Journal of Engineering Science and Technology
Vol. 14, No. 1 (2019) 059 – 072


# A TECHNIQUE FOR MEASURING QUANTITY OF OIL IN A FLOWING WATER-OIL MIXTURE

SANTHOSH K. V.

Department of Instrumentation and Control Engineering, Manipal Institute of Technology, Manipal Academy of Higher Education, India
E-mail: kv.santhu@gmail.com

**Abstract**

Often different liquids are mixed to achieve a desired product in process industry. Measurement of liquid quantity which is added is very essential as in appropriate mixture may lead to failure of desired product. The proposed paper makes an attempt to measure the quantity of liquid added when the flow is under dynamic condition. An orifice flow meter is used to measure the flow rate of the mixture. Output of flow meter after conditioning is transmitted to the PC. Using the relation of model parameters and density, density of mixture is calculated. On comparing the densities, amount of liquid added can be determined. The results computed were compared with actual values and is proved that the proposed technique has achieved its set objectives, with a root mean square percentage error of 0.039%.

Keywords: Density measure, Flow process, System identification, Two-phase liquid.

## 1. Introduction

Products are produced from a combination of two or more basic elements, combined together in appropriate quantities. The quality of product obtained totally depends on the type, process of combinations, accurate ratio's, and the quality of raw material. With the industry revolution and need for mass production has enabled automation in manufacturing domain. There exists several technique reported in past to control mixing of products. For controlling the process as per desired one should have constant monitoring [1, 2]. Many researchers such as Piuzzi et al. [3] have reported works on volumetric measurement, which is a method for computation of liquid density in a container by the technique of time domain reflectometry using a co-axial probe is reported. Raaf et al. [4] explained that the test for presence of mineral oil in a container is carried out using measurement of Kim et al. [6] reported that density is by a radiation type sensor. Montero and Vazquez [5] reported that the use of optical fibre for measurement of liquid volume is in a storage tank. Design of a doubly rotated resonator for measurement of liquid volume. According to Shigemi, [7], property of acoustic wave is used for measurement of small volume of liquid. Thurow et al. [8] commented that the image processing on the data acquired by optical sensor is carried out to determine the liquid level in micro plates. Many more works have been reported for measurement of liquid volume in static conditions. But there exists many applications where one needs to measure the liquid volume when the liquid is under flow/ dynamic condition.

Few researchers such as Lautscham et al. [9] have also reported works on measurement of liquid volume under flow, which is an acoustic sensor used to measure the volume of liquid flow using the radiation principle or Doppler principle. Heinicke [10] reported that the volumetric measurement of liquid metal when in flow is using the mass head flow sensor. According to Heinicke et al. [11], Lorentz force velocimetry is designed for measurement of liquid flow in the ducts. Like this, many authors have reported works on measurement of liquid volume in flow with a static ration of mixtures. These reported technique would definitely yield to inaccurate or uncontrolled process if any of the parameter is varied. But there are several characteristics which affect the flow rate of liquid and thus affect the measurement process. Franco et al. [12], reports the effect of liquid viscosity on flow; author also reports a radiation technique to measure the viscosity. Similar work is also reported on simulation study of a bubbly flow through a hydrofoil.

Many a times, there exists conditions where some solid additives are present in liquid and it needs to be measured. Blaz et al. [14] have also reported technique to identify these materials, a technique is reported to identify the amount of various liquids added into distilled water. Stroher et al. [15] reported a method to analyse the cholesterol contents in meat using temperature profile. Morrison and Driskell [16] stated that, flourometric detection is used to detect the quantity of B6 vitamin in human milk. Mandal et al. [17] reported that a liquid chromatographic method is used for determination of etoricoxib in human plasma. But when the mixture contains two different liquids, reported works fail to produce desired results. The proposed work makes an attempt to measure the amount of liquid added to the mixture based on variation of system gain and time response.

The paper is organised as follows: After introduction in Section 1, a brief description of experimental setup is discussed in Section 2. Analysis of system for variation of liquid density is reported in Section 3. Section 4 shows the results and analysis of proposed objectives. Finally, Section 5 deals with the conclusions and discussion.

## 2. Experimental Setup

To implement the proposed quantitative measurement technique for liquid added on to flow, a head type volumetric flow process station as shown in Fig. 1 is considered. Process station consists of a reservoir tank to store liquid under measure. Flow through pipe is controlled by a pneumatic linear control valve (air to open). The air required is pumped through a compressor controlled by current to pressure converter (I/P). To measure flow rate, a rotameter and orifice plate is used. Rotameter helps us to physically read liquid flow rate. Signal from orifice plate is feed to a differential pressure sensor to produce an electrical signal of 4-20 mA. By using suitable data converter and acquisition card, signal is transmitted to PC through RS 232 port. The control algorithm is designed on a PC using tools Simulink toolbox of MATLAB. Control signal calculated by controller is feed to I/P converter for driving the actuator (Pneumatic control valve in this case). The schematic connections of these elements in a flow process loop are as shown in Fig. 2 for complete process and open loop model respectively.

