Journal of Engineering Science and Technology
Vol. 13, No. 6 (2018) 1573 - 1586

VECTOR ALGEBRA QIBLA DETECTION IN AN INDOOR, SEMI-OPEN AND OUTDOOR ENVIRONMENT

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Abstract

There are myriad of methods used in determining the direction of Qibla and calculation using basic spherical trigonometric formula is the most accurate way that is used for centuries. Previous studies have successfully shown an alternative method using vector algebra formulas (VA) for Qibla determination to improvise the spherical trigonometric formula but only through the estimation software of Mathematica 8.0. Hence, this paper aims to extend the existing Qibla determination using VA on actual hardware implementation via integrated development environment (IDE) programming along with field test to confirm its capabilities. The prototype development in order to utilize of VA for Indoor Qibla Marking System comprises two modules: indoor and outdoor. The outdoor module includes a global positioning system (GPS) to provide location information, which is fed into the indoor module inside the building via wireless communication. The location information is then encoded into a format that is understood by the user and processed using VA to generate Qibla angle. While the indoor module consists of True North Finding System (TNFS) placed on the horizontal rotating laser mounted on the micro stepper motor to minimize motor disturbance as well as improving pointing accuracy. The results of the field test proved that the hardware and software implementation with VA method for Qibla determination obtained percentage accuracy of 99.95% compared to Qibla direction verified by Department of Survey and Mapping Malaysia (JUPEM) in collaboration with Department of Islamic Development Malaysia (JAKIM).

Keywords: Global positioning system, Qibla, Stepper motor, True north finding system, Vector algebra.

Nomenclatures	
Greek Symbols	
°	Angle, deg.
Abbreviations	
GPS	Global Positioning System
IDE	Integrated Development Environment
JAKIM	Department of Islamic Development Malaysia
JUPEM	Department of Survey and Mapping Malaysia
VA	Vector Algebra

1. Introduction

Qibla is the direction of the Kaaba in Mecca that Muslims faced to achieve unity and focus in their daily prayer [1]. The Qibla is used not only for prayer but also plays a part in various ceremonies such as slaughtering procedure of an animal for food [2] including after death where Muslim is required to be buried in the direction of Qibla [3], and this practice help the archaeologists to determine the remains of Muslims' cemetery if there are no other signs presented [4]. Property developers or a Muslim architect also advised to design and build the foundation with reference to the Qibla direction, where toilets are arranged in such a way that they do not end up facing towards or against the direction of Qibla. Hence, being consistent with Qibla direction reference is important for Muslims in order to fulfil their worship to Allah [5-6].

Approaches toward determining the direction of Qibla [7] can be classified into two conventional and modern [5]. For conventional way, the Qibla direction is determined by using a stick to see the shadow cast by it from the sun [8], the observation of the constellation of stars, and map projection method used by Islamic intellectuals such as al-Khwarizmi and al-Battani [9-11]. On the other hand, the calculation has been introduced to determine Qibla [5] via basic spherical trigonometric formula and this modern technology is the most accurate way that is used for centuries [12-13].

A number of devices have been developed to determine the Qibla direction using spherical trigonometric formula as control algorithm [8, 14-16]. However, spherical trigonometric formula involved complicated calculation and essentially challenging as it requires many angles in trigonometry on a spherical surface [17]. An alternative method using vector algebra (VA) has been proposed [5] to the improvised formula for Qibla direction determination. Simulation via estimation software of Mathematica 8.0 results proves that the method using Qibla vector algebra achieve better accuracies [5]. Nevertheless, these only been examined in simulations with unlimited memory and processing resources. Hence, it is recommended that simulations must be complemented with field test on actual hardware not only to assure their capabilities but also to permit continued exploration and evaluation.

Hardware implementation with the support of peripheral interface controller (PIC) named Quboque System was designed using GPS and compass [18] to extend the previous study [5] allowing to deal with real-time applications. The main objective of Quboque System is to solve direction of Qibla at Muslim's graveyard and to provide a better arrangement in Muslim cemetery. However, the major drawback of this approach is the usage of compass module that can be easily degraded by nearby ferrous materials, electromagnetic interference and including weather [19-21]. Hence, the compass is inaccurate and to solve the magnetic field effect towards heading is hard.

