hydrodynamic loads, Mathematical modelling.

1. Introduction

Meeting modern requirements for numerous vehicles that interact with a free fluid surface such as hydroplanes, airfoil boats, amphibians, ships, and others, demands to solve a set of scientific and technical problems. Among them, the determination of hydrodynamic load for preliminary strength analysis, which is required for further evaluation of vehicle motion parameters on the water surface, and it is considered to be the most time-consuming problem.

The increasing speed of vehicles leads up to the growth of hydrodynamic loads on the structural elements in contact with the water surface, causing the instability of motion, deformation, and destruction of the boat bottom. To avoid these negative factors, we have to calculate vehicle hydrodynamic characteristics that supply sufficient mechanical strength of vehicle construction, seaworthiness, and, therefore, to guarantee their reliability.

However, conducting a physical simulation is the time-consuming and financially costly procedure with expensive equipment and attendant personnel. At the same time, the availability of appropriate mathematical models that allow obtaining hydrodynamic characteristics of the vehicles under study with sufficient accuracy for practice will significantly reduce the time and costs for their production, optimize hydrodynamic vehicle layouts, and much more. It should be noted that the problem of determining the hydrodynamic loads on the hull of vehicles and the study of their hydrodynamic characteristics are not completely solved as a scientific and technical task. Firstly, this is due to the complexity of constructing appropriate adequate mathematical models, as well as the necessity of their numerical implementation via computer technology. In the last case, as a rule, we cannot solely rely on a general algorithm for a numerical realization of mathematical models, however, we should also rely on the use of engineering intuition in the algorithmization of the physical phenomenon and reasonably accepted limitations and assumptions.

Traditionally, two main approaches are used to solve this problem: calculation based on the semi-empirical or numerical models and physical modelling in hydro-pools or on open water by pulling geometrically similar models of the real objects under study. Final refinement of construction is carried out in the process of full-scale testing of designed boats. In this research, we propose a new mathematical model to study hydrodynamic characteristics of boats, which are based on solving bounded integral equations in core Cauchy and some numerical results and physical tests are examined; moreover, some calculations in real technical conditions are presented.

Wagner's [1] theory is the main background for the modern hydrodynamic research of flows with free surfaces is linear. In the former USSR, numerous research reports by Logvinovich [2], Pierson [3], Osipov [4] and Tihonov [5] were based on this concept beginning from the 30-ies of the last century in Central Aero-Hydrodynamic Institute (CAHI) named after Joukowski [2]. However, linear Wagner's theory can be applied only to flat-shape bodies. In addition, this concept ignores perturbations on the free stream boundary and supposes that body velocity is constant during its penetration into the fluid.

In fact, the hydrodynamics problem of flows with free stream boundaries is considerably nonlinear. Firstly, the fluid boundary shape is unknown and has to be determined. Secondly, the dynamic condition on this boundary is nonlinear and wetted shape changes and has to be determined. Not all of these questions are covered by the traditional theory of aerodynamics and ship hydrodynamics, which create serious mathematical difficulties when being solved in a general case.

The existing methods of hydroplanes hydrodynamics calculation are so inadequate, therefore, the physical model experiments or real tests are carried out in every particular case. As it is known, experiments with the physical model are very expensive; moreover, they have poor informative value and some difficulties in the law of motion determination. Real tests are too complicated from the technical and organizational point of view. Besides, they are limited by safety conditions during critical regimes testing and are inefficient in design.

Results of experimental studies by Qu-yuan [6] and Betayev [7], made possible to improve mathematical models of hydrodynamics. The number of scientific articles such as Jian-wei and Le-sheng [8], Ingber and Hailey [9], Zhi-rong et al. [10], Xu et al. [11], Gaudet [12], Chein and Chung [13], Wang and Jianmin [14], Chizhiumov et al. [15], Kumar et al. [16] and Petrova et al. [17] described a new approach to solving the problem of hydrodynamics with free boundaries, however, they assume that the flow is flat with numerous restrictions.

Unlike the well-known approaches, the task is considered here in a more general formulation, taking into consideration the following factors.

- An arbitrary law of motion, body geometry and its deformation changes.
- Splash-flushes influence and finite perturbations of free fluid boundaries.
- Geometry change of the wetted surface during the motion.
- Initial fluid velocity and free surface shape perturbation (waves of finite amplitude).

The model of a frictionless incompressible fluid is eligible for solving a number of technical hydrodynamics problems for motion modes and diverse body shape. Thus, our goal is to consider the peculiarity of problems in mathematical models of hydrodynamics that related to real nonlinear effects of dynamics.