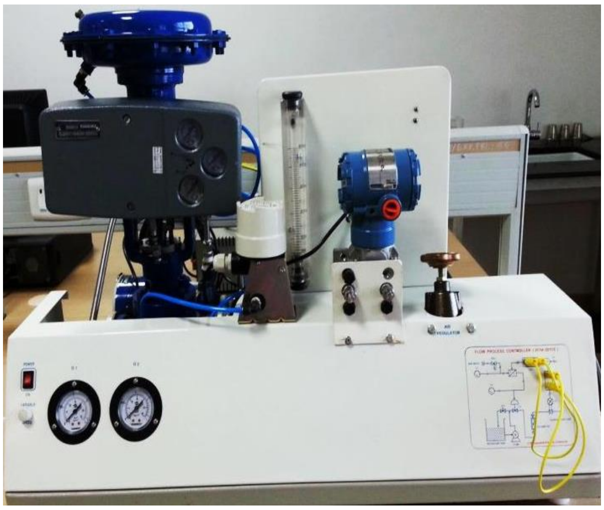

**Fig. 1. Flow process station.**

### Orifice flow meter

Orifice flow meter is the most commonly used head type flow meter consisting of a plate with a hole placed in the path of the flow [18, 19]. The flow meter works on the principle of Bernoulli's law. Whenever the flow reaches the hole of the orifice, it exhibits the pressure change because of the sudden change in dimension, and thus converges at a point called vena-contrata, where the pressure attained is maximum. The pressure difference across the vena-contrata and before orifice will be proportional to the velocity of the flow which can be derived by using Bernoulli's equation as given in Eq. (1).

$$Q = n * \sqrt{\Delta P} \tag{1}$$

where

$$n = \frac{C_d * A_2 * \sqrt{2}}{\sqrt{(1-\beta^4)(\rho)}} \tag{2}$$

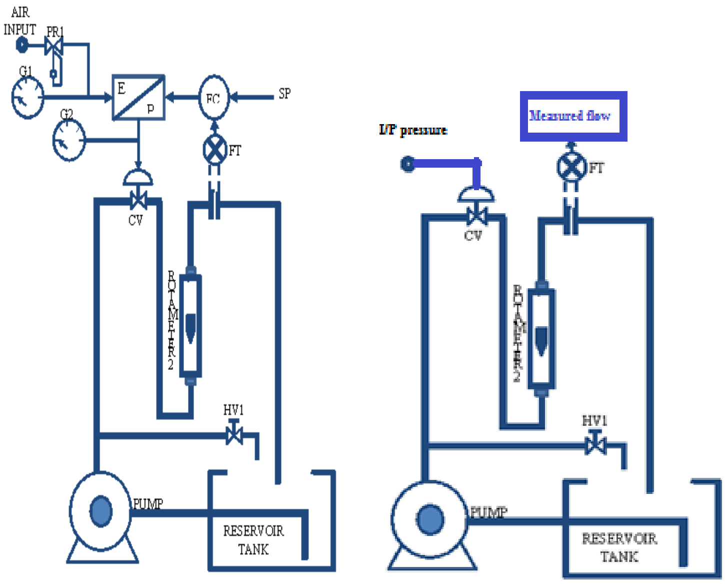


**Fig. 2. Schematic of flow process control.**

From the sensor signal, flow rate is measured using Eq. (1). Obtained pressure is converted to current signal of 4-20 mA. This current signal is transmitted to PC through RS232 port. Simulink block is used to obtain flow rate that is transmitted to controller and also proposed system identification program. Obtained 8 bit data corresponding to current, is normalized. The value provided is the product of process constant of orifice, gain of differential pressure transducer, and data converter circuits. The Simulink block for computation used is as shown in Fig. 3. This flow rate is used to design a controller for operating control value, for control of flow rate in the process. The same data is also used in system identification. On receiving data corresponding to actual flow, it is compared with set point value to take necessary control actions.

Once, the model is obtained, the next section discusses the methodology carried out in the proposed work.

