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BUCKLING LOAD MAXIMIZATION OF COMPOSITE LAMINATE USING RANDOM SEARCH ALGORITHM CONSIDERING UNIFORM THICKNESS AND VARIABLE THICKNESS APPROACH

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Abstract

In this article, the effectiveness of newly developed simplified version of Random Search Algorithm (RSA) based strategy for design optimization of a composite laminate is tested against few selected case studies of classical buckling load maximization using two-ply stack constraint. Variable Thickness Approach (VTA) and Uniform Thickness Approach (UTA) used during simulation prove that this algorithm is capable of handling the number of design variables of different nature in discrete form, simultaneously. RSA defined here is capable of accepting user-defined limit bounds with increment value for design variables and non-linear constraints. This will help the designer to obtain practically acceptable optimum designs. The obtained optimum results show that RSA outperforms Harmony Search Algorithm. The optimum results obtained using RSA for UTA are validated using FEA.

Keywords: Buckling load, Composite laminate, Genetic algorithm, OptiComp, Random search algorithm.

1. Introduction

Composite laminates have attracted the attention of many users in several engineering and other disciplines as a structural member because of the unlimited possibilities of deriving any characteristic material behaviour. To get required engineering properties for an application, multiple Fibre Reinforced Polymer (FRP) laminas/plies are connected together to constitute a structural element called a composite laminate.

Unforeseen failure of mechanical components can be grouped into two major categories: material failure and structural instability. The second one is often called buckling. Composite laminates are seeing increased usage as structural members in engineering applications, which are subjected to heavy compressive loads. While designing such structural components, the buckling load carrying capacity becomes a key point. For the structural elements subjected to high compressive stresses, the buckling failure mode is characterised by sudden sideways failure. The buckling load factor is nothing but the factor of safety against buckling and it can be calculated as the ratio of the buckling loads to the currently applied loads.

de Almeida [1] explored the effectiveness of Harmony search algorithm in buckling load maximization of the composite laminate plate. Harmony Search Algorithm (HAS) is inspired by the process of improvisation in music playing in order to search for the perfect set of harmony. A musician improvises a new melody by selecting a musical note on a random basis or improvises the previous melody with possible small pitch adjustments. HSA simulates this process by creating new solution vectors with the value of the design variables defined either on a random basis or selected from a pool of the best previously generated solutions and possibly applying small changes to the selected value, i.e., pitch adjustment.

Karakaya and Soykasap [2] in various load cases, done for buckling load maximization of the composite laminate using the genetic algorithm and generalised pattern search algorithm. The results obtained for different load cases in the paper are compared with the results obtained by previous researchers. Aymerich and Serra [3] and Rao and Arvind [4] respectively used ant colony optimization algorithm and scatter search algorithm for maximizing buckling load carrying capacity of the composite laminate using maximum strain theory as a constraint. Chang et al. [5], Topal and Uzman [6] and Ho-Huu et al. [7] respectively used the optimization techniques like permutation discrete particle swarm optimization technique, modified feasible direction method and cell-based smoothed discrete shear gap method for predicting buckling behaviour of composite laminates. Jing et al. [8] and Jing et al. [9] proposed a single criterion and multi-criteria optimization including buckling load factor by using Permutation Search (PS) algorithm and sequential permutation table, which resulted in a reduction of number evaluations in stacking sequence optimization. Ovesy et al. [10] studied the buckling analysis of rectangular composite laminate plate by using a finite strip method, while by Baba and Baltaci [11]. Kumar et al. [12] studied the effects of anti-symmetric laminate configuration, cutout and length/thickness ratio on the buckling behaviour of the composite laminate plate and investigated the buckling behaviour of laminated curved composite stiffened panels by considering different design parameters.

Many researchers developed different techniques for optimization of composite laminates as described in the earlier paragraph. For finding the optimum design of composite laminate using any one of such methods, the designer must have thorough knowledge about the optimization process and that particular technique. This may not be possible every time and become a hurdle to use these techniques. This article tries to overcome this hurdle by providing a simple and substantially accurate optimization strategy based on Random Search Algorithm (RSA).