2. Problem Formulation

Let the frictionless incompressible heavy fluid settle down in the lower half-space with a border, which contains the wetted surface $S(t)$ of an elastic body during motion, free surface (constant pressure) $\sum(t)$ and vortex wake following the body $\sigma(t)\subset\sum(t)$, Fig. 1 [18]. Further, we designate $\sum^{-}$, S^{-} and $\sum^{+}$, S^{+} as lower and upper borders and body sides, correspondingly.

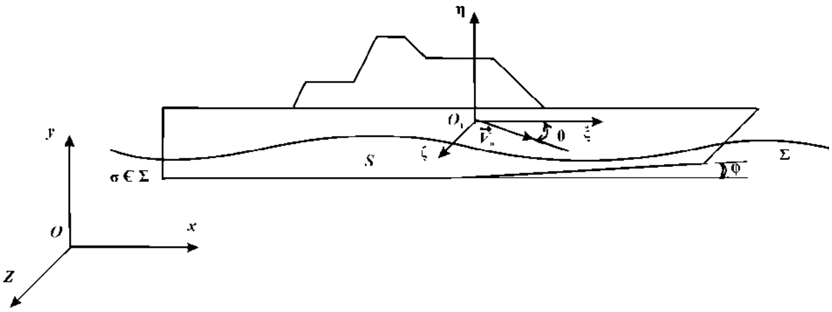

Fig. 1. Principal scheme of ship's motion on a liquid surface.

On waterline parts, where vortex wake following the body appears, Chaplygin-Joukowski (Ch-J) condition [2] should be performed. Spray-roots can be generated on some waterline parts. According to physical sense, the velocity here is finite, so Ch-J condition should be performed too. Mayboroda [18] performed the the edge condition, in which, body wetted surface may have lines of corner points (keel, cheekbones).The supposition of velocity field continuity at internal fluid points demands the performance of edge Ch-J condition in the Zommerfeld form on these lines.

Potential $\Phi(t,\vec{r})$ of perturbation velocities is supposed as a harmony function at inside points of fluid half-space.

$$\Delta\Phi(t,\vec{r}) = 0, \vec{r} \notin S(t) \cup \sigma(t) \cup \Sigma(t)$$

It should be satisfied in all time taking into consideration the following limit conditions:

- The condition of non-percolation at all points of a wetted surface of a body $S(t)$ is given in the form.

$$\frac{\partial\Phi(t,\vec{r})}{\partial n} = \vec{u}(t,\vec{r})\vec{n}(t,\vec{r}) - \frac{\partial\Phi_{н}(t,\vec{r})}{\partial n}, \quad \vec{r} \in S(t) \tag{1}$$

where $\vec{n}$ is an external normal to a body; $\Phi_{н}(t,\vec{r})$ is the potential of no turbulent velocities motion of liquid; $u(t,\vec{r})$ is the velocity vector at points of $S(t)$.

$$u(t,\vec{r}) = \vec{V}_0(t) + \vec{\Omega}(t) \times (\vec{r}(t) - \vec{r}_0(t)) + \vec{V}_d(t,\vec{r}), \vec{r}(t) \in S(t)$$

where $\vec{V}_d(t,\vec{r})$ is the deformation motion velocity vector at body surface points; $\vec{r}_0$ is the radius-vector at centre mass of the object.

- There is the kinematic condition of simultaneous motion of all points on the lower boundary of fluid surface in the form.

$$\frac{\partial\Sigma(t,\vec{r})}{\partial t} + \left(\text{grad}\Phi(t,\vec{r}), \text{grad}\Sigma(t,\vec{r})\right) + \left(\text{grad}\Phi_H(t,\vec{r}), \text{grad}\Sigma(t,\vec{r})\right) = 0 \tag{2}$$

where $\vec{r} \in \Sigma^-(t)$ and a dynamic condition may be presented in the Cauchy-Lagrange form.

$$\frac{\partial\Phi(t,\vec{r}')}{\partial t} + \frac{1}{2}\left|\text{grad}\Phi(t,\vec{r}')\right|^2 - \left(\text{grad}\Phi(t,\vec{r}'), \vec{V}_o(t)\right) - \vec{g}(\vec{r}' - \vec{r}_\infty) = 0 \tag{3}$$

where $\vec{r}' \to \vec{r} \in \Sigma^-(t)$ and $\vec{g}$ is the vector free-fall acceleration; $\vec{r}_\infty$ is the radius-vector at the point of an unperturbed free boundary. According to Belotserkovskii and Nisht [19], at points of a free boundary, Eq. (3) is solved together with the equation motion of liquid points in Euler's form.

$$\frac{\partial\vec{W}(t,\vec{r})}{\partial t} + \left(\vec{W}(t,\vec{r})\vec{\nabla}\right)\vec{W}(t,\vec{r}) = -\frac{1}{\rho}\text{grad } p(t,\vec{r}) + \vec{g} \tag{4}$$

where $\vec{r} \in \Sigma^{-}(t)$, $\vec{W}(t,\vec{r})$ is the absolute velocity vector at the points of lower fluid half-space; $p(t,\vec{r})$ is the pressure at the points of lower fluid half-space; ρ is the mass density of fluid; $\vec{\nabla}$ is the Hamilton operator.