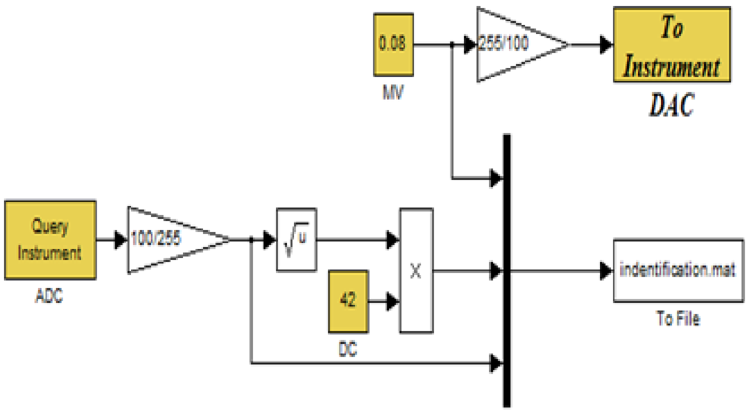


**Fig. 3. Simulink block for acquiring pressure and flow calculation.**

## 3. Methodology

The characteristics of flow loop for variations in liquid density is analysed, by subjecting process with test cases of liquid varying in densities. By measuring input and output response of flow loop, system identification by two point method of open loop response is used to compute variations in system model.

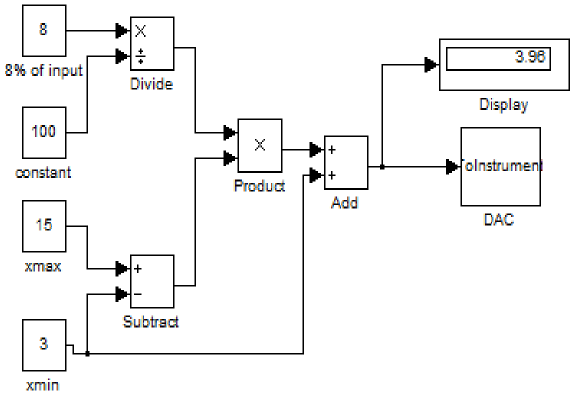


**Fig. 4. Simulink block for converter sub block.**

System identification is a mathematical model of a dynamic system based on experimental data. Several techniques are incorporated for identification of parameters of system. The proposed technique uses open loop response characterisation for system identification. Sundaresan et al. [20] commented that identifying the step response of system, computation of transfer function for proposed system is done by two point method.

The system is subjected with an input of 8% of max pressure value, system response in terms of change in flow is noted. A 'Converter' subsystem shown in Fig. 4 is used to convert the set point values given into a percentage value according to the formula given below in Eq. (3).

$$\frac{(X_{in}-X_{min})}{(X_{max}-X_{min})} * 100 \quad (3)$$

To test system response it is subjected to a step change in input from 0 to 8% and back to 0. The output flow is measured and plotted. Nine different cases of flow are tested for variations in densities. It is assumed that the quantity of water is known and oil quantity need to be calculated.

**Table 1. Summarizes the data acquired from model.**

| Liquid | Water in ml | Oil in ml | Density in kg/m$^3$ |
|---|---|---|---|
| **Sample 1** | 1000 | 0 | 999.97 |
| **Sample 2** | 4000 | 100 | 998.1706 |
| **Sample 3** | 4000 | 250 | 996.37 |
| **Sample 4** | 6000 | 250 | 996.441 |
| **Sample 5** | 4000 | 500 | 994.677 |
| **Sample 6** | 6000 | 500 | 994.7794 |
| **Sample 7** | 4000 | 600 | 993.179 |
| **Sample 8** | 4000 | 1000 | 990.154 |
| **Sample 9** | 6000 | 1000 | 991.63 |

Plot of response flow rate for a test input of 8% of full scale is shown in Figs. 5 to 7.