Global Positioning System (GPS) able to provide location and time information accurately with at least three to four satellites [22-24]. However, GPS does not work indoors [25-27] where the signals are heavily attenuated and scattered by roofs, walls and other objects [28-31]. Therefore, the development of Quboque System [18] is at best made only outdoor where the amount of disturbance is minimal and not reliable when using in buildings.

In Malaysia, Department of Islamic Development Malaysia (JAKIM) is the body fully responsible for the determination of the exact Qibla direction for main mosques and principal infrastructures across the country. JAKIM practices a variety of methods in determining the direction of Qibla including Kaabah sun Istiwaa that occurs on May 28 and July 16 each year, sunset scheduling, and sometimes the use of constellations. These conventional methods that have long been practised, complex and are not very practical for cases involving large, intricate structures such as hotels and offices, and at areas having harsh and hilly terrains.

Therefore, building the above, there is a need for a system and method for translating the direction and orientation from the outdoor area to the indoor area of a building to overcome the raised issue. The utilization of Vector Algebra for Qibla detection divided into modules outside and inside the building. The outdoor modules include a GPS for locating location information to be sent to indoor module stationed in building via a wireless communication technology medium on 433MHz radio frequency waves. Coordinate location data is encoded into a format that is understood by the user and processed using algebraic vector formula to generate Qibla angle.

The remainder of the paper is organized as follows: Section 2 presents system hardware design, followed by Section 3 describing the Qibla vector algebra supported by integrated development environment (IDE) software. Thereafter, Section 4 demonstrates the implementation of the micro-stepping unit, including calibration assembly by using polygon mirrors and autocollimator. The measurements and field tests are presented in Section 5 and finally, Section 6 summarizes and concludes the findings of the proposed study.

2. Hardware Integration

The implementation of the project comprises indoor and outdoor modules. The outdoor module transmits coordinate information to the indoor module through 433MHz radio frequency wave. The current position in form of latitude and longitude are decoded, calculated using VA and displayed in a readable format (angle of Qibla). This is illustrated in Fig. 1.

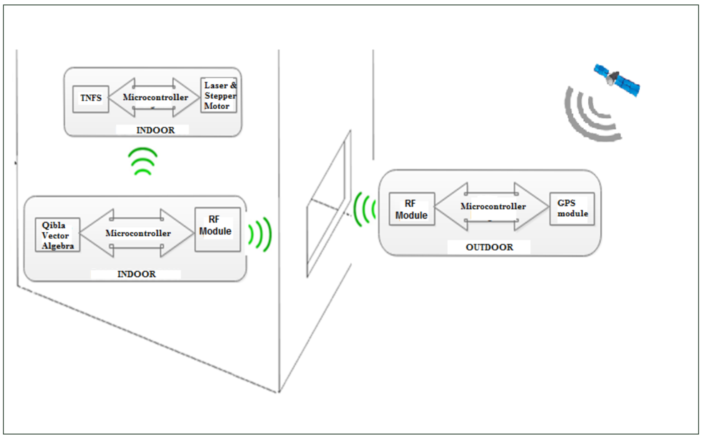

Fig. 1. llustration of the whole system.

2.1. Outdoor module

ATmega microcontroller serves as a processing unit takes data from the GPS module and displays the data on an organic light-emitting diode (OLED) display before being transmitted to the indoor module through 433 MHz radio frequency module (RF). The processing unit will receive this data on an ongoing basis whether the same or different data where it depends on the programming. The outdoor module is designed by using some basic hardware that includes but not limited to microprocessors, crystal oscillators, OLED display, GPS module and RF module as depicted in the following Fig. 2.

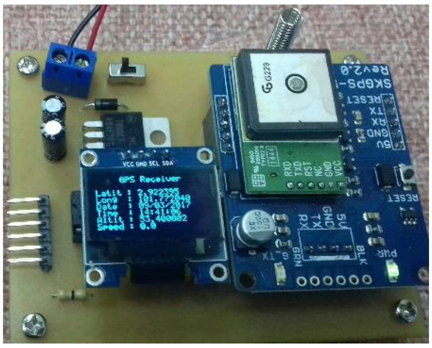

Fig. 2. Outdoor module.