A composite laminate can be designed using two approaches, Uniform Thickness Approach (UTA) and Variable Thickness Approach (VTA). All laminas in a laminate will have the same thickness in UTA, while in VTA, the laminas in the laminate may have the same or different thicknesses. The use of ply thickness as a user-defined discrete variable is rarely observed [13-15] so far in the available literature because of manufacturing difficulty and mathematical complexity. The comparison of both the approaches yields that the number of design variables in VTA becomes more than the design variables in UTA. Moreover, the nature of variables, i.e., ply angle and ply thickness is different. A ply angle is an integer number while ply thickness is a real number. Fabrication of variable-thickness composite structures can be done by ply drops and splicing and can be preferred for designing highly critical aerospace components because of high manufacturing cost [16].

de Almeida [1] defined in the present study, in which, RSA is initially applied for maximizing buckling load carrying capacity of composite laminates for the different problems. Then capability of RSA to handle different design variables simultaneously is demonstrated using Variable Thickness Approach. As results obtained using RSA show significant improvement over the reference results, the optimum results obtained using UTA are validated at the end. RSA is simple to understand and any common designer can implement it. The obtained results show that RSA is capable of finding a near-optimal solution and its performance in this regard depends on sample size. The mathematical development of the problem is described in the next section.

2. Development of Optimization Problem

A composite laminate plate with length '*a*' and width '*b*'; subjected to in-plane normal loads N_{xx}, N_{yy} and in-plane shear load N_{xy} is shown in Fig. 1. In the figure, *X*, *Y* and *Z* denote a global coordinate system of the composite laminate, while 1 and 2 represent a local coordinate system for individual lamina. Axis 1 of the local coordinate system is along the length of the fibre and axis 2 is perpendicular to local axis 1 [17].

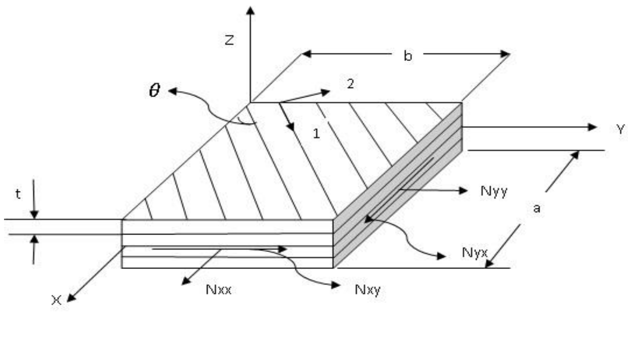


Fig. 1. Global and local coordinate systems for composite laminate.

The laminate selected for optimization in the problem under consideration is balanced symmetric laminate as these laminates avoid strength reducing bending-stretching effects by virtue of mid-plane symmetry. At the same time, mid-plane symmetry results in a reduction of the number of design variables and ultimately minimizes the computational time.

2.1. Development of objective function

Buckling of the composite laminate plate into '*e*' and '*f*' half-waves (along with the length and width directions, respectively) occurs when the load multiplier γ_{cr} reaches to the value [4].

$$\gamma_{cr}(e,f) = \pi^2 \left[\frac{D_{11}\frac{e^4}{a^4} + 2(D_{12}+2D_{66})\left(\frac{e}{a}\right)^2\left(\frac{f}{b}\right)^2 + D_{22}\frac{f^4}{b^4}}{N_{xx}\left(\frac{e}{a}\right)^2 + N_{yy}\left(\frac{f}{b}\right)^2 + N_{xy}\left(\frac{ef}{ab}\right)}\right] \quad (1)$$

The lowest value of γ_{cr} is the critical buckling load, which can be obtained by substituting *e* and *f* equal to 1.

$$\gamma_{cr} = \pi^2 \left[\frac{\frac{D_{11}}{a^4} + \frac{2(D_{12}+2D_{66})}{a^2 \times b^2} + \frac{D_{22}}{b^4}}{\frac{N_{xx}}{a^2} + \frac{N_{yy}}{b^2} + \frac{N_{xy}}{ab}}\right] \quad (2)$$

