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COMPARISON OF PI CONTROLLER PERFORMANCE FOR FIRST ORDER SYSTEMS WITH TIME DELAY

RAMAKOTESWARA RAO ALLA*, J. S. LATHER, G. L. PAHUJA

Electrical Engineering Department, National Institute of Technology Kurukshetra, Kurukshetra, Haryana, India-136119
*Corresponding Author: ramnitkkr@gmail.com

Abstract

Delays appear often in all real world engineering systems. Delay systems have the property that the rate of variation in the system state depends on the previous states also. They are frequently a source of instability and poor system performance. In order to get the required performance from the delay system controller design plays a vital role. Because of the robust nature, easy structure Proportional Integral Derivative (PID) controllers are extensively used in many industrial loops. Parameter tuning of the PID controller is an essential task. Numerous industrial processes, whose transfer function is of first order, can be easily controlled with PI controllers. This paper presents the comparative analysis of an approach based on Lambert W function for PI controller design for first order systems with time delay among Smith predictor (SP) and Zeigler-Nichols (ZN) methods of design. Performance of the considered methods in terms of various performance specifications through simulation results has been illustrated. Results demonstrate that the Lambert W function based PI tuning results in adequate performance compared to other methods with respect to parameters settling time, overshoot, errors, etc.

Keywords: Time delay, Lambert W approach, Smith predictor, Zeigler Nichols, PID Controller.

1. Introduction

Most of the physical dynamical systems exhibit delays either due to inherent nature or because of delayed measurements. Feedback control itself may also be an important source of delay induced by the actuators, sensors and modern digital meters. Time delay systems are infinite dimensional in nature, represented by the delay differential equations and are difficult to control. Presence of delay in the system can cause undesirable system performance and may lead to instability. It is

Nomenclature

K_I	Integral controller gain
K_M	Steady state gain
K_P	Proportional controller gain
Q_0	Unknown Matrix
S_o	Solution Matrix
W_0	Matrix Lambert W function

Greek Symbols

λ	Eigenvalue
ζ	Damping ratio
λ_d	Desired eigenvalue
τ_M	Time constant
ω_n	Natural frequency

Abbreviations

DDE	Delay Differential Equation
PID	Proportional Integral Derivative
IAE	Integral Absolute Error
ISE	Integral Square Error
ITAE	Integral Time Absolute Error
MSE	Mean Square Error
PI	Proportional Integral
SP	Smith Predictor
ZN	Zeigler-Nichols

difficult to design controller and analyse stability for such systems. There exist a number of books and monographs devoted to this field of active research since two decades [1-5]. Because of the infinite number of poles of the characteristic equation of delay system, it is difficult to analyse such systems with classical methods. So, the systems with delay were usually analysed by taking the approximation to the delay term in the past. Approximation methods like Taylor series and Pade's approximation suggested in the literature [6] have drawbacks of limited precision and unsteadiness of the real time system.

Smith predictor is a popular and useful delay time compensator. This approach was the initial proposal to remove the delay part from the feedback loop and therefore avoid the difficulty of infinite dimensional and leads to finite [7]. Later there have been many modifications proposed to the basic smith predictor approach for dead time processes. Hägglund [8] proposed PI controller tuned Smith configuration for first order plus time delay plants. The controller was named as predictive PI controller (PPI). For improving the robustness with the addition of a filter another extended PPI has been proposed by Normey-Rico et al. [9].

Recently an analytic solution approach based on Lambert W function for delay systems was given by Yi et al. [10]. This approach gives the solution of delay differential equation just similar to the solution of an ordinary differential equation. The eigenvalue assignment of dominant eigenvalues from infinite Eigen spectrum with PI control of first order time delayed system has been done using the Lambert W approach by Yi et al. [11]. DC motor control with proportional plus integral and proportional plus velocity control using Lambert W approach was given by Yi et al.

[12] and it is extended with proportional plus integral plus velocity control, lead compensator design using the Lambert W function for rotary motions of SRV 02 plant for regulating the speed of a DC motor in the presence of time delays by Ana et al. [13]. Shinozaki et al. [14] derived stability conditions assuring robustness using the Lambert W function for linear time-delay systems. In case of uncertainties in the coefficients of the quasi-polynomial are set in suitable regions, then it is proved in [14] that extreme point results can be undertaken. Whereas in [15], with the help of eigenvalue assignment an observer based state feedback controller using Lambert W function approach was used to address this complexity by Sun Yi et al. and robust stability analysis of neural networks considering time-delays and parametric uncertainty have been addressed in [16]. For stability analysis, the dominant characteristic roots have been obtained using Lambert W function based approach. A review on this approach has been given in [17] and also cited different applications of its use.