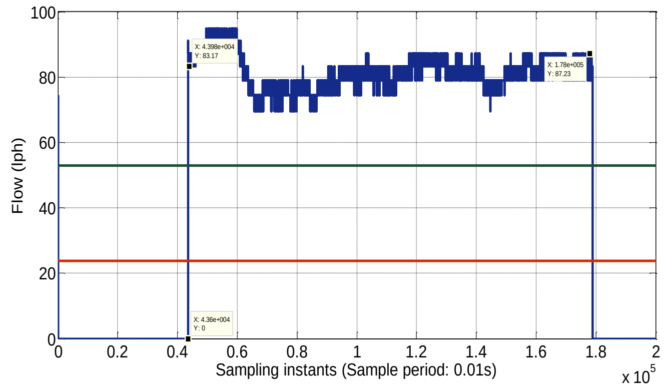


**Fig. 5. Output response flow rate when the set point is varied Sample 9.**

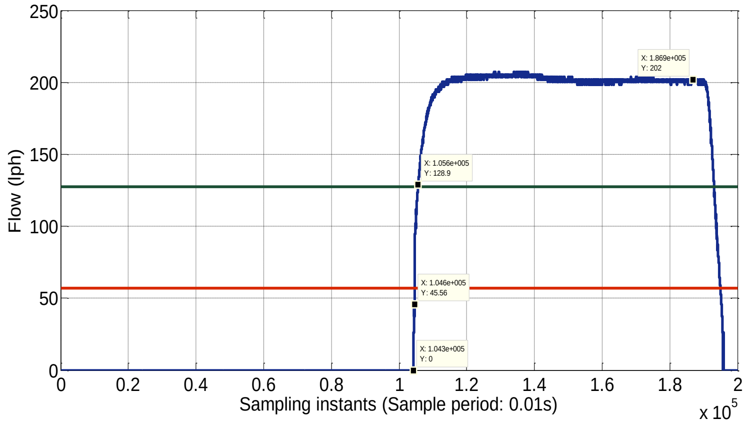


**Fig. 6. Output response flow rate when the set point is varied for Sample 1.**

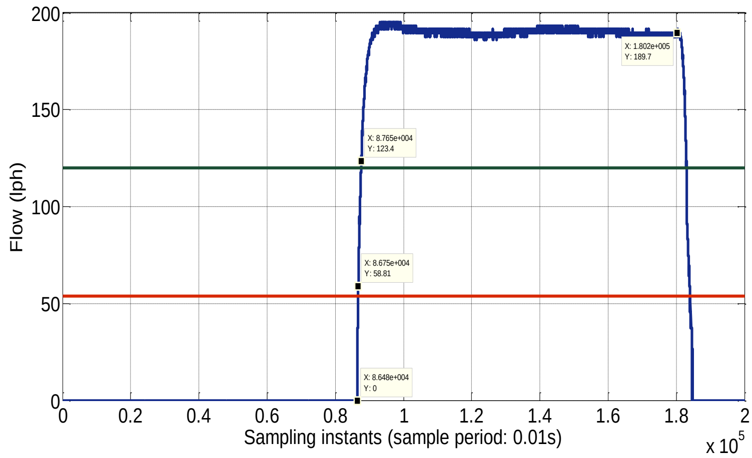


**Fig. 7. Output response flow rate when the set point is varied for Sample 8.**

## Two point method

Flow characteristics are analyzed for step change in input, time response characteristic resemble the first order system response. Several researchers have also carried out similar exercises in their research. The flow process is assumed to be a first order plus time delay system, and in the present study the concentration is only on Newtonian flows of low viscosity. The system response will be very quick and will have a time constant which will be very minimal once its exponential is computed. Therefore, for all practical situations, we ignore the exponential part [20-30]. To compute the values of system coefficients so as to analyze the behavior, system is subjected to a test.

$$G(s) = \frac{K}{Ts+1} e^{-\tau s} \tag{4}$$

Since the flow process is a quick process, τ will be very small and thus $e^{-\tau s}$ can be neglected. Two point method is a model identification technique based on open loop output response of system, for a step change in input. Two point method states that when the first order system is subjected with the step input, it produces output as shown in Fig. 8, which will have a transfer function of the form given by Eq. (5).

$$G_p(s) = \frac{K}{Ts+1} \tag{5}$$

Calculation of constants $T$ and $K$ is done using Eqs. (6) and (7).

$$T = 1.5(T2 - T1)\ s \tag{6}$$

$$K = \frac{\%\ \Delta output}{\%\ \Delta input} \tag{7}$$

T1 and T2 are the derived time for the process to produce 28.3% and 63.2% of change in output, and is denoted by $t_{28.3\%}$ and $t_{63.2\%}$ respectively. For computation of $K$ the relative output change y' is used to compute the relative output as given in Eq. (8).

$$y' = \frac{y-y1}{y2-y1} * 100 \tag{8}$$

Let's take an example of Sample 8, from Fig. 7 the output time characteristics for a step change in input is derived. From the output characteristics it is seen that the time taken to reach 28.3 % and 63.2% of output is 8.294 and 9.286 seconds. T is computed by the Eq. (8) which will be 1.5. (9.286-8.294)=1.488.

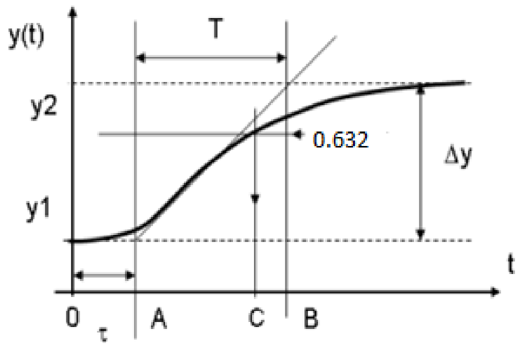


**Fig. 8. Open loop response of first order system for step input [20].**

For computation of *K*, we need to compute the change in percentage in output and input which is

% change in output (lph) = (122-54/122)*100= 54.83

% change in input (mA) = (8-4.18/8)*100 = 47.76

So, *K* = 54.83/47.76 = 1.1439.