- At all points of *S*(*t*) the Ch - J condition is performed in the form.

$$\left|\text{grad}\Phi(t,\vec{r}\,')\right| = 0(1),\ \vec{r}\,' \to \vec{r} \in S(t) \tag{5}$$

- On vortex free wake $\sigma(t,\vec{r}) \subset \Sigma(t)$ behind *S*(*t*) the conditions Eqs. (2) to (4) are also carried out.
- At infinity in front of the body, the condition of the absence of perturbations at all points of fluid is.

$$\lim\Phi(t,\vec{r}) = \lim\nabla\Phi(t,\vec{r}),\ |\vec{r}| \to \infty \tag{6}$$

The problem in the formulation Eqs. (1) to (6) in common case has to be completed by the condition of Tomson theorem [18] for points in lower fluid half-space, including the area of fluid perturbation motion written in the form.

$$\frac{\partial}{\partial t}\oint_{S(t),\Sigma^{-}(t,\vec{r})\cup\sigma(t,\vec{r})} \vec{W}(t,\vec{r})d\vec{r} = 0$$

3. Mathematical Models

The analytical solution Eqs. (1) to (6) is impossible, we will propose the numerical method of their solution [18].

3.1. Singular-integral equation of carrier surface

Let us imagine the border of fluid $S(t)\cup\Sigma(t), \sigma(t,\vec{r})\cup\Sigma(t)$ of vortex system surface $\vec{\gamma} = \{\vec{\gamma}_{\Gamma}(S(t)), \vec{\gamma}_{\delta}(\sigma(t,\vec{r})), \vec{\gamma}_{G}(\Sigma(t))\}$. Fluid perturbed motion velocity is described by integral in Cauchy's form.

$$\vec{W}(t,\vec{r}) = \frac{1}{4\pi}\int_{S\cup\Sigma} \frac{\vec{\gamma}(t,\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^{3}} dS(\vec{r}\,') \tag{7}$$

It suits almost all free surface shapes and should be understood as the main meaning.

Let us assume that $|\vec{\gamma}|$ satisfies Helder condition and $S(t)\cup\Sigma(t)$ must be smooth according to Lyapunov [18]. Only lines of corner points (sector spray-root on waterline $S(t)\cap\Sigma(t)$, keel and cheekbones on *S*(*t*)) are assumed on $S(t)\cup\Sigma(t)$. The velocity field continuity at fluid internal points is supposed, and the performance of condition $\vec{\gamma} = 0$ in Eq. (7) on lines of a corner, points are required. It corresponds to the edge condition in Zommerfeld form [18] for keel and submerged cheekbones of the body. At some waterline sectors $S(t)\cap\Sigma(t)$, the formation of spray-roots is possible. Tops of these spray-roots are corner points with finite fluid velocity. The density $\vec{\gamma}$ of Cauchy's integral may also tend to zero in this case.

The performance of condition Eqs. (1) with (7) at points of wetted body surface S(t) gives the singular-integral equation of the first sort.

$$\vec{n}(t,\vec{r})\ \{\int_S \frac{\vec{\gamma}_\Gamma(t,\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dS(\vec{r}\,')+\int_\Sigma \frac{\vec{\gamma}_G(t,\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}d\Sigma(\vec{r}\,')$$
$$+\int_\sigma \frac{\vec{\gamma}_\delta(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}d\sigma(\vec{r}\,')\}=4\pi\vec{n}(t,\vec{r})[\vec{V}_o(t)+\vec{\Omega}(t)\times(\vec{r}(t)-\vec{r}_o(t)) \tag{8}$$
$$+\vec{V}_d(t,\vec{r})-\vec{V}_H(t,\vec{r})]\,,\quad \vec{r}\in S(t)$$

where $V_H(t,\vec{r})$ is the velocity of fluid unperturbed motion.

Equation (8) is incorrect in steady metric and its solution can be implemented only numerically. The unknown vortices sheet $\vec{\gamma}_\Gamma(S(t))$ of the body-wetted surface $S(t)$ contains the required vortices $\vec{\gamma}_G(t,\vec{r})$ on the free water surface Σ. In accordance with Helmholtz theorems [18], the density $\vec{\gamma}_\delta(t,\vec{r})$ of free vortexes is known and remains constant.

