

ANALYSIS AND COMPARISON OF WAVE PROPAGATION IN MICROSTRIP LINES AND COPLANAR WAVEGUIDES

KIM HO YEAP¹, KOON CHUN LAI^{1,*}, SOO KING LIM², CHONG YU LOW¹

¹Faculty of Engineering and Green Technology

²LKC Faculty of Engineering and Science
Universiti Tunku Abdul Rahman, Malaysia

*Corresponding Author: laikc@utar.edu.my

Abstract

Despite their obvious advantages over microstrips, coplanar waveguides (CPWs) have not been widely used as probes for wave coupling in mixer circuits. Microstrips are still the preferred option. One major reason is because CPWs are generally believed to exhibit higher attenuation than microstrip probes. Here, we present numerical analysis between the loss of waves in normal and superconducting CPWs and microstrips. The results show that the loss in microstrips is generally higher than that found in CPWs at dimensions larger than the wavelength. At dimensions comparable and smaller than the wavelength (i.e. $q < 2.2$), however, the loss in a CPW appears to be significantly lower. The results suggest strongly that, with careful design, CPWs can be a better alternative for the coupling of waves in Superconductor Insulator Superconductor (SIS) mixer circuits.

Keywords: Microstrip, Coplanar waveguides, Quasiparticles, Surface impedance.

1. Introduction

The conventional coplanar waveguide (CPW) proposed by Wen [1] is a planar device consisting of a dielectric substrate with a layer of conductor deposited at the top surface, as shown in Fig. 1. The metallization layer is separated into three sections – a centre strip with narrow gap at both sides separating it from two ground planes. To simplify the analysis, Wen has assumed the thickness of the dielectric substrate b to approach infinity. For practical application, however, the thickness of the CPW has to be finite. Indeed, it is the width w and thickness t_s of the centre strip, the gap between the strip and the ground plane w_c , the permittivity ϵ_d , and the height b of the dielectric substrate which would determine

Nomenclatures

b	Height of the dielectric substrate of a CPW, μm
f	Operating frequency, s^{-1}
$K(k_1)$	Elliptic integral of the first kind
$K'(k_1)$	Complement of the elliptic integral of the first kind
q	Multiplication factor
R_s	Surface resistance of the conductor, ohm-m
t_s	Strip thickness, μm
w	Width of the strip, μm
w_c	Gap between the strip and the ground plane, μm
Z_0	Characteristic impedance of the wave, Ω
Z_S	Surface impedance of the strip and ground plane, Ω

Greek Symbols

α	Attenuation of wave
Δ	Energy gap parameter, eV
ϵ_d	Permittivity of dielectric, F/m
ϵ_{eff}	Effective dielectric constant
ϵ_w	Permittivity of conductor, F/m
μ_w	Permeability of conductor, H/m
σ	Conductivity of conductor, S/m
τ	Mean free time, s
ω	Angular frequency, rad/s
\hbar	Planck's constant, $\text{m}^2\text{kg/s}$

Abbreviations

CPW	Coplanar Waveguide
FETs	Field Effect Transistors
MESFETs	Metal Semiconductor Field Effect Transistors
MMICs	Monolithic Microwave Integrated Circuits
MOSFETs	Metal Oxide Semiconductor Field Effect Transistors
SIS	Superconductor Insulator Superconductor

the characteristic impedance Z_0 and the attenuation α of wave in the CPW.

CPWs have been extensively used in the design of monolithic microwave integrated circuits (MMICs). In fact, CPWs offer a few advantages over microstrip lines. Unlike microstrip structures which require via holes to ground active devices, CPWs allow ground connections to be made at the substrate edge [2, 3]. It is worth noting that at high frequencies, via holes can introduce significant inductance and degrade circuit performance [4]. Besides, CPWs are uniplanar devices which eliminate the additional steps for backside wafer processing and this significantly reduces the fabrication cost. CPWs also allow easy connection of shunt and series circuit elements [5-7]. Hence, it is well suited for use with field effect transistors (FETs) such as metal oxide semiconductor field effect transistors (MOSFETs) and metal semiconductor field effect transistors (MESFETs), which are coplanar [8] in nature as well.