2.2. Indoor module

ATmega microcontroller Fig. 3 depicts the indoor system that comprises of True North Finding System (TNFS) [32-33] placed on the horizontal rotating laser mounted on a stepper motor. The two layer of printed circuit board as shown in Fig.

4 is developed to control the rotation of the micro stepper motor. Essentially, the goal of using stepper motor is to improve low-speed smoothness and minimize motor disturbance as well as improving pointing accuracy [34-35] running on a specially developed stepper motor shield. A driver is selected to control the rotation in the micro mode of 0.1125° per step and switch between different positions.

The microcontroller Arduino controls the rotating stepper motor in sub-degree stepping stage, while the TNFS is tasked to collects multiple data for determining the direction of true north (reference point). The true north information is used as a reference input to appropriately steers the laser source forming the directional line pointing toward Qibla direction based upon outdoor systems via Qibla vector algebra formula.

Fig. 3. Outdoor module.

Fig. 4. Custom made micro stepper driver shield.

3. Software Integration

This study introduces a new approach where the VA performed using Arduino IDE software due to its advantage of less CPU intensive and advantageous in actual application. The user's current location and Qaaba location in Mecca must first be defined in order to calculate the Qibla angle using VA. In this case, the user's current location information will be tracked using the GPS outdoor module. As in Fig. 5, the "Serial.available" is used to read all the available information from the GPS outdoor module.

```
//We first check if there is any data available in the receiver.
while (Serial.available())
```

Fig. 5. Programming code to read GPS information.

The following code in Fig. 6, the algorithm used to extract and encode into readable character when the data is available in the receiver.

```
{
  //Then read and encode the data
  message+=(char)Serial.read();
}
```

Fig. 6. Programming code to extract the received data into readable character.

The location data of Kaabah already provided by Department of Survey and Mapping Malaysia (JUPEM); 21°25'15.6"N (latitude origin) and 39°49'29.1"E (longitude origin) as shown in Fig. 7.

HINTUNGAN ARAH KIBLAT

Seksyen Graviti Dan Falak
Bahagian Ukur Geodetik
Jabatan Ukur Dan Pemetaan Malaysia

Masjid / Surau	: UKM		
Daerah / Lokasi	: BANGI		
Negeri	: SELANGOR		KIBLAT-1970-01-26-V1
Latitud Origin, $Ø_O$	: 03° 40' 48" U	Longitud Origin, λ_O	: 101° 30' 24" T
Latitud Stesen, $Ø_S$	: 2° 55' 22.1" U	Longitud Stesen, λ_S	: 101° 46' 23.3" T
Latitud Mekah, $Ø_M$	: 21° 25' 15.6" U		
Longitud Mekah, λ_M	: 39° 49' 29.1" T		

Fig. 7. Field test location information and coordinates of Kaabah generated by JUPEM.

From the obtained coordinate, the vector $\vec{u}$ can be computed by the following equation:

$$\vec{u} = \langle -Zp, 0, Xp \rangle \tag{1}$$

The vector $\vec{u}$ is a unit vector that directs to the North and also tangent for the point of interest, p. Vector $\vec{v}$ is a vector that has a direction to the Qaaba, m and therefore, the vector $\vec{v}$ is given as follows:

$$\vec{v} = \langle -Zpb, Zpa - XpC, Xpb \rangle \tag{2}$$

Algorithm for coordinate p and m as shown in Fig. 8 while both vector $\vec{u}$ and $\vec{v}$ demonstrated in the algorithm as in Fig. 9 and Fig. 10 respectively. The vector of $\vec{N}$ can be obtained using the dot product of $\overrightarrow{OM}.\overrightarrow{OP}$, where it can be computed by the following equation:

$$\vec{N} = \langle -ZpYm, ZpXm - XpZm, XpYm \rangle \tag{3}$$

```
//Coordinate Kaabah
//latitude origin Mekah = 21°25'15.6"N (21.421°) by JUPEM
//longitude origin Mekah = 39°49'29.1"E (39.82475°) by JUPEM
float LAT_MAKKAH = 21.4210;
float LONG_MAKKAH = 39.82475;

//User's location
//IF LAT = 5.41deg = 0.09442231 radians
//IF LONG = 100.31 = 1.75073977 radians
float Xp = cos(radians(latitude));
float Yp = 0;
float Zp = sin(radians(latitude));

float Xm = cos(radians(LAT_MAKKAH)) * cos(radians(longitude - LONG_MAKKAH));
float Ym = cos(radians(LAT_MAKKAH)) * sin(radians(longitude - LONG_MAKKAH));
float Zm = sin(radians(LAT_MAKKAH));
```