Body waterline $S(t)\cap\Sigma(t)$ during motion with rather a high velocity contains two types of points. The first one coincides with the corner points of surface $S(t)$ (rear profile of a body, cheekbones, and others), keeps their position during body motion and may serve as a place of the body vortex free wake $\sigma(t,\vec{r})\in\Sigma(t)$. The second type belongs to the spray-root generating zone, where body load decreases asymptotically to zero and these points may be considered as the corner points of the boundary $S(t)\cup\Sigma(t)$ with $\vec{\gamma}(t,\vec{r})=0$.

The potential theory admits here the solution with the power peculiarity and infinite loading on a waterline. However, the experimental data admit searching the solution (8) in a class of functions with the finite loading on the waterline.

Equation (8) is not determined because it contains unknown $\vec{\gamma}_\Gamma(S)$ on the body and it contains the required $\vec{\gamma}_G(t,\vec{r})$ on the free boundary $\Sigma(t)$. Thus, for the solution (8) it is necessary to create previously the mathematical model of heavy fluid free surface perturbed motion (ship waves), which allows the determination $\vec{\gamma}_G(t,\vec{r})$ through $\vec{\gamma}_\Gamma(S)$.

3.2. Mathematical model of ship wave of finite amplitude

Let us consider the nonlinear problem of ship wave's hydrodynamics for a fluid of infinite depth [18]. Velocity potential $\Phi(t,\vec{r})$ of waves, propagating from the motion of source with constant velocity $\vec{V}_0$ is the harmonic function at internal points of fluid half-space with a free border $\Sigma(t)$ and has to satisfy the border conditions Eqs. (2) to (4) and (6).

Dynamic condition Eq. (3) on $\sum^{-}(t)$ allows determining the limit value $\vec{V}_{\Sigma}(t,\vec{r})$ on $\sum(t)$ in Cartesian's coordinate system, which looks as follows

$$\left|\vec{V}_{\Sigma}(t,\vec{r})\right| = \left\{V_0^2(t) - \left[W_n(t,\vec{r}) - V_{0n}(t,\vec{r})\right]^2 - 2\left[\frac{\partial \Phi(t,\vec{r})}{\partial t} + g(y(t) - y_{\infty}(t))\right]\right\}^{\frac{1}{2}}, \quad (9)$$

$$\vec{r} \in \Sigma^{-}(t)$$

where W_n is the absolute velocity normal component at $\sum(t)$ points; $\vec{V}_0(t)$ is the free stream velocity and V_{0n} is the normal component at $\sum(t)$ points.

Substituting fluid free surface $\sum(t)$ by vortex system $\vec{\gamma}(t,\vec{r}) \in \Sigma(t)$ and determining the perturbed velocities potential in upper half-space additionally, one gets space analogy of Sokhotsky equation system [18] as:

$$\begin{aligned} &\vec{W}^{+}(t,\vec{r}) - \vec{W}^{-}(t,\vec{r}) = \vec{\gamma}(t,\vec{r}) \times \vec{n}(t,\vec{r}), \\ &\vec{W}^{+}(t,\vec{r}) + \vec{W}^{-}(t,\vec{r}) = \frac{1}{2\pi}\int_{\Sigma} \frac{\vec{\gamma}(t,\vec{r}^{'}) \times (\vec{r} - \vec{r}^{'})}{\left|\vec{r} - \vec{r}^{'}\right|^3} d\Sigma(\vec{r}^{'}) \end{aligned} \quad (10)$$

where $\vec{W}^{+}(t,\vec{r})$, $\vec{W}^{-}(t,\vec{r})$ are the perturbed velocities limit meanings for points $\vec{r}$ on the fluid boundary $\sum(t)$ (" + " from above," − " from below).

Lemma 1. In accordance with Eqs. (9) and (10), the free boundary shape at points at all time determined by both $\vec{\gamma}(t,\vec{r}) \in \Sigma(t)$ is represented by the nonlinear equations system in the form.