Nevertheless, CPWs have not been commonly used in the coupling of millimeter and submillimeter waves in Superconductor-Insulator-Superconductor

(SIS) mixer circuits. Microstrips are still the preferred option for wave coupling. One most general reason for this is that CPWs are believed to inherently exhibit higher conduction loss than microstrips [4]. According to Gopinath [9] and Itoh [10], however, the conduction loss in CPWs can be significantly lower than that in microstrip lines under special circumstances. In fact, Kittara et al. [11] has performed a theoretical study by applying the formulations developed by Gupta [12] and Yassin and Withington [13]. According to [11], it was reported that when a CPW is designed with a higher dielectric constant and at dimensions much larger than the microstrip, the loss turns out to be much lower than that in a microstrip. It is not indicated, however, which structures would exhibit lower loss when the dielectric constant and dimensions (such as the width of the strip w , the thickness of the substrate b , etc.) of the structures are similar to each other. Hence, in this paper, the authors present a comparison and analysis between the losses of waves in normal and superconducting CPWs and microstrip lines – both of which are designed at similar dimensions. In order to show that the performance of both devices at large dimensions (such as those used in printed circuit technology) and small dimensions (such as those used in SIS mixer circuits) could be different, an analysis is made with the dimensions of both devices multiplied with a factor q varying from 0 to 5.

It is difficult to determine the conduction loss for planar lines in an absolute sense. The loss depends on a large extent on the conductor surface roughness, which can vary from device to device, albeit being designed with the same geometry and dimensions [4]. In addition, the loss is highly affected by the behaviour of the current crowded at the edge of the strip with different thickness [13-15]. Therefore, the surface roughness for both CPWs and microstrip structures has been assumed to be zero in the comparison of microstrip and CPWs.

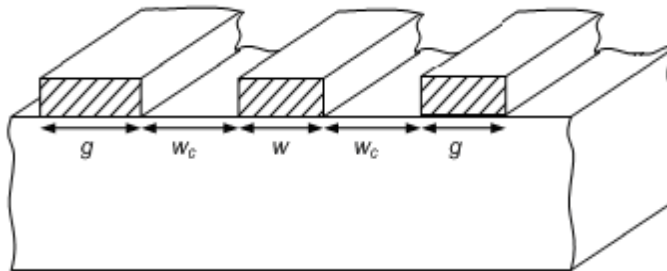


Fig. 1. The cross section of a coplanar waveguide.

2. Mathematical Model

Three types of losses can be identified in coplanar waveguides, namely dielectric, ohmic and radiation/surface wave losses. Very often dielectric loss can be taken to be negligibly small by choosing a low loss substrate material. Power leakage due to surface waves and radiation from unwanted modes can also be avoided by carefully designing the CPW circuit. Since the loss from the dielectric and radiation/surface waves could be suppressed, the ohmic loss in coplanar waveguides would only be considered in the subsequent analysis.

2.1. Ohmic loss

To calculate the ohmic losses in CPWs, the analytical solution published by Ghione [16] shall be applied. The power dissipated in the line is evaluated through a conformal approximation of the current density of the finite thickness structure, with the width of the ground plane g tending to infinity. The analytical relation of the attenuation constant is then derived using the power-loss method as Eq. (1)

$$\alpha = \frac{R_s \sqrt{\epsilon_{eff}(f)} \left\{ \frac{\left[\pi + \ln \left(\frac{8\pi w(1-k_1)}{2t_s(1+k_1)} \right) \right]}{w} + \frac{\left[\pi + \ln \left(\frac{4\pi(w+2w_c)(1-k_1)}{t_s(1+k_1)} \right) \right]}{(w+2w_c)} \right\}}{240\pi K(k_1)K'(k_1)(1-k_1^2)} \tag{1}$$

where R_s is the surface resistance of the conductor, $\epsilon_{eff}(f)$ the frequency dependent effective dielectric constant, w the width of the strip, and $K(k_1)$ and $K'(k_1)$ are the complete elliptic integrals of the first kind and its complement, respectively. The argument of the elliptic integrals k_1 can be solved using a pair of conformal transformations [17]

$$k_1 = w(w+2w_c)^{-1} \tag{2}$$

Here, the series expansion for $K(k_1)$ illustrated by Hilberg [18] has been implemented, as given below