Majority of modern industrial systems are controlled via proportional integral derivative controllers. First order time delay unstable systems analysis and associated P and PI control with subsequent stability conditions have been derived by Venkatashankar and Chidambaram [18]. Many methods exist in literature to tune the parameters of the PID controller gains, i.e. proportional, integral and derivative gains [18-20]. One of the most important and basic tuning method is Ziegler Nichols method [19]. In this paper PI controller design using ZN, Smith predictor and Lambert W function based methods have been considered for first order delay systems. Two examples were considered for illustration of results. Performance in terms of time response specifications like maximum overshoot, rise time, settling time, peak time and errors, integral absolute error (IAE), mean square error (MSE), integral square error (ISE), integral time absolute error (IATE) have been considered and compared.

The remaining paper is organized as: section 2 presents ZN method of PI controller tuning and smith predictor approach for dead time systems, Section 3 describes the Lambert W analysis of time delay systems and PI controller design using Lambert W approach. Section 4 presents the simulation results and discussions followed by concluding section.

2. Proportional Integral controller Design

In this section PI controller design using Ziegler Nichols method and Smith predictor configured PI controller design is presented.

2.1. Ziegler Nichols method

In process control for tuning PID parameters, Ziegler Nichols method is one of the popular methods. Two methods of tuning, proposed by Ziegler and Nichols, are step response method and frequency response method. Step response method is an experimental tuning method for open loop plants. From the step response of the plant two parameters are calculated and then by using the measured parameters and with the help of existing formulae the controller gains can be tuned [19].

Frequency response method is a closed loop tuning method. By keeping the controller integral and derivative gains zero and by changing the proportional gain the response of the process is observed. This process is continued till the response results in periodic oscillations. After achieving the required periodic oscillations the value of corresponding gain and time period are termed as ultimate gain and ultimate time period. Based on the ultimate gain and ultimate time period, Ziegler Nichols frequency method [20] has simple formulae to calculate the remaining parameters for different controller types PI, PID, etc. as required by the decision maker. The controller structure with time delay process is as shown in Fig. 1.

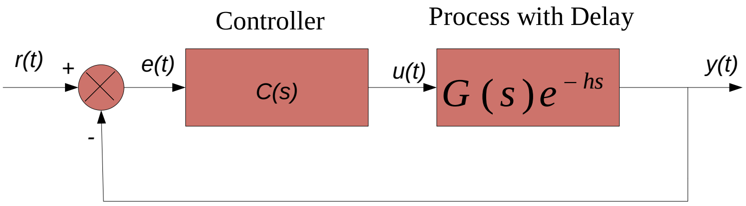

Fig. 1. Controller structure with time delay.

2.2. Smith predictor Design

Smith-predictor controller is a useful technique for dead time compensation. By taking the time delay term outside the feedback loop, Smith predictor results in a delayed response of a delay free system. With the help of the delay free plant in smith configuration the controller for the delay plant can be designed. The smith predictor configuration with the controller D(s) is as shown in Fig. 2. The controller is to be designed in such a way that, if the time delay h in the process is absent, then feedback around the controller ensures that the system with the time delay will give satisfactory performance. For the PI controller, the gains can be obtained as given below,

Consider a first order plant with time delay as

$$G(s) = G_P(s)e^{-sh} = \frac{K_M}{\tau_M s + 1} e^{-sh} \tag{1}$$

where τ_M is the time constant, h is the time delay, and K_M represents the steady-state gain.

Consider the PI controller

$$D(s) = K_P + \frac{K_I}{s} \tag{2}$$

With the purpose of getting required time domain specifications, desired natural frequency ω_n and desired damping ratio ζ are chosen, and the desired eigenvalues can be calculated as:

$$\lambda_d = -\omega_n \zeta \pm \omega_n \sqrt{\zeta^2 - 1} i = -\sigma \pm \omega_d i \tag{3}$$

Assuming no time delay, the controller gains K_P and K_I can be chosen as

$$K_P = \frac{2\zeta\omega_n\tau_M - 1}{K_M} \quad \text{and} \quad K_I = \frac{\omega_n^2\tau_M}{K_M}$$

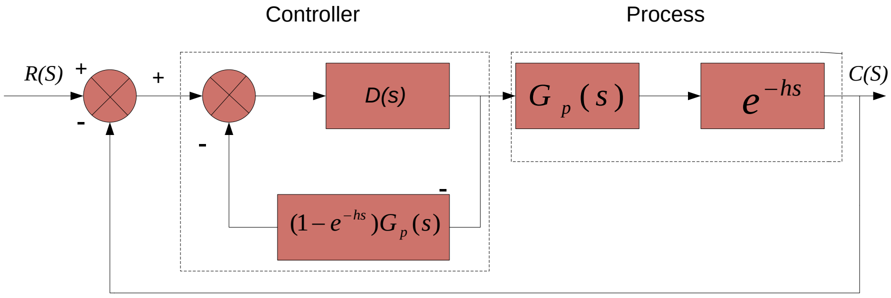

Fig. 2. Smith predictor configuration with controller.