$$\left.\begin{aligned} &\vec{\gamma}(t,\vec{r}) = 2\vec{n}(t,\vec{r}) \times (E - \vec{n}(t,\vec{r})\vec{n}^T(t,\vec{r}))[\frac{1}{4\pi}\int_{\Sigma} \frac{\vec{\gamma}(t,\vec{r}) \times (\vec{r} - \vec{r}')}{\left|\vec{r} - \vec{r}'\right|^3} d\Sigma(\vec{r}') \\ &- \frac{\operatorname{sgn}(W_n(t,\vec{r})}{\left|\vec{n}(t,\vec{r}) \times \vec{g}\right|} \times [V_0^2(t) - (W_n(t,\vec{r}) - V_{0n}(t,\vec{r}))^2 \\ &- 2(\frac{\partial \Phi(t,\vec{r})}{\partial t} + g(y(t) - y_{\infty}(t))]^{\frac{1}{2}} \vec{g} - \vec{V}_0(t)], \\ &W_n(t,\vec{r}) = \frac{\vec{n}(t,\vec{r})}{4\pi}\int_{\Sigma} \frac{\vec{\gamma}(t,\vec{r}) \times (\vec{r} - \vec{r}^{'})}{\left|\vec{r} - \vec{r}^{'}\right|^3} d\Sigma(\vec{r}^{'}), \quad \vec{r} \in \Sigma(t), \\ &\frac{d\vec{r}}{dt} = \frac{1}{4\pi}\int_{\Sigma} \frac{\vec{\gamma}(t,\vec{r}) \times (\vec{r} - \vec{r}^{'})}{\left|\vec{r} - \vec{r}^{'}\right|^3} d\Sigma(\vec{r}^{'}) - \frac{1}{2}\vec{\gamma}(t,\vec{r}) \times \vec{n}(t,\vec{r}) - \vec{V}_0(t), \quad \vec{r} \in \Sigma(t) \end{aligned}\right\} \quad (11)$$

where $(E - \vec{n}(t,\vec{r})\vec{n}^T(t,\vec{r}))$ is the operator of vector decision on the plane at a point with $\vec{n}(t,\vec{r})$; E is the unit matrix; $\vec{n}^T(t,\vec{r})$ is the transposed of a normal vector $\vec{n}(t,\vec{r})$ and $\vec{n}(t,\vec{r})\vec{n}^T(t,\vec{r})$ is the multiplication of matrixes.

Equation (11) system in Lemma 1 is the mathematical model of free boundary motion in accordance with the kinematic condition Eq. (2). Realisation Eq. (11) is performed by discrete vortexes method [13, 20] and it showed good results in comparison with exact analytic solutions (two-dimensional and three-dimensional), as well as with numerical and experimental results of some authors in the field of ship waves theory for Froude's numbers *Fr*> 1.0 [21].

3.3. Mathematical model of hydrodynamics of carrier surface

Equation (8) with the system, Eq. (11) allows constructing hydrodynamics mathematical model for vehicles that interact with a free surface of fluid and permit to formulate the next lemma.

Lemma 2. If take Eq. (8), the system Eq. (11) with conditions Eqs. (1) to (6), the vortex sheets density and shape of a free boundary, wake and body-wetted surface at points at all time is described by the nonlinear equations system in the form.

$$\left.\begin{aligned}
&\vec{n}(t,\vec{r})\int_{S\cup\Sigma}\frac{\vec{\gamma}(t,\vec{r})\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dS(\vec{r}\,') = 4\pi\,\vec{n}(t,\vec{r})[\vec{V}_0(t)+\vec{\Omega}(t)\times(\vec{r}(t)-\vec{r}_0(t))\\
&+\vec{V}_d(t,\vec{r})-\vec{V}_N(t,\vec{r})],\ \vec{r}(t)\in S(t),\\
&\vec{\gamma}_G(t,\vec{r}) = 2\vec{n}(t,\vec{r})\times(E-\vec{n}(t,\vec{r})\vec{n}^T(t,\vec{r}))[\frac{1}{4\pi}\int_{S\cup\Sigma}\frac{\vec{\gamma}(t,\vec{r})\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dS(\vec{r}\,')\\
&-\frac{\operatorname{sgn}(W_n(t,\vec{r})}{|\vec{n}(t,\vec{r})\times\vec{g}|}\times[V_0^2(t)-(W_n(t,\vec{r})-V_{0n}(t,\vec{r}))^2\\
&-2(\frac{\partial\Phi(t,\vec{r})}{\partial t}+g(y(t)-y_\infty(t))]^{\frac{1}{2}}\vec{g}-\vec{V}_0(t)],\ \ \vec{r}(t)\in\Sigma(t),\\
&W_n(t,\vec{r}) = \frac{\vec{n}(t,\vec{r})}{4\pi}\int_{\Sigma}\frac{\vec{\gamma}(t,\vec{r})\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}d\Sigma(\vec{r}\,'),\qquad \vec{r}\in\Sigma(t),\\
&\frac{d\vec{r}}{dt} = \frac{1}{4\pi}\int_{\Sigma}\frac{\vec{\gamma}(t,\vec{r})\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dS(\vec{r}\,')-\frac{1}{2}\vec{\gamma}_G(t,\vec{r})\times\vec{n}(t,\vec{r})-\vec{V}_0(t),\qquad \vec{r}\in\Sigma(t)
\end{aligned}\right\}\quad (12)$$

The system Eq. (12) assumes that $|\vec{\gamma}(t,\vec{r})|$ satisfies Helder condition and strives to zero at boundary corner points. Thus, Ch-J condition on waterline $S(t)\cap\Sigma(t)$ is performed, condition on edges of the border $S(t)\cup\Sigma(t)$ (keel, cheekbones, crests of nonlinear waves, spray-roots tops) in Somerfield form and the destruction mechanism of nonlinear waves and spray-roots is reproduced [18].