For $0 \leq k_1 \leq 0.707$,

$$K(k_1) = \frac{\pi}{2} \left\{ 1 + 2 \left(\frac{k_1^2}{8} \right) + 9 \left(\frac{k_1^2}{8} \right)^2 + 50 \left(\frac{k_1^2}{8} \right)^3 + 306.25 \left(\frac{k_1^2}{8} \right)^4 + \dots \right\} \tag{3a}$$

For $0.707 \leq k_1 \leq 1$,

$$K(k_1) = p + (p-1) \left(\frac{k_1'^2}{4} \right) + 9 \left(p - \frac{7}{6} \right) \left(\frac{k_1'^4}{64} \right) + 25 \left(p - \frac{37}{30} \right) \left(\frac{k_1'^6}{256} \right) + \dots \tag{3b}$$

where $p = \ln(4/k_1')$ and $k_1' = (1-k_1^2)^{1/2}$

A simple but accurate expression which relates $K'(k_1)$ to $K(k_1)$ can be found in [19] as

for $0 \leq k_1 \leq 0.707$,

$$K'(k_1) = \frac{K(k_1)}{\pi} \ln \left[\frac{2(1+\sqrt{k_1'})}{(1-\sqrt{k_1'})} \right] \tag{4a}$$

and for $0.707 \leq k_1 \leq 1$,

$$K'(k_1) = \frac{K(k_1)\pi}{\ln \left[\frac{2(1+\sqrt{k_1})}{(1-\sqrt{k_1})} \right]} \tag{4b}$$

The surface resistance R_s in Eq. (1) can be obtained by computing the surface impedance Z_s of the strip and ground plane, followed by extracting the real part of Z_s . The surface impedance Z_s can be expressed in terms of the electrical properties of the conductor [15, 20]

$$Z_s = \left(\frac{\mu_w}{\epsilon_w} \right)^{1/2} \tag{5}$$

where μ_w and ϵ_w are the permeability and permittivity of the conducting material (i.e. strip and ground planes), respectively. ϵ_w is complex and is given as

$$\epsilon_w = \epsilon - j \frac{\sigma}{\omega} \tag{6}$$

where σ is the conductivity of the conducting material, ϵ the permittivity of free space and ω is the angular frequency.

2.2. Attenuation of waves

In order to estimate the loss of waves in millimeter and submillimeter wavelengths more accurately, the authors have applied Drude’s model for the frequency dependent conductivity σ [20]

$$\sigma = \sigma_n (1 + j\omega\tau)^{-1} \tag{7}$$

where σ_n is the conventional constant conductivity and τ the mean free time. For most conductors [21], the mean free time is in the range of 10^{-13} to 10^{-14} s. According to the Bardeen-Cooper-Schrieffer (BCS) theory [22], energy level equivalent or higher than the energy gap $2\Delta(T)$ is necessary in order to break Cooper pairs into quasiparticles in superconductors. The complex conductivity developed by Mattis and Bardeen from the microscopic analysis of Bardeen-Cooper-Schrieffer (BCS) theory [15, 23] is defined as Eqs. (8) and (9)