**Table 2. Summarizes the data acquired from model.**

| Liquid | T1(s) | T2(s) | T(s) | K | Model |
|---|---|---|---|---|---|
| **Sample 9** | 5.52 | 6.578 | 1.87 | 1.135 | $\frac{1.135}{1.87s+1}$ |
| **Sample 8** | 8.294 | 9.286 | 1.488 | 1.144 | $\frac{1.144}{1.488s+1}$ |
| **Sample 7** | 2.552 | 3.346 | 1.191 | 1.15 | $\frac{1.15}{1.191s+1}$ |
| **Sample 6** | 6.834 | 7.613 | 1.169 | 1.172 | $\frac{1.172}{1.169s+1}$ |
| **Sample 5** | 9.783 | 11.14 | 1.135 | 1.201 | $\frac{1.201}{1.135s+1}$ |
| **Sample 4** | 3.472 | 4.218 | 1.129 | 1.26 | $\frac{1.26}{1.129s+1}$ |
| **Sample 3** | 5.529 | 6.474 | 1.11 | 1.29 | $\frac{1.29}{1.11s+1}$ |
| **Sample 2** | 7.609 | 8.667 | 1.058 | 1.35 | $\frac{1.35}{1.058s+1}$ |
| **Sample 1** | 1.18 | 2.16 | 1.047 | 1.44 | $\frac{1.44}{1.047s+1}$ |

From the responses of liquid, the constants are calculated and tabulated in Table 1. By using the Eq. (6), Eq. (7), and Eq. (8) model transfer function are computed. Similar study is carried on to all the nine samples and Table 2 shows the transfer function obtained from two point method.

From the responses and estimated model shown in Table. 1, it is clear that flow process model varies with changes in liquid densities. From the transfer function model, the relation between the density and gain or density and time constant can be computed.

Curve fitting algorithms are used to compute the relation between gain (*K*), time constant (*T*) and density of liquid ($\rho$) and is given by Eq. (9) to (12) for different orders.

Using the second order curve fitting function, relation between density and gain can be given by Eq. (9).

$$\rho = -62.38K^2 + 186.9K + 859.9 \quad (9)$$

Similarly, equation can be computed using a third order equation as in Eq. (10)

$$\rho = 896.8K^3 - 3523K^2 + 4621K - 1027 \quad (10)$$

Relation between the density and time constant can be written using second order equation as in Eq. (11)

$$\rho = 23.5T^2 - 78.02T + 1054 \quad (11)$$

For a third order expression,

$$\rho = -87.88T^3 + 404.2T^2 - 614.7T + 1301 \tag{12}$$

From the above equations one can compute the density of the liquid, which in-turn can be used to compute the additional volume of unknown liquid.

To validate the transfer function obtained from FOPDT technique, system identification toolbox of MATLAB is used to model the process system using the input output relation.

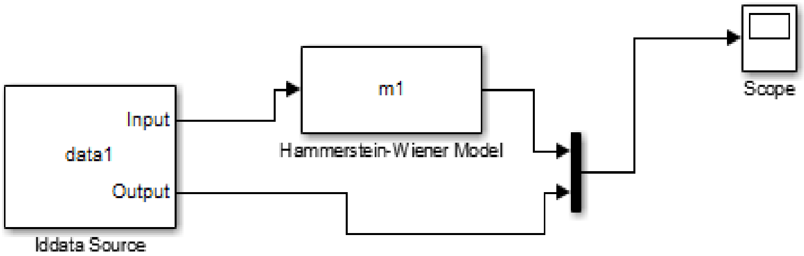


**Fig. 9. Simulink block diagram of proposed identification model.**

Considering Sample 8 for the computing the transfer function, input and output data were imported after creating data objects. These were used in the Simulink block diagram as shown in Fig. 9.

Figure 9 shows the Simulink model for comparison using the Hammerstein-Wiener model. From the working set of data, Hammerstein Weiner model was created and from the validation set of data 'data1' was created. The input of the data1 is fed to the model for the prediction of the output and the output of data1 and 'm1' is fed to the multiplexer and the output can be compared through the scope.