Realisation of the mathematical model Eq. (12) (two-dimensional and three-dimensional) is performed by the method of discrete vortexes [13, 20]. While constructing the wetted body surface and wake vortex model, both Tomson and Helmholtz theorems are executed.

Spray-roots are simulated by additional vortex sheets in waterline $S(t)\cap\Sigma(t)$ vicinity according to Joukowski scheme in order to get the solution with limited loading in a spray-root zone.

4. Results and Discussion

Particular cases of approbating mathematical models [18] through numerical realization by the method of discrete vortexes [13, 20] for flat and spatial arrangements have been reflected in a number of research works for example [21]. However, it should be noted that the experience of the mathematical model's implementation [18] led to the conclusion that they should be used primarily for local hydrodynamic problems (local strength estimation). This led to the necessity of considering the variable shape of the wetted surface even when motion is simulated on calm water.

Further, the presence of an agitated water surface in the numerical model including irregular waves is taken into consideration. It causes a massive calculation of parameters in the nodes, as well as continuous vortexes of the wetted surface of the boat bottom, which is described by recalculation mesh at every time moment. In addition to the above mentioned, particular methodologies for implementing mathematical models for planar and spatial layouts are proposed to attribute to the block of problems of local hydrodynamics. These are mainly studies related to the determining the distribution of pressures on the wetted bottom surface on the initial stage of landing (landing impact and gliding) when the interaction with the water surface on maximum speeds occurs. In other words, these studies are carried out under the conditions of the effect of maximum hydrodynamic loads with the purpose of analysing the strength of the bottom, providing the initial information for its calculations, determining the limitations of kinematic parameters, and solving other problems.

The research of the dynamics of objects behaviour on a calm and agitated water surface, including irregular waves, was carried out in the interest of solving problems of motion stability, determining balancing characteristics, total strength, as well as optimizing the hydrodynamic layout. This research was performed based on the synthesis of various methods [22], namely flat sections [2, 5] and discrete vortices [20]. To determine the hydrodynamic characteristics of the cross-sections the technique of implementing the mathematical model [18] for a particular case of vertically immersed contours into an ideal weighty fluid [21] is used. The model is free from restrictions of the bottom frames geometry of the boat bottoms. This technique has shown the high efficiency of calculations. In the general ideology of research, this is attributed to the problems of general hydrodynamics. In problems of this type, there is an essential change in the kinematic parameters, which requires solving the corresponding system of differential equations of motion.

Aerodynamic characteristics in the right parts of the motion equations are determined due to the synthesis of methodologies that realise linear and nonlinear mathematical models of aerodynamics using the method of boundary integral equations. Pressure resistances are defined as components of the total aerodynamic and hydrodynamic forces. To consider friction resistance, well-known approaches of the ship and aerodynamics theories [23] have been used. The Neumann spectrum, which is widely utilised in the ship's rolling theory [23] has been used in the simulation of the water surface state. For comparing the hydrodynamic characteristics that are calculated by developing mathematical models we used a well-known generalization of the experimental and numerical materials [20, 22, 23]. Examples of numerical realisation of mathematical models (12) for some actual technical problems are given in Figs. 2 to 6.

Figure 2 illustrates some theoretical, experimental and numerical results presented in works by Wagner [1], Pierson [3] and Zhao and Faltinsen [20], where C_n is the coefficient of normal hydrodynamic force for the different dead rise angles β of bottom rigid plates during immersion into the fluid with constant velocities and large Froude numbers (Fr_Δ> 5).

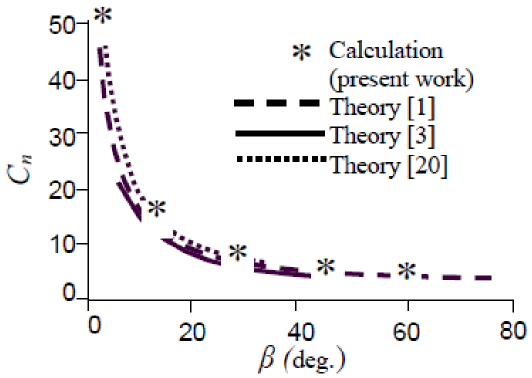

Fig. 2. **Coefficient of normal hydrodynamics forces on flat dead rise wedge for different meanings dead rise angles during immersion into liquid with constant velocities.**

Some numerical results of distributed hydrodynamic loads by water landing have been obtained and illustrated in Fig. 3 (numbers of steady plate profiles along the keel to the bottom-nose). These calculations were performed for unsteady gliding of rigid dead rise bottom of amphibian flying-boat «Lutama» with β = 22.5° on step for the full mass of amphibian 2500 kg, the trim angle φ = 5.5° and flight-path angles θ = 0° under the condition of constant velocity, amphibian flying-boat dead rise bottom for wave altitude $h_w = 0.25$ m and various data wavelength $\lambda_w = 15$ m, 25 m. Numerical values of common hydrodynamic forces are obtained on distributed loads during steady gliding on long regular waves for an angle of deadrise flying-boat bottom and match to physical CAHI test [21].