In this equation, D_{11}, D_{12}, D_{22} and D_{66} represent elements of bending stiffness matrix D. The matrix D can be calculated using stiffness matrices $[\bar{Q}]$ of individual laminas. The elements of $[\bar{Q}]$matrix depend on the material properties and ply angle of the lamina under consideration. According to Malliac [17], matrix *D* can be calculated as the following.

$$D = \frac{1}{3}\sum_{j=1}^{n} (\bar{Q})(h^3{}_j - h^3{}_{j-1}) \quad (3)$$

where $[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}$,

$\bar{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta,$

$\bar{Q}_{12} = Q_{12}(\sin^4\theta + \cos^4\theta) + (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta,$

$\bar{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta,$

$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos\theta\sin^3\theta,$

$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta,$

$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta),$

And $Q_{11} = \frac{E_{11}}{1-\gamma_{12}\gamma_{21}}$, $Q_{22} = \frac{E_{22}}{1-\gamma_{12}\gamma_{21}}$, $\gamma_{21} = \left(\frac{E_{22}}{E_{11}}\right)\gamma_{12}$,

$Q_{12} = Q_{21} = \frac{\gamma_{21}E_{11}}{1-\gamma_{12}\gamma_{21}} = \frac{\gamma_{12}E_{22}}{1-\gamma_{12}\gamma_{21}}$, $Q_{66} = G_{12}$.

In Eq. (3), h_{j-1} is the distance from the mid-plane to the top of the j^{th} lamina and h_j is the distance from the mid-plane to the bottom of the j^{th} lamina. These additional geometric parameters required during laminate analysis are shown in Fig. 2. The laminate is made of '*n*' number of laminas. In Fig. 2, h_0 denotes the distance from the laminate mid-plane to the top of the first lamina while h_1 is the distance from the laminate mid-plane to the bottom of the first lamina. The total thickness of the laminate is denoted with the letter '*T*'. Z_j is the distance from the laminate mid-plane to the mid-plane of the j^{th} lamina.

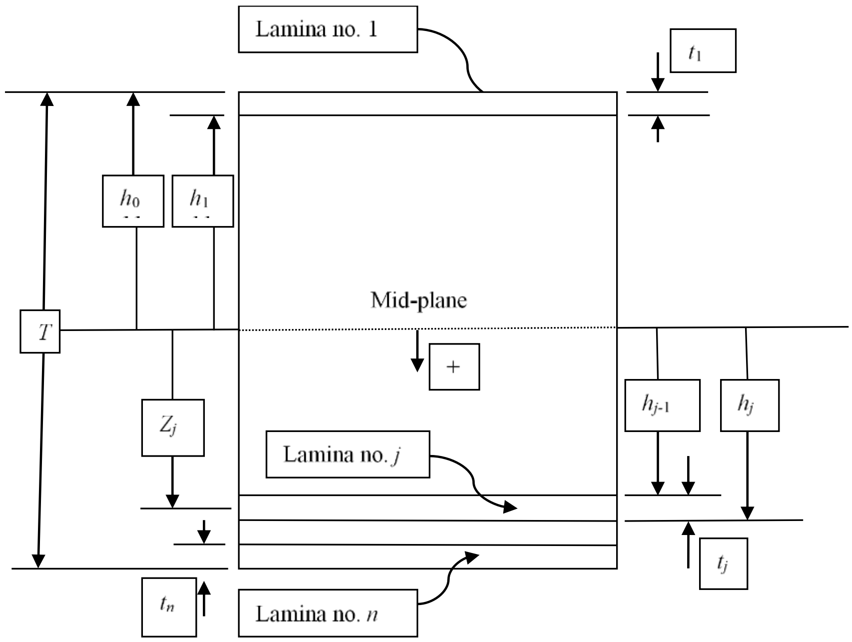


Fig. 2. Additional geometric parameters of laminate.