Fig. 8. Algorithm for coordinate *p* and *m* to calculate Vector $\vec{u}$ and $\vec{v}$.

Finally, substitute the vector $\vec{u}$ and $\vec{v}$ into $\left(\vec{u}.\vec{v} / \|\vec{u}\| \|\vec{v}\|\right)$ to yield the value of theta, θ that will be substracted with 360° to get the Qibla angle as shown in Fig. 11.

```
float U = (-Zp , 0 , Xp); //(U1,U2,U3)

//vector u*vector u
float U1 = (-Zp);
Serial.println("");
Serial.print("vector U1 is: "); Serial.print(U1);

float U2 = 0;
Serial.println("");
Serial.print("vector U2 is: "); Serial.print(U2);

float U3 = (Xp);
Serial.println("");
Serial.print("vector U3 is: "); Serial.print(U3);

float UU = sqrt(pow (U1, 2) + pow(U2, 2) + pow(U3, 2));
Serial.println("");
Serial.print("vector UU is: "); Serial.print(UU);
```

Fig. 9. Programming code for vector $\vec{u}$.

```
float V = (-Zp * n2 , Zp * n1 - Xp * n3 , Xp * n2);

//vector v*vector v
float V1 = (-Zp * n2); //V1
Serial.println("");
Serial.print("vector V1 is: "); Serial.print(V1);

float V2 = (Zp * n1) - (Xp * n3); //V2
Serial.println("");
Serial.print("vector V2 is: "); Serial.print(V2);

float V3 = (Xp * n2); //V3
Serial.println("");
Serial.print("vector V3 is: "); Serial.print(V3);

float VV = sqrt(pow (V1, 2) + pow(V2, 2) + pow(V3, 2));
```

Fig. 10. Programming code for vector $\vec{v}$.

```
float Theta = acos((UV) / (UU * VV));
float Y = degrees(Theta);
float Degree = ( 180 - Y);
float Kiblat = (360 - Degree);

Serial.println("");
Serial.print("KIBLAT = ");
Serial.println(Kiblat, 2);

int degree = Kiblat;
Serial.println("");
Serial.print("KIBLAT in degree = ");
Serial.println(degree);
```

Fig. 11. Algorithm of equation to find Qibla angle.

4. Test Scenario

The tests conducted at three different scenarios: indoor, semi-open and outdoor to verify the prototype implementation.

4.1. Indoor

The prototype tested indoor environment at prayer room of Centre for Collaborative Innovation (PIK) Bangi with JAKIM. JAKIM officer determines and marked the Qibla direction using the Theodolite as shown in Fig. 12. These processes rely on the concept of points and degree calculations and must be carried out outside under the sun. For marking of Qibla direction inside the building, the points are firstly marked outside and then transferred point-by-point into the PIK's prayer room.

Fig. 12. JAKIM officer observed the sun and equipped with theodolite.

4.2. Semi-open

Semi-open scenario test was accomplished with JUPEM at Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia (FKAB UKM). JUPEM performed similar process where the points are firstly marked outside and then transferred point-by-point to the semi-open space as depicted in Fig. 13. The performance result of this test shown in next section.

Fig. 13. JUPEM observed the difference of Qibla direction marked by prototype.

4.3. Outdoor

An outdoor experiment was conducted at three different faculties at Universiti Kebangsaan Malaysia (UKM) Bangi for the outdoor scenarios: Fakulti Kejuruteraan dan Alam Bina (FKAB), Kolej Burhanuddin Helmi (KBH), and Fakulti Pendidikan (FPEND) between 10.00 a.m. to 2.30 p.m. at latitude of $\varphi = 2.9300°$ N. The measured data are then compared with the Sun Compass Application (SCA) as in Fig. 14 developed by Universiti Sultan Zainal Abidin (UniSZA) which certified of its accuracy by JAKIM.