3. Lambert W function approach of PI controller design

PI controller design using Lambert W function based approach is given in this section. Because of infinite dimensionality of delay systems, classical methods available for delay-free systems are not directly applicable to delay systems. Numerical, graphical, or approximate approaches have been used for delay systems in the literature. However, these methods have limitations in terms of accuracy and/or robustness leading to unsteadiness of the real system and induce non-minimum phase leads towards high-gain problems. Thus, to overcome these problems an analytic approach for the control and stability of systems with delay was developed termed as Lambert W function based approach. A complete solution in terms of system parameters can be obtained for delay differential equations using the concept of the Lambert W function and is analogous to the state transition matrix approach in case of linear ordinary differential equations (ODE).

The advantages of this approach are it can algebraically solve the characteristic equation of scalar linear time-delay systems, helps to study the qualitative features of the characteristic roots of the system and always supplies an exact analysis being free of conservativeness. The finite number of rightmost or dominant eigenvalues can be obtained and stability can be determined using this dominant subset without the need for considering the location of other infinite eigenvalues. The exponential terms in the characteristic equation due to time-delays are not approximated, (e.g., Pade approximation). Hence, the obtained result using the Lambert W approach is more precise, accurate and robust.

The main benefit of this solution approach is that the derived solution has an analytical form expressed in terms of the system parameters. Hence, the parameters affect in the solution can be determined and, also, how each parameter affects every eigenvalue. By choosing appropriate proportional and integral gains the right most eigenvalues of the system can be assigned to required positions using Lambert W approach.

Considering an open loop transfer function of the system with PI controller as

$$G(s) = \frac{K_M}{\tau_M s + 1} e^{-sh} \left(K_P + \frac{K_I}{s} \right) = \frac{y}{e} \tag{4}$$

where $e = -y + r$. The time domain representation of the overall closed loop system becomes.

$$\ddot{y} = -\frac{1}{\tau_M}\dot{y} - \frac{K_M K_P}{\tau_M}\dot{y}(t-h) - \frac{K_M K_I}{\tau_M} y(t-h) + K_M K_P \dot{r}(t-h) + K_M K_I r(t-h) \tag{5}$$

To convert the same into state space form, assume $x_1 \equiv y, x_2 \equiv \dot{y}$, the above equation can be rewritten as $\dot{x}_1 \equiv x_2$

$$\dot{x}_2 = -\frac{1}{\tau_M} x_2 - \frac{K_M K_P}{\tau_M} x_2(t-h) - \frac{K_M K_I}{\tau_M} x_1(t-h) + K_M K_P \dot{r}(t-h) + K_M K_I r(t-h) \tag{6}$$

In state space form, it can be expressed as follows

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & \frac{-1}{\tau_M} \end{bmatrix}}_{A} x(t) - \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{K_M K_I}{\tau_M} & \frac{K_M K_P}{\tau_M} \end{bmatrix}}_{A_d} x(t-h) \tag{7}$$

Because of the time delay, the system becomes infinite dimensional leads to an infinite number of eigenvalues. This creates a problem in controlling systems with time delay. It is necessary to place right most dominant eigenvalues to control such type of systems. The approach based on Lambert W function is the most appropriate method for eigenvalue assignment through controller gains. Lambert solution matrix S_0 and its eigenvalues can be calculated using the system matrices A and A_d from

$$S_0 = \frac{1}{h} W_0(A_d h Q_0) + A \tag{8}$$

where W_0 represents the matrix Lambert W function and Q_0 is unknown matrix introduced and can be calculated by solving

$$W_0(A_d h Q_0) e^{W_0(A_d h Q_0) + Ah} = A_d h \tag{9}$$

To obtain the matrix Q_0, Eq. (9) can be solved numerically for the principal branch with the help of nonlinear solvers (i.e., fsolve in MATLAB) and by replacing the matrix Q_0 into (8), S_0 and its eigenvalues can be obtained. From the eigenvalues the stability can be analysed and performance can be improved by changing the controller gains. For detailed explanations of the Lambert W function method of eigenvalue assignment the reader can refer to [21].