2.2. Optimization algorithms

In the current article, the stacking sequence of the laminate, which will provide the maximum buckling load factor is obtained using 'OptiComp' developed in MATLAB [13]. OptiComp is a comprehensive optimization procedure developed for design optimization of the composite laminate, which can handle a variety of laminate design problems effectively with little selection effort. RSA demonstrated here is a part of OptiComp developed by the authors of this article.

It provides a choice of two approaches, namely, Uniform Thickness Approach (UTA) and Variable Thickness Approach (VTA) to design a composite laminate. In UTA, all the plies will have uniform thickness while ply angles and a number of plies will be treated as design variables. In VTA, the number of plies, ply angles and ply thicknesses are treated as design variables.

Random Search method (RSA) is based on the use of random numbers in finding the optimum point. Because of the availability of random number generators in most of the computer libraries, this method can be used quite effectively. Even though it is an old technique of optimization, its use for design optimization of a composite laminate is yet not observed in the available literature. In OptiComp, original RSA is moulded to accept various design variables of different nature in discrete form simultaneously. The process flow of RSA is explained in Fig. 3.

Along with the necessary geometric parameters and material properties, the user has to provide a number of solutions (samples) to be generated as user input, to begin with, RSA. The number of possible solutions (samples) to be generated depends on the criticality of constraints, available computational time and required accuracy. RSA initiates with the development of a trial design vector using one random number for each design variable as shown in Fig. 3. The size of the trial design vector is '$2n$' while designing the composite laminate made of '$2n$' laminas using VTA. The trial design vector is nothing but one of the possible configurations of the composite laminate in terms of ply angles and thicknesses.

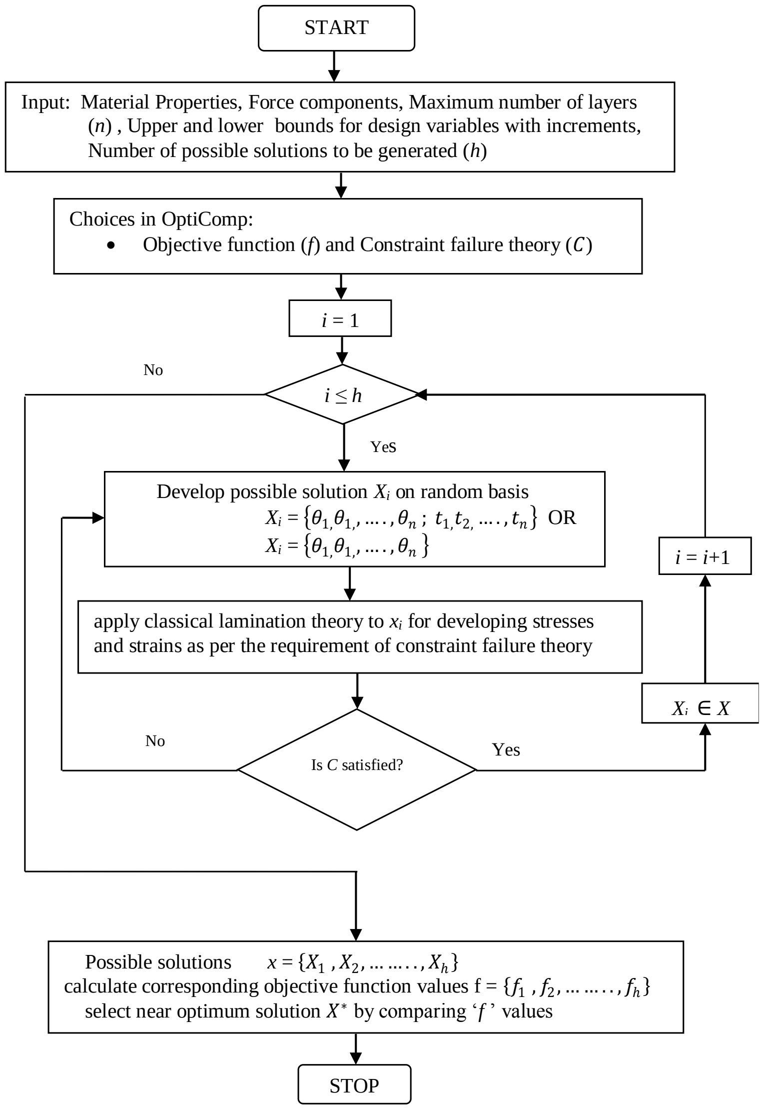


Fig. 3. Flowchart of random search algorithm for design optimization of composite laminate.