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega} \int_{\Delta}^{\infty} [f(E) - f(E + \hbar\omega)] \frac{E^2 + \Delta^2 + \hbar\omega E}{(E^2 - \Delta^2)^{1/2} [(E + \hbar\omega)^2 - \Delta^2]^{1/2}} dE \tag{8}$$

$$+ \frac{1}{\hbar\omega} \int_{\Delta - \hbar\omega}^{-\Delta} [1 - 2f(E + \hbar\omega)] \frac{E^2 + \Delta^2 + \hbar\omega E}{(E^2 - \Delta^2)^{1/2} [(E + \hbar\omega)^2 - \Delta^2]^{1/2}} dE$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega} \int_{\Delta - \hbar\omega, -\Delta}^{\Delta} [1 - 2f(E + \hbar\omega)] \frac{E^2 + \Delta^2 + \hbar\omega E}{(\Delta^2 - E^2)^{1/2} [(E + \hbar\omega)^2 - \Delta^2]^{1/2}} dE \tag{9}$$

where \hbar is the reduced Planck’s constant, σ_n the normal conductivity and $\Delta = \Delta(T)$ the energy-gap parameter. The function,

$$f(E) = (1 + \exp(E/kT))^{-1} \tag{10}$$

gives the Fermi-Dirac statistics and k is the Boltzmann’s constant. The first integral in Eq. (8) describes the effect of the thermally excited quasiparticles. The second integral denotes the generation of quasiparticles by fields with frequencies f corresponding to energies above the gap energy. Thus, the second integral is zero for $\hbar\omega < 2\Delta$. Since σ_2 indicates the contribution due to the Cooper pairs, the

lower integration limit in Eq. (9) becomes $-\Delta$ when $\hbar\omega > 2\Delta$. Δ depends on temperature and is obtained from the relation [15, 25]

$$\ln(\tilde{\Delta}) = -2 \int_0^{\infty} \left(E^2 + \tilde{\Delta}^2 \right)^{-1/2} \left\{ 1 + \exp \left[\left(\pi / \gamma_E \tilde{T} \right) \left(E^2 + \tilde{\Delta}^2 \right)^{1/2} \right] \right\}^{-1} dE \quad (11)$$

where $\tilde{\Delta} = \Delta(T) / \Delta(0)$, $\tilde{T} = T / T_c$ and $\gamma_E = 1.781$ is the Euler's constant. Ghione [16] has applied an effective dielectric constant ϵ_{eff} which does not vary with frequency f in its calculation of loss. Here, to account for the dispersive effect in CPWs, a frequency dependent effective dielectric constant $\epsilon_{eff}(f)$ has been incorporated into Eq. (1). The frequency dependent effective dielectric constant $\epsilon_{eff}(f)$ is found by curve fitting the results of simulation [24]

$$\epsilon_{eff}(f) = \left[\sqrt{\epsilon_{eff}} + \frac{\sqrt{\epsilon_r} - \sqrt{\epsilon_{eff}}}{1 + G(f / f_{TE})^{-1.8}} \right]^2 \quad (12)$$

where f_{TE} is the cut-off frequency for the substrate. The variables G , u_A , and v_A are given respectively as

$$G = \exp \left[u_A \ln \left(\frac{w}{w_c} \right) + v_A \right] \quad (13a)$$

$$u_A = 0.54 - 0.64 \ln(2w/b) + 0.015 [\ln(2w/b)]^2 \quad (13b)$$

$$v_A = 0.43 - 0.86 \ln(2w/b) + 0.54 [\ln(2w/b)]^2 \quad (13c)$$

The effective dielectric constant ϵ_{eff} is dispersionless and can be derived using quasi-static methods. Hence, the ϵ_{eff} formulated by Veyres and Hanna [17] using the quasi-static conformal transformation has been applied in Eq. (12). The effective dielectric constant ϵ_{eff} is expressed as

$$\epsilon_{eff} = 1 + \frac{\epsilon_r - 1}{2} \frac{K(k_2) K'(k_1)}{K'(k_2) K(k_1)} \quad (14)$$

The argument k_2 of the elliptic integral is given as

$$k_2 = \sinh \left(\frac{\pi w}{4b} \right) \left\{ \sinh \left[\frac{\pi(w/2 + w_c)}{2b} \right] \right\}^{-1} \quad (15)$$