A second order transfer function was obtained from the identification techniques as given by

$$G(s) = \frac{1.139 - 0.023s}{1 + 1.496s + 0.0296s^2} \tag{13}$$

This equation can be rewritten as

$$G(s) = \frac{1.139(1 - 0.0201s)}{(1.476s + 1)(1 + 0.0202s)} \tag{14}$$

On considering Pade's approximation Eq. (14), can be rewritten as

$$G(s) = \frac{1.139}{(1.476s + 1)} e^{-0.04s} \tag{15}$$

which is the form of FOPDT equation, and on comparison with the results tabulated by two-point method, the transfer function model is almost the same.

## 4. Results and Analysis

Once the relation between the gain, time constant, and density is obtained, the designed model is subjected to test in real life application to validate. For testing, the proposed technique is subjected to different sample like 3 litres of water is mixed with 50 ml of oil and incremented every time with 50 ml, every time percentage of error is calculated. For calculation two methods are used i.e. density computation by

K and T. Results obtained by computation using K and T are shown in Table 3. Input output response obtained from reported technique is plot in Fig. 10. Figure 11 shows the percentage error for computation of density from reported technique.

For testing the performance of proposed density measurement technique, it is subject to test in real life with 105 data sets. Density calculated from the proposed technique for every case is compared with actual density and is plot in the graph shown in Fig. 9. From Fig. 10 it is clearly seen that proposed technique is able to measure density accurately. Percentage error computed shows the results are well within the permissible range and the root mean square percentage error for 105 test samples is found to be 0.03941%.