The first '*n*' elements of the trail design vector denote ply angle stacking sequence of laminate and remaining '*n*' elements denote ply thickness stacking sequence of the laminate. The first element of trial design vector, i.e., first ply angle is associated with $(n+1)^{th}$ element, i.e., first thickness value, the second element of trial design vector, i.e., second ply angle value is associated with $(n+2)^{th}$ element, i.e., second thickness value and so on. In the case of composite laminate design optimization using UTA, the size of the trial design vector will be '*n*'. In UTA, all the elements of trial design vector define one of the possible ply angle stacking sequences of the laminate and each lamina possess uniform thickness '*t*'.

Each element of trial design vector is initially defined in terms of random numbers ranging from '0' to '1' and it is necessary to convert these random numbers in acceptable values of ply angles and ply thicknesses. Letθ_L and θ_U be the lower and upper limiting values of the ply angles, while $t_L$and $t_U$be the lower and upper limiting values of the ply thicknesses. Let the increment values of ply angles and ply thicknesses within the given limit bounds be $\Delta\theta$ and Δt respectively as provided by user. Each random number generated is converted into required angle value or thickness value using relations given below.

$$\text{Expected angle value} = \left\{\Delta\theta \times round\left(random\ number \times \left(\frac{\theta_U - \theta_L}{\Delta\theta}\right)\right)\right\} + \theta_L \quad (4)$$

$$\text{Expected thickness value} = \left\{\Delta t \times round\left(random\ number \times \left(\frac{t_U - t_L}{\Delta t}\right)\right)\right\} + t_L \quad (5)$$

The developed trial design vector for VTA is shown in Table 1.

Table 1. Trial design vector for VTA.

Element no.	**1**	**2**	***n***	***n*+1**	***n*+2**	**2*n***
Trial vector	θ_1	θ_2	θ_n	t_1	t_2	t_n

Equations (4) and (5) facilitate the designer to choose own values for limit bounds and increments for different design variables as per the need and availability. Developed trial design vector indicates one of the possible stacking sequences of the laminate.

If this trial design vector does not satisfy one or more constraints selected by the user, then the algorithm will continue generating new trial vectors until a trial vector that satisfies all the constraints is found. This trial vector, which satisfies all the constraints becomes the first possible solution for the problem under consideration and becomes one of the elements of solution set '*X*'. This procedure is repeated to generate the number of possible solutions (samples) as prescribed by the user for the given problem.

The function values of all the solutions stored in solution set '*X*' are then compared and best out of them will be approximate near the optimum solution of the problem. Apart from the flowchart, a time constraint is also applied as termination criteria while solving the problem under consideration.

As per this constraint, the optimization will be terminated if the algorithm cannot develop a single possible solution in 15 minutes. The best individual found for one sample size is stored and forwarded to the next sample size as one of the individual solutions.

RSA defined in OptiComp differs the basic random search algorithm for constrained optimization [18] in the following ways.

- The developed trial design vector is immediately discarded if it does not satisfy the constraint defined by the user instead of using any penalty approach. This will ensure that all the design vectors stored in the solution set at the end of the process are the feasible solutions for the given problem. This will increase the probability of getting near the optimal solution with reduced sample size.
- Intermediate comparison between the design vectors is eliminated. Instead of that, all the feasible solutions are stored in the solution set and objective function evaluation for all of them will take place in one stroke. This means that the constraint violation checking process and objective function evaluation process are completely separated in the current strategy. Both these changes are helpful in reducing the required computational time.

Considering UTA, VTA, and RSA, it is now possible to develop two optimization problems of buckling load maximization. These two problems can be mathematically expressed as given below.