3. Implementation and Results

In order to compare the loss in microstrip lines and CPWs at different dimensions, the strip width and substrate thickness of both devices operating at $f = 100$ GHz are varied. For the CPW, the conduction loss is computed using Eq. (1). Whereas, the conduction loss is computed using the full wave equation in [15] for the microstrip line. In the analysis, the loss of the strip width w at 750 nm and substrate thickness b at 250 nm is first computed. Both strip width and substrate thickness are then increased by a multiplication factor q – i.e. strip width $w = 750 \text{ nm} \times 10^q$ and substrate thickness $b = 250 \text{ nm} \times 10^q$. Here, q is allowed to vary

from 0 to 5. The strip thickness t_s for both the microstrip and CPW is taken to be 300 nm, while the ground plane thickness t_g for the microstrip is the same as the strip thickness – i.e. $t_g = t_s = 300$ nm. The strip and ground plane are made of Niobium (Nb) with constant conductivity $\sigma_n = 1.57 \times 10^7$ S/m at room temperature and the dielectric constant of the substrate ϵ_r is given as 3.8 for both the microstrip line and CPW. The distance between the strip and the ground plane for the coplanar waveguide w_c is taken to be 5 μm .

From Fig. 2, it can be clearly seen that as q increases, the conduction loss of the microstrip line decreases at a higher rate than the CPW. Both curves intersect at $q = 2.2$. At large dimensions where $q > 2.2$, the loss of the microstrip line is much lower than the CPW. At $q < 2.2$, however, it can be observed that the conduction loss of the CPW turns out to be considerably lower. Such results give a strong implication especially in the design of SIS mixer circuits for the detection of millimeter and submillimeter waves, where microstrips are usually used for the coupling of waves. The dimensions of an SIS circuit are small. In [25] for instance, the substrates cross section for a microstrip used to couple a 100 GHz signal is around $610 \times 150 \mu\text{m}^2$. Due to the fact that a CPW features much lower attenuation in small dimensions (where $q < 2.2$), the result in Fig. 2 actually suggests that CPWs can, hence, be considered as a better alternative for waves coupling.

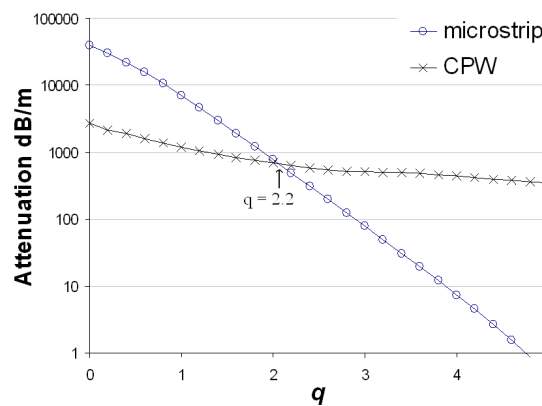


Fig. 2. Comparison of conduction loss between microstrips and CPWs ($w = 750 \text{ nm} \times 10^q$, $b = 250 \text{ nm} \times 10^q$).

The characteristic impedance of a CPW or microstrip line is determined by its strip width-to-substrate thickness (w/b) ratio. When scaling the dimension of the structure, it is important to scale both w and b accordingly. In order to investigate the influence of these parameters and to find out the more dominant parameter (w or b) to the variation in loss, one of these parameters was fixed while the other was varied. Figure 3 depicts the change in loss in both microstrips and CPWs when the substrate thickness b was fixed at 250 nm whereas the strip width w was initially set at 750 nm. The multiplication factor which varies from 0 to 5 was then applied to the strip width, changing it at $750 \text{ nm} \times 10^q$. Likewise, Fig. 4 shows the loss in both structures when the width was fixed at 750 nm and the thickness varied at $250 \text{ nm} \times 10^q$. It can be observed from Fig. 3 that the loss in the coplanar waveguide CPW is more susceptible to the change in the strip width. The loss in the microstrip, on the other hand, stays almost invariant as q increases.