Fulfilling the calculations for water-landing and unsteady gliding of amphibian flying-boat elastic bottom, we assumed its hydro elastic vibration for the first tone as a keel and waterline clamped-edge diaphragm. Own frequencies and their amplitudes were obtained out of a real amphibian unit of vibration test. Summary and distributed hydrodynamic loads for various vibration phases, deadrise angles, flight-path angles, and trim angles were calculated. Wetted surface vibration in the studied range of its parameters increases maximal local and summary loading approximately in twice and 1.7 times correspondingly compared to rigid flying-boat bottom.

Application of the described above mathematical models for surface ship hydrodynamics studies is illustrated in Fig. 4. Calm water towing resistance (ε is the draft-lift ratio) as a function of Froude number Fr_Δ is considered for the well-known water glider model BK-1 [23] with the force-displacement 1050 N, the load coefficient C_Δ=0.598 and various ship centrings $\bar{x}_g$ ($\bar{x}_g = x_g / L$, where x_g is the distance of gravity centre from the stern, L= 2.94 m is the ship cheekbone line length). In Fig. 4, the results are given for:

$$1 - \bar{x}_g = 0.35;\ 2 - \bar{x}_g = 0.4;\ 3 - \bar{x}_g = 0.45.$$

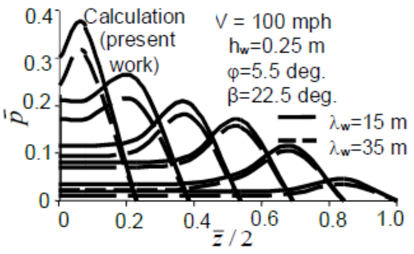

Fig. 3. Distributed hydrodynamic loads on dead rise flying-boat bottom of amphibian during steady gliding on calm water.

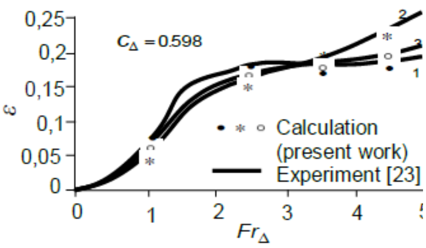

Fig. 4. Towing resistance of ship surface during motion on calm water.

The well-known approach to friction resistance determination was used here. Unlike model tests in towing tank, by means of mathematical modelling one can predict ship version on an arbitrary confused sea in the self-propelled regime with known propulsion characters.

Figure 5 shows the calculated balancing characteristics (calm water) such as balancing trim angle φ_{bal} in the function of the relative speed Fr_Δ (the Froude number in terms of displacement) for the average centring ($\bar{x}_g = 0.29$ of the average aerodynamic chord of the wing) of a light amphibious aircraft «Lutama» with the full take-off mass 2500 kg [22].

Normal hydrodynamic acceleration n_y of amphibian flying-boat deadrise bottom «Lutama» with the take-off mass of amphibian 2250 kg on regular waves with h_w = 0.45 m was calculated. Figure 6 illustrates its numerical results.

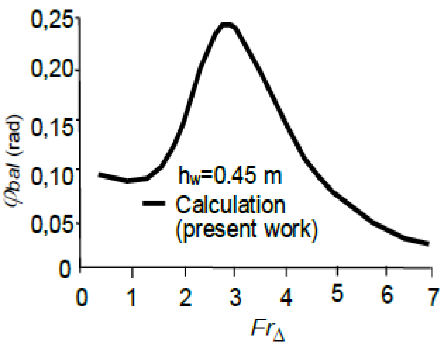

Fig. 5. Amphibians trim angles of balance during take-off on calm water.

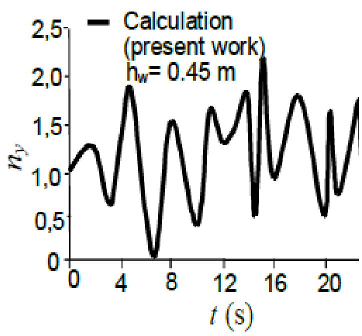

Fig. 6. Normal hydrodynamic acceleration of amphibian during take-off on a regular wave.