2.2.1. Problem statement for UTA

Find $[\theta_i]$:

For maximizing:

$$\gamma_{cr} = \pi^2 \left[\frac{\frac{D_{11}}{a^4} + \frac{2(D_{12}+2D_{66})}{a^2 \times b^2} + \frac{D_{22}}{b^4}}{\frac{N_{xx}}{a^2} + \frac{N_{yy}}{b^2} + \frac{N_{xy}}{ab}} \right]$$

Subjected to: $\text{Two} - \text{ply stack constraint.}$

$$-45^0 \leq \theta_i \leq 90^0 (\text{Ply angle increment value } 45^0).$$

$$i = 1,2, \ldots\ldots, n \text{ and } t_i = 0.127\ mm.$$

Using a random search algorithm.

2.2.2. Problem statement for VTA

Find $[\theta_i, t_i]$.

For maximizing

$$\gamma_{cr} = \pi^2 \left[\frac{\frac{D_{11}}{a^4} + \frac{2(D_{12}+2D_{66})}{a^2 \times b^2} + \frac{D_{22}}{b^4}}{\frac{N_{xx}}{a^2} + \frac{N_{yy}}{b^2} + \frac{N_{xy}}{ab}} \right]$$

Subjected to: $\text{Two} - \text{ply stack constraint.}$

$$-45^0 \leq \theta_i \leq 90^0 (\text{ Ply angle increment value } 45^0).$$

$$0.125 \leq t_i \leq 0.129. \text{ (Ply thickness increment value 0.002 mm)*.}$$

$$i = 1,2, \ldots\ldots, n$$

Using a random search algorithm.

*Discrete ply thickness values used in the simulation are only for demonstration purpose and do not possess any practical relevance. The results obtained in terms of stacking sequences and buckling load carrying capacity of composite laminate for both these problems are provided in the next section.

3. Results

While designing a composite laminate plate subjected to in-plane compressive loads, buckling load carrying capacity becomes a crucial factor as it may result in premature failure of the structure. The random search algorithm explained in the earlier section is used for maximizing buckling load carrying capacity of a simply supported plate subjected to biaxial in-plane compressive loading.

de Almeida [1] considered a symmetric simply supported laminate having length 500 mm and width 1000 mm, for buckling load maximization purpose. Optimization in this reference article is carried out using only two ply stack constraint (two consecutive plies must have same ply angle). The material properties of graphite/epoxy composite laminate plate used in the simulation are given in Table 2.

Table 2. Material properties of graphite epoxy by de Almeida [1].

Property	Value
Elastic modulus E_{11} GPa	127.6
Elastic modulus E_{22} GPa	13
Shear modulus G_{12} GPa	6.4
Poisson's ratio γ_{12}	0.3

This laminate is subjected to biaxial compressive forces of magnitude Nxx = 0.333 N/m and Nyy = 1 N/m as specified in the reference article. The uniform lamina thickness under consideration is 0.127 mm for UTA. The different design problems are formulated by varying number of plies associated with laminate as 32, 48 and 64.

3.1. Results for problem 2.2.1

The buckling load carrying capacities obtained by random search algorithm considering UTA are shown in Table 3 for various numbers of possible solutions (samples). For each sample size, best of five results are provided in Table 3 and the buckling load factors obtained for all the cases using random search algorithm are compared with the results obtained by de Almeida [1] in the same table.

Table 4 represents optimum ply angle stacking sequences obtained by random search algorithm for various load cases.

Table 3. Buckling load factors using random search algorithm for UTA.

Number of Laminas (n)	Buckling load factor for			Buckling load factor by de Almeida [1]	% rise considering maximum value obtained by RSA
	Samples 8000	Samples 16000	Samples 32000		
32	52258	52258	52258	41250	26%
48	173050	174210	175520	139348	25%
64	403530	407440	408460	330326	22%

Table 4. Optimum stacking sequences obtained for different number of laminas using RSA for UTA.