Figure 4 shows that the loss in the CPW varies very little, whereas the microstrip line decreases sharply as the substrate thickness increases. Clearly, a coplanar waveguide CPW with a wider width tends to reduce losses in the structure. Since the ground plane of a CPW structure lies at the same surface as the strip and that the fields interact directly between the strip and the ground plane, the phenomenon i.e. wider strip results in lower loss, could be attributed to the ability a wider strip in containing the fields within the strip.

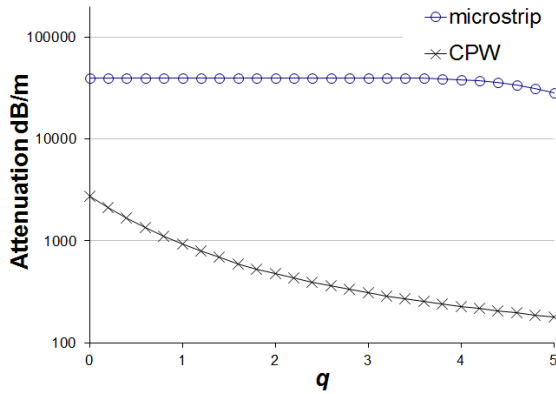


Fig. 3. Comparison of conduction loss between microstrips and CPWs ($w = 750 \text{ nm} \times 10^q$, $b = 250 \text{ nm}$).

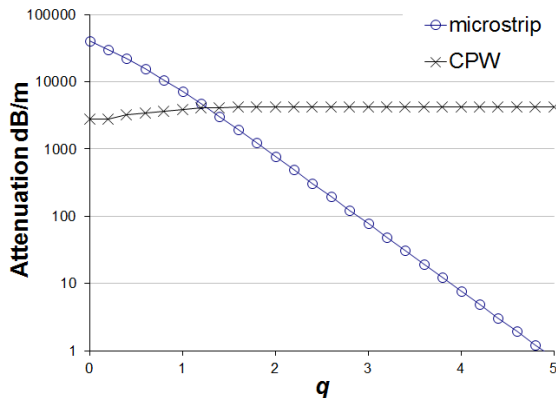


Fig. 4. Comparison of conduction loss between microstrips and CPWs ($w = 750 \text{ nm}$, $b = 250 \text{ nm} \times 10^q$).

4. Comparison between Microstrips and Coplanar Waveguides

An SIS mixer circuit usually operates under the critical temperature of the superconductor. Hence, the performance of superconducting coplanar waveguides and microstrips must be investigated as well. Figures 5 and 6 show the losses of waves in superconducting CPWs and microstrips operating at temperature $T = 4.2 \text{ K}$, for “large” and “small” dimensions, respectively. For “large” dimensions where $q > 2.2$, the following parameters for both CPWs and microstrips have

been taken: $w = b = 200 \mu\text{m}$, $t_s = t_g = 300 \text{ nm}$. On the other hand for “small” dimensions where $q < 2.2$, the parameters are: $w = 750 \text{ nm}$, $b = 250 \text{ nm}$, $t_s = t_g = 300 \text{ nm}$. The distance between the strip and ground plane in a coplanar waveguide w_c for “large” and “small” dimensions are $5 \mu\text{m}$ and $2 \mu\text{m}$ respectively.

From Fig. 5, it can be clearly seen that the loss of wave in a superconducting CPW with “large” dimensions turns out to be higher than those in a microstrip line. The loss in the CPW is, however, much lower at “small” dimension, as depicted in Fig. 6. Hence, this suggests that the result shown in Fig. 2 for normal structures is also valid for the case of a superconductor. In other words, it can be seen that at “small” dimensions – i.e. the size of a probe usually used for wave coupling in an SIS mixer, a CPW exhibits much lower loss if compared to a microstrip probe of a similar size.