4. Results and Discussions

In this section, two examples of a first order system with delay have been considered to illustrate the performance of the PI controller design using Lambert W function based approach, Smith predictor (SP) method of compensating delay and Zeigler-Nichols method of tuning.

Example 1: Transfer function of the system considered is

$$G(s) = \frac{e^{-0.2s}}{s+1} \tag{10}$$

By adding the PI controller transfer function, and converting the system into state space form can be represented in delay differential equation as

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}}_{A} x(t) - \underbrace{\begin{bmatrix} 0 & 0 \\ K_I & K_P \end{bmatrix}}_{A_d} x(t-0.2) \tag{11}$$

The controller gains using Smith predictor method and Lambert W function based methods are given in Table 1.

Table 1. PI controller gains obtained by Lambert W function approach and Smith predictor.

ω_n	ζ	Dominant Pole	Smith Predictor		Lambert W Function	
			K_I	K_P	K_I	K_P
1.02	**0.26**	$-0.2724 \pm 0.989i$	1.0404	-0.4696	1.1214	-0.2012
1.05	**0.27**	$-0.2866 \pm 1.017i$	1.1025	-0.4330	1.1822	0.1623
1.26	**0.36**	$-0.4603 \pm 1.1738i$	1.5876	-0.0928	1.5652	0.2235
1.55	**0.46**	$-0.7174 \pm 1.3815i$	2.4025	0.4260	2.1368	0.7236

For example, the response for $\omega_n = 1.55$ and $\zeta = 0.46$, corresponding eigenvalues $-0.7174 \pm 1.3815i$ the proportional and integral gains are chosen to see the response of the system. For selected ω_n one can assign the desired eigenvalues to desired locations by tuning the controller gains using Lambert W function. The gains for ZN method calculated are $K_P = 4.5$ $K_I = 0.666$. Figure 3 shows the simulation result of all the three methods.

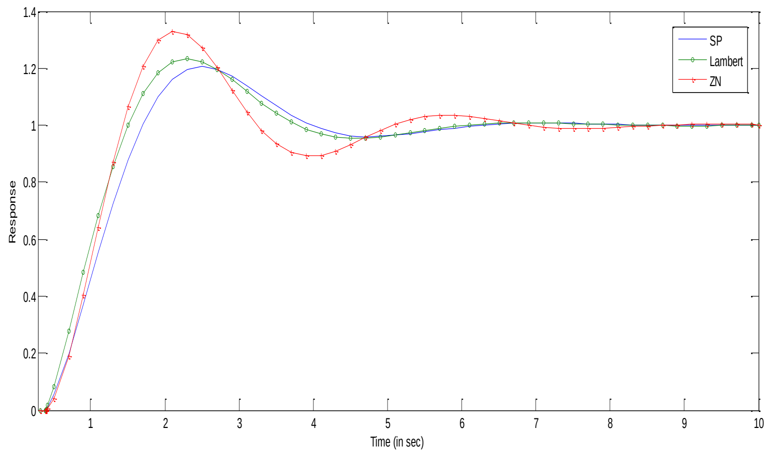

Fig. 3. Response of Smith predictor, Lambert W function based approach and Zeigler-Nichols method.

The comparison among these tuning methods in terms of various performance measures like rise time, settling time, peak time and overshoot has been given in Table 2. Feedback control systems are essentially used to decrease the error $e(t)$ among the measured output and the reference input to obtain quick response. For getting error free required performance or to optimize it, the controller parameters are to be adjusted. The errors integral square error (ISE), integral time absolute error (ITAE), integral absolute error (IAE) and mean square error (MSE) have been calculated and tabulated in Table 3.

Table 2. Transient response specifications.

Transient Response Characteristics	Smith Predictor (SP)	Ziegler Nichols (ZN)	Lambert W Function(LWF)
Rise Time (Sec)	0.95	0.73	1.06
Settling Time (Sec)	5.19	5.98	3.67
Maximum Overshoot (%)	20.50	33.2	11.9
Peak Time (Sec)	2.11	1.73	2.26

Table 3. Results of different error indices.

Method	IAE	ITAE	ISE	MSE
Smith Predictor	1.198	1.701	0.697	0.505
Zeigler Nichols	1.126	1.913	0.529	0.438
Lambert W Function	1.125	1.646	0.616	0.443

From the results, it can be observed that various performance specifications have been improved greatly with Lambert W approach in terms of settling time, overshoot. It can be clearly seen that the overshoot is very less in case of Lambert W function method compared to other methods. From the errors point of view also the Lambert W function results in better performance in terms of IAE and IATE. The ZN method gives superior results with respect to errors ISE and MSE.