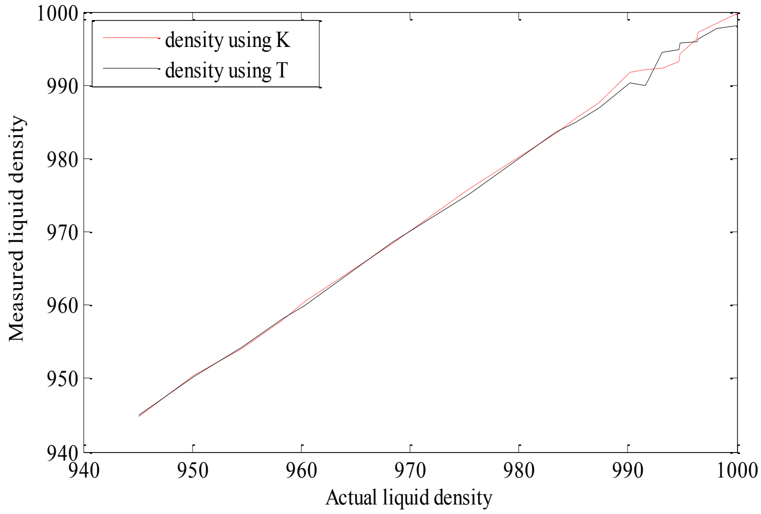


**Fig. 10. Actual vs. measured density of proposed technique.**

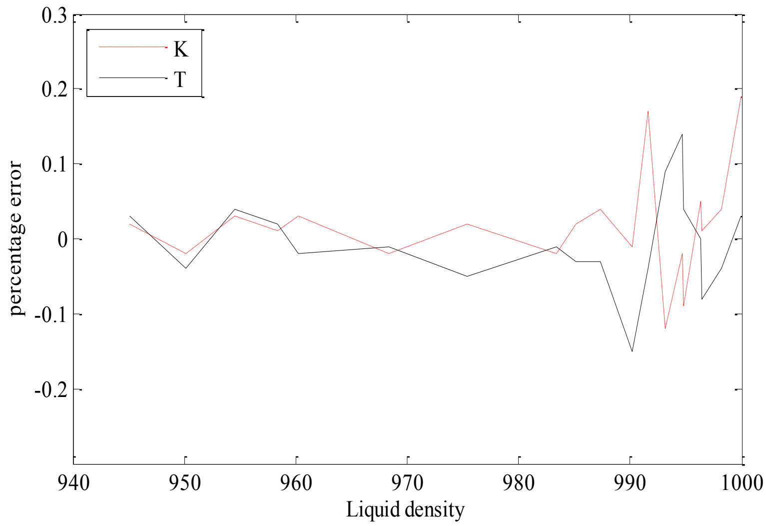


**Fig. 11. Percentage error in density computation from proposed technique.**

**Table 3. Results obtained from proposed technique.**

| Theoretical Density in kg/m$^3$ | Density computed from proportional gain in kg/m$^3$ | Error Percentage | Density computed from time constant in kg/m$^3$ | Error Percentage |
|---|---|---|---|---|
| 945.12 | 944.88 | 0.03 | 944.91 | 0.02 |
| 950.11 | 950.45 | -0.04 | 950.31 | -0.02 |
| 954.54 | 954.12 | 0.04 | 954.27 | 0.03 |
| 958.32 | 958.10 | 0.02 | 958.27 | 0.01 |
| 960.21 | 960.44 | -0.02 | 959.89 | 0.03 |
| 968.32 | 968.44 | -0.01 | 968.55 | -0.02 |
| 975.34 | 975.84 | -0.05 | 975.11 | 0.02 |
| 983.33 | 983.42 | -0.01 | 983.56 | -0.02 |
| 985.12 | 985.44 | -0.03 | 984.88 | 0.02 |
| 987.32 | 987.63 | -0.03 | 986.92 | 0.04 |
| 990.15 | 991.67 | -0.15 | 990.27 | -0.01 |
| 991.63 | 992.07 | -0.04 | 989.93 | 0.17 |
| 993.18 | 992.33 | 0.09 | 994.41 | -0.12 |
| 994.68 | 993.25 | 0.14 | 994.90 | -0.02 |
| 994.78 | 994.38 | 0.04 | 995.72 | -0.09 |
| 996.37 | 996.35 | 0.00 | 995.86 | 0.05 |
| 996.44 | 997.19 | -0.08 | 996.35 | 0.01 |
| 998.17 | 998.52 | -0.04 | 997.75 | 0.04 |
| 999.97 | 999.67 | 0.03 | 998.07 | 0.19 |

## 5. Conclusions

Density computation of liquid in flow is computed in proposed technique. In this technique, liquid density is computed by modeling the flow process station using flow rate of liquid. Quantity of unknown liquid added to the mixture is calculated using the density of known liquid and flow rate of the entire mixture. Results show the efficiency of the proposed technique for determination of unknown quantity of liquid mixed in real life application.

Further, the process can be extending by incorporation of intelligence technique like neural network of support vector machine.

### Nomenclatures

| | |
|---|---|
| $A_2$ | Cross section area at vena contracta |
| $C_d$ | Discharge coefficient |
| $K$ | Process gain |
| $T$ | Time constant |
| $Xin$ | Set Point input given to system. |
| $Xmax$ | Maximum operating pressure (15 psi). |

| | |
|---|---|
| *Xmin* | Minimum operating pressure (3 psi). |
| ***Greek Symbols*** | |
| $\beta$ | Ratio of cross section area of orifice plate to pipe |
| $\Delta P$ | Differential pressure |
| $\tau$ | Time taken for the system to produce output for the given input. |
| $\rho$ | Density of liquid |
| **Abbreviations** | |
| I/P | Current to pressure converter |

## References


1. Cornish, D. (1994). Instrument performance. *Journal of Measurement and Control*, 27(10), 323-328.
2. Miller, R.W. (1996). *Flow Measurement Engineering Handbook (*3rd ed.) Boston, United States of America: McGraw-Hill Professional.
3. Piuzzi, E.; Merla, C.; Cannazza, G.; Zambotti, A.; Appollonio, F.; Cataldo, A.; D'Atanasio, P.; De Benedetto, E.; and Liberti, M. (2013). A comparative analysis between customized and commercial systems for complex permittivity measurements on liquid samples at microwave frequencies. *IEEE Transactions on Instrumentation and Measurement*, 62(5), 1034-1046.
4. Raaf, J.L.; Bazarko, A.O.; Garvey, G.T.; Hawker, E.A.; Johnson, R.A.; Mills, G.B.; Pla-Dalmau, A.; and Toyloe, R. (2002). Mineral oil tests for the miniboone detector. *IEEE Transactions on Nuclear Science*, 49(3) 957-962.
5. Montero, D.S.; and Vazquez, C. (2012). Polymer optical fiber intensity-based sensor for liquid-level measurements in volumetric flasks for industrial application. *ISRN Sensor Networks*, Article ID 618136, 7 pages.
6. Kim, Y.; Vig, J.R.; and Ballato, A. (1998). Sensing the properties of liquids with doubly rotated resonators. *Proceedings of IEEE International Frequency Control Symposium*. Pasadena, California, United States of America, 660-666.
7. Shigemi, S. (2007). Measurement of acoustic property of small-volume liquid sample beyond acoustic window. *Japanese Journal of Applied Physics*, 46(7B), 4549-4554.
8. Thurow, K.; Stoll, N. and Ritterbush, K. (2011). A fast optical method for the determination of liquid level in microplates. *Journal of Automated Methods and Management in Chemistry*, 6, Article ID 805153, 6 pages.
9. Lautscham, K.; Wente, F.; Schrader, W.; and Kaatze, U. (2000). High resolution and small volume automatic ultrasonic velocimeter for liquids. *Journal of Measurement Science and Technology*, 11(10), 1432-1439.
10. Heinicke, C. (2014). Contact-free measurement of the flow field of a liquid metal inside a closed container. *EPJ Web of Conferences,* 67, 4 pages.
11. Heinicke, C.; Thess; A. and Rahneberg, I. (2011). Towards local resolution in turbulent liquid metal duct flows. *Proceedings of the 13th European Turbulence Conference.* Warsaw, Poland, 5 pages.
12. Franco, E.E.; Adamowski, J.C.; and Buiochi, F. (2010). Ultrasonic viscosity