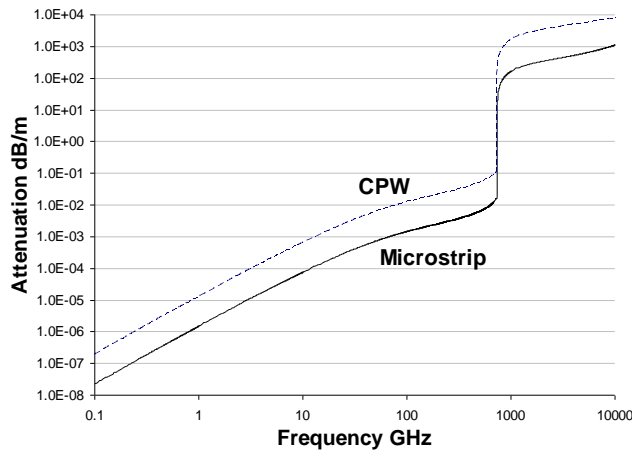


Fig. 5. Conduction loss of superconducting microstrips and CPWs for “large” structures where $q > 2.2$.

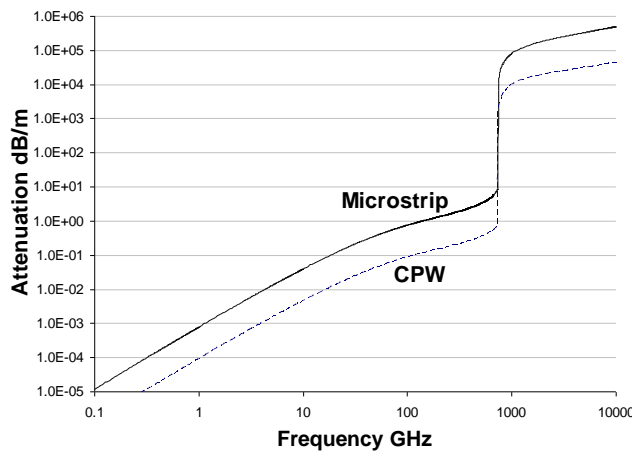


Fig. 6. Conduction loss of superconducting microstrips and CPWs for “small” dimensions where $q < 2.2$.

5. Conclusion

The modelling and comparison between the attenuation of waves propagating in a coplanar waveguide (CPW) and microstrip line are performed. The results for both the normal and superconducting cases show that the conduction loss of a microstrip line decreases at a higher rate than that of the CPW as the dimensions for both devices increase. The analysis apparently shows that the belief that microstrips exhibit lower loss than CPWs is not generally true. Losses in microstrips designed at dimensions larger than the wavelength (such as those used in printed circuit boards) is much lower. However, as the dimensions reduce to that comparable to or smaller than the wavelength (i.e. $q < 2.2$), the loss in a CPW appears to be significantly lower. Such finding is very useful especially in the design of mixers. As reported in [26], the size of a probe used in SIS mixer lies in the range where $q < 2.2$. The result illustrated in Figs. 2 and 4 suggests that CPWs can be considered as a better alternative device in the coupling of millimeter and submillimeter waves.