Example 2: Transfer function of the system considered is

$$G(s) = \frac{e^{-0.21s}}{s} \tag{12}$$

The controller gains for selected ω_n and ζ, tuned by Smith predictor method and Lambert W function based methods are given in Table 4.

For example, the response in the case, $\omega_n = 1.53$ and $\zeta = 0.74$, corresponding eigenvalues $-1.1364 \pm 1.0347i$, the proportional and integral gains are chosen to see the response of the system. Figure 4 depicts the simulation result of the response of the considered system.

The comparison among different tuning methods in terms of various performance measures like rise time, settling time, peak time and overshoot has been given in Table 5. Also errors integral square error (ISE), integral time absolute error (ITAE), integral absolute error (IAE) and mean square error (MSE) have been calculated and tabulated in Table 6.

Table 4. PI controller gains obtained by Lambert W function approach and Smith Predictor

ω_n	ζ	Dominant Pole	Smith Predictor		Lambert W Function	
			K_I	K_P	K_I	K_P
1.21	0.52	$-0.6355 \pm 1.0383i$	1.4641	1.2584	1.2132	1.1123
1.32	0.44	$-0.5928 \pm 1.1796i$	1.7424	1.1616	1.3233	1.2069
1.82	0.43	-0.7923 ± 1.6435	3.3124	1.5652	2.2467	1.6275
1.53	0.74	$-1.1364 \pm 1.0347i$	2.3409	2.2644	1.7372	1.4170

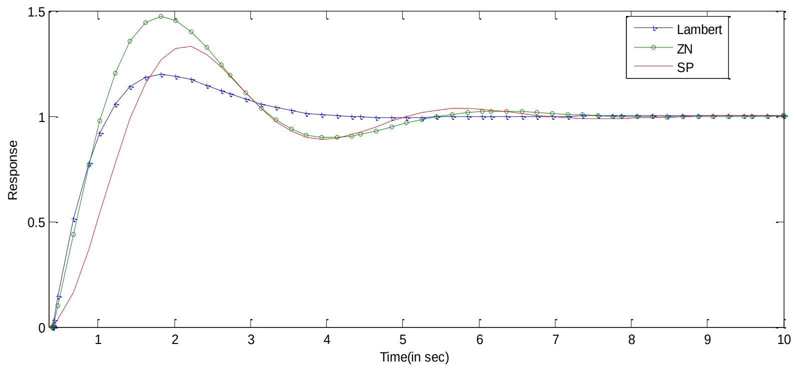

Fig. 4. Response of Smith predictor, Lambert W function based approach and Zeigler-Nichols method.

Table 5. Transient Response Specifications.

Transient Response Characteristics	Smith Predictor (SP)	Ziegler Nichols (ZN)	Lambert W Function(LWF)
Rise Time (Sec)	0.70	0.72	0.54
Settling Time (Sec)	5.58	5.83	3.23
Maximum Overshoot (%)	27.8	32.6	19.7
Peak Time (Sec)	1.8	1.69	1.46

Table 6. Results of different error indices.

Method	IAE	ITAE	ISE	MSE
Smith Predictor	1.326	2.353	0.658	0.677
Zeigler Nichols	1.124	1.873	0.498	0.468
Lambert W Function	0.806	0.942	0.430	0.251

It can be observed from the results of the time response specifications that the performance specifications rise time, settling time and overshoot have been enhanced significantly with Lambert W approach. From the errors point of view also the errors, integral absolute error, integral time absolute error, integral square error and mean square error are less for Lambert W method compared to others. From the overall performance, it can be observed that the Lambert W function results in adequate performance compared to other methods.

5. Conclusion

PI controllers are widely used for the performance improvement of first order systems with time delay. In this paper, three methods of tuning of the PI controller including an approach which is based on Lambert W function, Smith predictor and ZN method were described. The gains for the controller were calculated. The gains in Lambert W based approach were chosen for the required natural frequency and damping ratio. Disparate to prediction based approaches, the Lambert W based method is independent on pole-zero cancellation. So, the controller obtained in such way can effectively improve the performance and easily stabilize the delay system. Simulation results show the performance of three methods considered in terms of time response specifications. From the results, it can be concluded that the Lambert W function based approach results give enhanced performance compared to Smith predictor and ZN methods. The advantage of the Lambert W function based method is it is not a prediction or approximation based approach.

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