Fig. 14. The sun compass application by UniSZA.

5. Results and Discussion

Various tests at three different scenarios successfully done to compare the accuracy of the developed prototype with the existing method and equipment. The performance evaluations are categorized as follows.

5.1 Indoor scenario

On 24th October 2016 in collaboration with JAKIM, Indoor Qibla Marking System has been tested. The measurement results as in Table 1 proven an accuracy of 99.98% compared to Qibla direction marked by JAKIM as can be seen in Fig. 15.

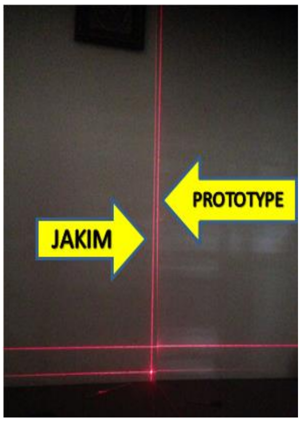

Fig. 15. Comparison of Qibla marking between the developed prototype and JAKIM.

Table 1. Comparison between developed prototype and JUPEM.

Indoor Scenario	Prototype	JAKIM	Percentage accuracy
Qibla angle	292.6944°	292.6261°	99.98%

5.2 Semi-open scenario

On 14th March 2017, the second indoor test was conducted with JUPEM and the obtained Qibla angle are successfully accurate and verified with an accuracy of 99.98% as in Table 2. Fig. 16 captured the comparison Qibla direction marked by JUPEM and the developed prototype.

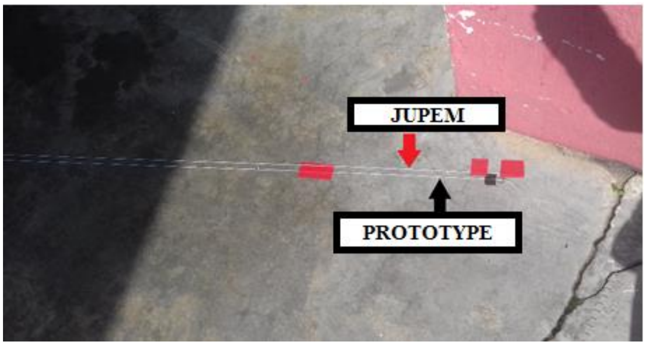

Fig. 16. Comparison of Qibla marking between the developed prototype and JUPEM

Table 2. Comparison between developed prototype and JUPEM.

Semi-open Scenario	Prototype	JUPEM	Percentage accuracy
Qibla angle	292.6944°	292.6261°	99.98%

5.3 Outdoor scenario

The measured Qibla angle through prototype is compared with the Sun Compass Application (SCA) as Table 3.

Table 3. Comparison between SCA and prototype.

Field Test	Prototype	SCA	Percentage accuracy
College Burhanuddin	292.6292°	292.6301°	99.99%
Faculty of Engineering	292.7508°	292.7801°	99.66%
UKM Mosque	292.4812°	292.4829°	99.99%

Table 3 demonstrates that the utilization of Vector Algebra for outdoor Qibla Marking achieve an average accuracy of 99.88% compared with the Qibla orientation of the SCA. Hence, the developed system is accurate and stable compared to the available Qibla direction scheme and equipment.

6. Conclusions

The presented work demonstrates indoor and outdoor module with the utilization of vector algebra (VA) for Qibla marking system. The conducted field tests of indoor, semi-open and outdoor with Department of Survey and Mapping Malaysia (JUPEM) and Department of Islamic Development Malaysia (JAKIM) successfully proven that the building prototype achieves percentage accuracy of 99.95% compared to the actual Qibla direction.

Acknowledgements

Authors would like to thank Centre for Collaborative Innovation (PIK), Universiti Kebangsaan Malaysia (UKM) and Ministry of Higher Education (MOHE) for financial support through grants INOVASI-2017-10 and FRGS/1/2016/TK04/UKM/02/7.

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