References

1. Wen, C.P. (1969). Coplanar waveguide: A surface strip transmission line suitable for non-reciprocal gyromagnetic device application. *IEEE Transactions on Microwave Theory and Techniques*, 17, 1087-1090.
2. Browne, J. (1987). Broadband amps sport coplanar waveguide. *Microwaves RF*, 26(2), 131-134.
3. Browne, J. (1987). Coplanar MIC amplifier bridges 0.5 to 18.0 GHz. *Microwave RF*, 26(6), 194-195.
4. Jackson, R.W. (1986). Considerations in the use of coplanar waveguide for millimeter-wave integrated circuits. *IEEE Transactions on Microwave Theory and Techniques*, 34, 1450-1456.
5. Browne, J. (1989). Coplanar waveguide supports integrated multiplier systems. *Microwaves RF*, 28(3), 137-138.
6. Browne, J. (1990). Coplanar circuits arm limiting amp with 100-dB gain. *Microwave RF*, 29(4), 213-220.
7. Browne, J. (1992). Broadband amp drops through noise floor. *Microwave RF*, 31(2), 141-144.
8. Ahmad, I.; Ho, Y.K.; and Majlis, B.Y. (2006). Fabrication and characterization of a 0.14 μm CMOS device using ATHENA and ATLAS simulators. *International Science Journal of Semiconductor Physics, Quantum Electronics, and Optoelectronics*, 9, 40-46.
9. Gopinath, A. (1982). Losses in coplanar waveguide. *IEEE Transactions on Microwave Theory and Techniques*, MTT-30, 1101-1104.
10. Itoh, T. (1989). Overview of quasi-planar transmission lines. *IEEE Transactions on Microwave Theory and Techniques*, MTT-37, 275-280.
11. Kittara, P.; Yassin, G.; and Withington, S. (2002). Analysis of superconducting coplanar waveguides for SIS mixer circuits. *Proceedings of the 13th International Symposium on Space Terahertz Technology*, Harvard University, US, 571-580.
12. Gupta, K.C.; Garg, R.; Gahl, I.; and Bhartia, P. (1996). *Microstrip Lines and Slotlines* (2nd ed.). Boston: Artech House.

13. Yassin G.; and Withington, S. (1995). Electromagnetic models for superconducting millimetre-wave and submillimetre-wave microstrip transmission line. *Journal of Physics D: Applied Physics*, 28, 1983-1991.
14. Heitkamper, P.; and Heinrich, W. (1991). On the calculation of conductor loss on planar transmission lines assuming zero strip thickness. *IEEE Transactions on Microwave Theory and Techniques*, 39, 586-588.
15. Yeap, K.H.; Tham, C.Y.; Yeong, K.C.; and Lim, E.H. (2010). Full wave analysis of normal and superconducting microstrip transmission lines. *Frequenz Journal of RF Engineering and Telecommunications*, 64, 56-66.
16. Ghione, G. (1993). A CAD-oriented analytical model for the losses of general asymmetric coplanar lines in hybrid and monolithic MICs. *IEEE Transactions on Microwave Theory and Techniques*, 41, 1499-1510.
17. Veyres C.; and Hanna, V.F. (1980). Extension of the application of conformal mapping techniques to coplanar lines with finite dimensions. *International Journal of Electronics*, 48, 47-56.
18. Hilberg, W. (1969). From approximations to exact relations for characteristic impedances. *IEEE Transactions on Microwave Theory and Techniques*, 17, 259-265.
19. Jahnke, E.; Emde, F.; and Losch, F. (1960). *Tables of Higher Functions*. New York: McGraw-Hill.
20. Yeap, K.H.; Tham, C.Y.; Yassin, G.; and Yeong, K.C. (2011). Attenuation in rectangular waveguides with finite conductivity walls. *Radioengineering Journal*, 20(2), 472-478.
21. Kittel, C. (1986). *Introduction to Solid State Physics*. New York: John Wiley and Sons Inc.
22. Bardeen, J.; Cooper, L.N.; and Schrieffer, J.R. (1957). Theory of superconductivity. *Physical Review*, 108(5), 1175-1204.
23. Kautz, R.L. (1978). Picosecond pulses on superconducting striplines. *Journal of Applied Physics*, 49, 308-314.
24. Hasnain, G.; Dienes, A.; and Whinnery, J.R. (1986). Dispersion of picosecond pulses in coplanar transmission lines. *IEEE Transactions on Microwave Theory and Techniques*, 34, 738-741.
25. Vassilev, V.; Belitsky, V.; Risacher, C.; Lapkin, I.; Pavolotsky, A.; and Sundin, E. (2004). Design and characterization of a sideband separating SIS mixer for 85-115 GHz. *Proceedings of 15th International Symposium on Space Terahertz Technology*, Massachusetts, US, 1-8.
26. Vassilev, V.; and Belitsky, V. (2001). Design of sideband separation SIS mixer for 3 mm band. *Proceedings of 12th International Symposium on Space Terahertz Technology*, California, US, 373-382.