measurement using the shear-wave reflection coefficient with novel signal processing technique. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, 57(5), 1133-1139.

13. Uchlyama, T.; and Degawa, T. (2007). Vortex simulation of the bubbly flow around a hydrofoil. *International Journal of Rotating Machinery,* Article ID 72697, 9 pages.
14. Blaz, N.; Maric; A.; Toskov, S.; Miskovic, G.; Radosavljevic, G.; and Zivanov, L. (2014). Capacitive sensor for quantity detection of known liquid present in distilled water. *Proceedings of the 37$^{th}$ International Spring Seminar on Electronics Technology*. Dresdane, Germany, 438-441.
15. Stroher, G.L.; Rodrigues, A.C.; Dias, L.F.; Pedrao, M.R.; de Paula, L.N.; Visentainer, J.V.; and de Souza, N.E. (2012). Comparative analysis and validation methodologies of GC and HPLC for analysis of cholesterol in meat products. *American Journal of Analytical Chemistry,* 3(4), 306-311.
16. Morrison, A.L.A.; and Driskell, J.A. (1985). Quantities of $B_6$ vitamers in human milk by high-performance liquid chromatography: Influence of maternal vitamin $B_6$ status. *Journal of Chromatography B: Biomedical Sciences and Applications*, 337, 249-258.
17. Mandal, U.; Senthil, Rajan, D.; Bose, A.; Gowda, A. K.; Ghosh, A.; and Pal, T. K. (2006). Development and validation of an HPLC method for analysis of etoricoxib in human plasma. *Indian Journal of Pharmaceutical Sciences*, 68(4), 485-489.
18. Doebelin, E.O. (2003). *Measurement systems - application and design (5$^{th}$ ed.*). Boston: McGraw-Hill Higher Education.
19. Liptak, B.G. (2003). *Instrument engineers' handbook: Process measurement and analysis (4$^{th}$ ed.*). Boca Raton, Florida: CRC Press.
20. Sundaresan, K.R.; Prasad, C.C.; and Krishnaswamy, P.R. (1978). Evaluating parameters from process transients. *Industrial and Engineering Chemistry Process Design and Development*, 17(3), 237-241.
21. Nygaard, G.; and Naevdal, G. (2005). Modelling two-phase flow for control design in oil well drilling. *Proceedings of the International Conference on Control Applications.* Toronto, Ontario, Canada, 675-680.
22. Bi, Q.; Cai, W.-J.; Lee, E.-L.; Wang, Q.-G.; Hang, C.-C.; and Zhang, Y. (1999). Robust identification of first-order plus dead-time model for step response. *Control Engineering Practice*, 7(1), 71-77.
23. Marlin, T.E. (2000). *Process control. Designing processes and control systems for dynamic performance (2$^{nd}$ ed.*). Singapore: McGraw-Hill Book Co.
24. Seborg, D.E.; Edgar, T.F.; and Mellichamp, D.A. (2004). Process dynamics and control (2$^{nd}$ ed.). *American Institute of Chemical Engineers*, 54, 3026.
25. Anderson Jr., J.D. (1995). *Computational fluid dynamics. The basics with applications*. New York: McGraw-Hill Inc.
26. Aris, R. (1962). *Vectors, tensors, and the basic equations of fluid mechanics*. Englewood Cliffs, New Jersey, United States of America: Prentice Hall., Inc.
27. Jakobsen. H.A. (2014). *Numerical solution methods, chemical reactor modeling*. New York, United States of America: Springer International Publishing.

28. Erlach, K. (2013). *Value stream design: The way towards a lean factory*. Berlin, Germany: Springer Science + Business Media.
29. Obrazovskii, E.G. (2006). Diffusion-controlled first-order surface reactor in turbulent flow. *Journal of Experimental and Theoretical Physics*, 103(1), 119-125.
30. Jiang, R.; and Wu, Q.-S. (2005). First order phase transition from free flow to synchronized flow in a cellular automata model. *European Physical Journal B - Condensed Matter and Complex System*, 46(4), 581-584.