Number of Laminas (*n*)	Buckling load factor	Stacking sequence
32	52258	$[(0_8)_2]s$
48	175520	$[(0_9/\text{-}45/45_2)_2]s$
64	408460	$[(0_7/45/0_3/45/0_2/90/45)_2]s$

Results provided in Table 3 show improvements in buckling load factor values with increased sample size. The obtained best result is reaching closer to the optimal solution for that problem with increased sample size. At the same time, the increased sample size results in more computational time. It can be seen that the optimum results obtained by RSA are better than the reference results for all the test cases.

3.2. Results for problem 2.2.2

In the earlier case, it is observed that the sample size 32000 provides better results. The capability of RSA to handle different design variables simultaneously is demonstrated by solving earlier optimization problems using VTA. Ply thicknesses are varying in discrete form along with ply angles in VTA. Ply angles are having integer nature in solution space, while ply thicknesses are the real numbers in discrete form. The buckling load carrying capacities obtained by random search algorithm considering VTA and stacking sequences obtained for ply angles and ply thicknesses are shown in Table 5 for 32000 possible solutions (samples). Best of five results obtained for each test case is provided in Table 5.

Table 5. Buckling load factors using random search algorithm for VTA.

Number of Laminas (*n*)	Buckling load factor for samples 32000	Stacking sequence (Ply angles and ply thicknesses)
32	53816	$\theta = [(0_8)_2]s$ $t = [(0.129/0.127/0.129/0.127/0.129/0.127/0.129_2)_2]s$
48	179300	$\theta = [(0_{11}/90)_2]s$ $t = [(0.127_6/0.129/0.127/0.129_2/0.127_2)_2]s$
64	419540	$\theta = [(0_9/\text{-}45/0_3/\text{-}45/0/45]s$ $t = [(0.0.127_4/0.129_2/0.125/0.129/0.127/0.129_2/0.127_3/0.129/0.127)_2]s$

The buckling load factors obtained using VTA are greater than UTA as well as reference results**.** It is expected as the maximum value of discrete ply thickness used in VTA is greater than the uniform ply thickness used in UTA.

The ply thickness stacking sequences show that most of the optimum ply thicknesses are reached to this maximum value. Further improvement in the results can be achieved by increasing the sample size.

The buckling load factors obtained by reference article (R), UTA (*U*) and VTA (*V*) for different test cases are compared in Fig. 4.

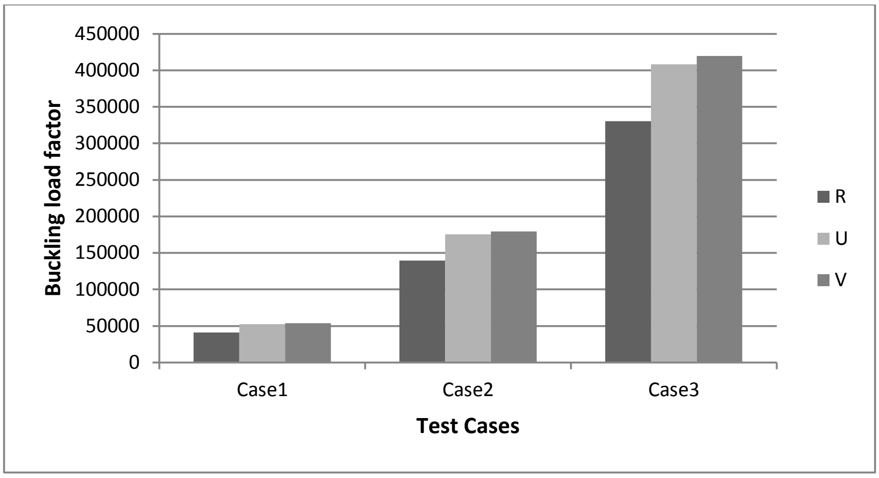


Fig. 4. Comparison of buckling load factors obtained by different considerations.