5. Conclusions

Nonlinear unsteady hydrodynamics mathematical models of bodies during their interaction with free a fluid surface are examined in this paper. New mathematical models are presented, which allow obtaining the hydro-aerodynamic characteristics of vehicles (amphibians, hydroplanes, ekranoplanes, ships and other objects during their interaction with the free fluid surface).

In addition, such analysis allows us to estimate the following: local strength; peculiarities in the behaviour of object under study with a different state of the water surface; the required power of the power plant; determine the dynamic characteristics during acceleration; possible overloads in the sections of vehicles, which are interesting for research; form restrictions on the kinematic and many other parameters.

In general, it allows increasing the efficiency of hydrodynamic design, which is very important for creating reliable high-performance hydro-vehicles in contact with the water surface and increasing the safety of their operation.

Mathematical models and approaches considered in this paper will be the most prospective for solving nonlinear problems in hydrodynamics. The usage of computational experiment results on the basis of a reliable mathematical model and methods of its realization could be taken as a scientific basis for modern technical applications.

Nomenclatures

C_Δ	Load coefficient (Fig. 4)
C_n	Coefficient of normal force (Fig. 2)
E	Unit matrix
Fr	Froude number
Fr_Δ	Froude number in terms of displacement (Figs. 4 and 5)
$\vec{g}$	Vector of free-fall acceleration
h_w	Wave altitude (Figs. 3 and 6), m
L	Ship cheekbone line length, m
$\vec{n}(t,\vec{r})$	External normal to HV
n_y	Normal hydrodynamic acceleration (Fig. 6)
$p(t,\vec{r})$	Pressure at the points of lower fluid half-space, Pa
$\overline{p}(t,\vec{r})$	Dimensionless pressure (Fig. 3)
$\vec{r}$	Radius-vector at the point of $\Sigma(t)$ and $S(t)$
$\vec{r}_0$	Radius-vector at centre mass of HV
$\vec{r}_\infty$	Radius-vector at the point of an unperturbed free boundary
S^-, S^+	Lower and upper body sides
$S(t)$	Wetted surface of elastic HV body (Fig. 1)
T	Index of transpose
t	Time (Fig. 6), s
$\vec{u}(t,\vec{r})$	Velocity vector at points of HV wetted body surface.
$\vec{V}_d(t,\vec{r})$	Deformation motion velocity vector at HV body
$\vec{V}_H(t,\vec{r})$	Velocity of fluid unperturbed motion
$\vec{V}_0(t,\vec{r})$	Free stream velocity

$\vec{V}_\Sigma(t,\vec{r})$	Limit value $\vec{V}_\Sigma(t,\vec{r})$ of $\Sigma(t,\vec{r})$
$V_{0n}(t,\vec{r})$	Normal component of velocity at free border
$\vec{W}(t,\vec{r})$	Absolute velocity vector
$W_n(t,\vec{r})$	Absolute velocity normal component at free border
$\vec{W}^{\pm}(t,\vec{r})$	Perturbed velocities limit meanings on fluid boundary (" + " from above, " − " from below)
x_g	Distance of gravity center from the stern, m
$\bar{x}_g$	Ship centring $\bar{x}_g = x_g / L$ (Fig. 4)
$\bar{z}/2$	Dimensionless semi-width of HV bottom (Fig. 3)

Greek Symbols

∇	Hamilton operator
β	An angle of flying-boat rigid deadrise bottom (Fig. 2), deg.
$\vec{\gamma}_\Gamma(S(t))$	Vortices sheet of body-wetted surface $S(t)$
$\vec{\gamma}_G(t,\vec{r})$	Vortices on the free water surface $\Sigma(t,\vec{r})$
$\vec{\gamma}_\delta(t,\vec{r})$	Free vortexes on the water surface $\Sigma(t,\vec{r})$
ε	Towing resistance (Draft-lift ratio) (Fig. 4)
θ	Flight-path angle (Figs. 1 and 2), deg.
λ_W	Wavelength (Fig. 3), m
ρ	Mass density of fluid, kg/m^3
Σ^-, Σ^+	Lower and upper borders
$\Sigma(t,\vec{r})$	Free surface (constant pressure) of water
$\sigma(t,\vec{r})$	Vortex wake following the body
φ	Trim angle (Figs. 1 and 2), deg.
φ_{bal}	Balancing trim angle (Fig. 5), deg.
$\Phi(t,\vec{r})$	Potential of perturbation velocities
$\Phi_n(t,\vec{r})$	Potential of no turbulent velocities of liquid motion

Abbreviations

BK-1	Water glider model
CAHI	Central Aero-Hydrodynamic Institute
Ch-J	Chaplygin-Joukowski condition
HV	Hydro-vehicles
USSR	The Union of Soviet Socialist Republics

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