3.3. FEA validation of results obtained by RSA for problem 1

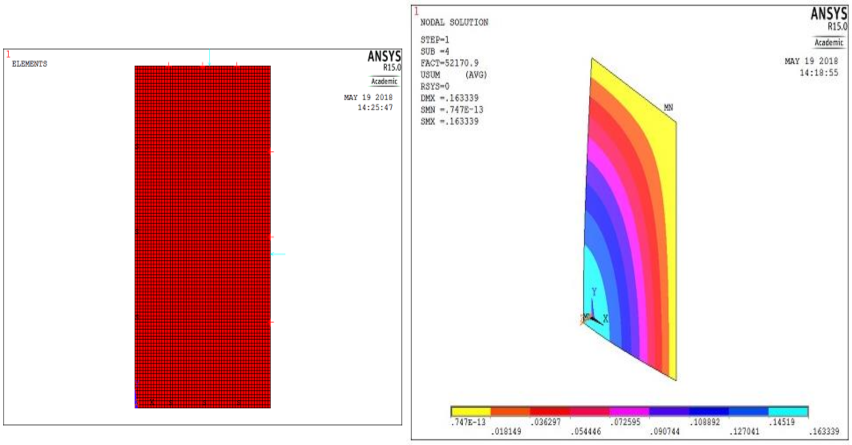
It is necessary to validate the results obtained by RSA (provided in Tables 3 and 4) as these results show significant improvement in buckling load factors (minimum 22%) over the reference results. For this purpose, a quarter model of the simply supported plate with given dimensions is prepared in Ansys software.

The optimum stacking sequences provided in Table 4 are separately applied to this model. The quarter model of the plate is meshed using Shell 181, four noded elements. The Finite Element Analysis (FEA) model of the plate is shown in Fig. 5(a). Symmetric boundary conditions are applied on the lower and left edge of the model while remaining edges are simply supported.

Already defined compressive forces are applied on simply supported edges in pressure form. The buckling load factors obtained for all the cases using FEA are provided in Table 6 and one representative result of the buckling load analysis using FEA is shown in Fig. 5(b). It is observed that buckling load factors obtained by FEA for optimum stacking sequences of all the test cases are closely matching with the buckling load factors shown by RSA.

Table 6. Buckling load factors obtained using RSA and FEA for UTA.

Number of Laminas	Buckling load factor using RSA for UTA (Samples 32000)	Buckling load factor using FEA
32	52258	52171
48	175520	172320
64	404860	393284



(a)Boundary conditions used. **(b) Buckling load factor for laminate with 32 plies.**

Fig. 5. FEA analysis for buckling load factor using quarter plate model.

4. Conclusions

The current study demonstrates the effectiveness of random search algorithm based optimization strategy in design optimization of the composite laminate plate. Buckling load maximization of the composite laminate using this strategy results in the following conclusions.

- The buckling load factors and optimum results obtained by RSA (UTA as well as VTA) for all the case studies are far better than the optimum results, in which, obtained by Harmony Search Algorithm.
- The computational accuracy of RSA improves with an increase in the number of samples at the cost of computational time.
- The results obtained by VTA are better than the results obtained by UTA. This is because the upper limit of ply thickness used in VTA is more than the uniform lamina thickness used in UTA. Ply thickness stacking sequences show that all the laminas are trying to reach to the upper limit of thickness in VTA. This shows that RSA is capable of handling a large number of design variables in a discrete form with different nature.
- Even though RSA is simple to implement, it possesses substantial computational accuracy. Therefore, RSA has the potential to become famous in the researchers' community working in the field of design optimization of the composite laminate.

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Nomenclatures

a	Length of the laminate, mm
b	Width of the laminate, mm
$D_{i,j}$	Elements of bending stiffness matrix
E_{11}	Elastic Modulus in longitudinal direction, GPa
E_{22}	Elastic modulus in transverse direction, GPa
G_{12}	In plane shear modulus, GPa
n	Number of plies
$\overline{Q_{i,j}}$	Elements of stiffness matrix of individual lamina
t	Thickness of lamina
γ_{12}	Major poisson's ratio
γ_{cr}	Buckling load factor

Greek Symbols

Δt	Ply thickness increment value
$\Delta\theta$	Ply angle increment value
θ	Ply orientation angle

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