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APPLICATION OF CURVE FITTING TECHNIQUES USING SOFT COMPUTING TECHNIQUES (GA, GSA, POS AND NR) IN MULTILEVEL INVERTERS FOR HARMONIC MINIMIZATION

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Abstract

With the advent of low cost and powerful computers, the soft computing (SC) approach seems to be a better approach and well suited to handle the complexity of the HEPWM problem. This paper presents an approach to the application of soft computing techniques for selective harmonic elimination from output voltage of Single phase multilevel inverter by proper selection of switching angles. Newton Raphson method (NR), Genetic algorithm (GA), Particle swarm optimization (PSO) & Gravitational search algorithm (GSA) are prominent soft computing techniques which are implemented to obtain the minimum total harmonic distortion (THD) of multilevel inverter. To obtained experiment results a new modulation method is used to change these angles into PWM switching. Another approach in the determination of switching angles for Selective Harmonic Elimination technique of reduction of harmonics is the use of Curve Fitting technique has been reported. Some new curve fitting techniques mainly piecewise curve fitting technique using spline has been worked out and analyzed. Comparison of performances of soft computing and curve fitting techniques is done. Simulation and experimental results prove the powerful merits of the proposed techniques and improvements done on previous works.

Keywords: Multilevel inverters, Selective harmonic elimination, Total harmonic distortion, PWM switching, curve fitting, SPLINE.

1. Introduction

Nowadays, multilevel inverters have become very popular for their use in high-voltage and high-power applications [1], because of many advantages like low

Nomenclatures

G	Gravitational constant
m	Modulation index
M_{ii}	Inertial mass
$M\alpha i$	Active Gravitational Mass
Mpi	passive Gravitational Mass
X_i^d	Position

Greek Symbols

α, θ	Switching angle, deg.

Abbreviations

CHB	Cascaded H-bridge
GA	Genetic algorithm
GSA	Gravitational Search Algorithm
HEPWM	Harmonic Elimination Pulse Width Modulation
NR	Newton Raphson
PSO	Particle Swarm Optimization
SC	Soft Computing
THD	Total Harmonic Distortion

switching stress, low total harmonic distortion (THD). In multilevel inverters, the required output voltage is achieved by suitable combination of multiple low dc voltage sources used at the input side. One of the main challenges in multilevel inverters is the control of switching angles to eliminate low order harmonics from output voltage. The selective harmonic elimination techniques perform calculation of feasible switching angles to eliminate the dominant low-order line-to-line voltage harmonics in a cascaded multi-cell multilevel power converter. Higher order harmonics can easily be removed by filters and in some applications filters cannot be used. So SHE is very useful for reducing THD [2].

In multilevel inverters, waveform analysis is performed using Fourier theory. An iterative procedure, mostly Newton-Raphson method is used to obtain the solution of derived non-linear transcendental equations [3-4]. This is a derivative dependent method and there is a probability that it ends in local optima; further, to guarantee convergence, a good choice of the initial values is required. The transcendental equations which are used in SHE method are transformed into polynomial equations where with the help of some relevant theories determination of the switching angles is performed for the elimination of specific harmonics [5]. However, there is an increase in the degrees of the polynomials in these equations, as the amount of targeted harmonics, and therefore, the number of switching angles increase, which further increases the difficulty in computation. Recently a combination of interval search technique and Newton's method has been described to find all the solutions to non-linear transcendental equations [6]. Even though the solutions for both single and three phase wave forms is given by the method, but it has not been tested with higher harmonics where the method appears to be much complicated, and therefore, much computational expense is expected. Over the years, numerous works on Soft-Computing (SC) for HEPWM are published [7-9]. But there is deficiency in comparative analysis and work on these techniques. The most important advantage of SC techniques is its ability to handle non-linear mathematical problems in a non-conventional way. Adding to that, the availability of powerful personal computers has made complex computation feasible. One of the popular SC based optimization technique is genetic algorithm [10-12]. GA is a type of artificial intelligence approaches, and it origins from optimization of problems. GA is used to replace numerical algorithms to overcome their drawbacks. GA is used in order to reduce the computational burden that is associated with the solution of the non-linear transcendental equations of the SHE method for reducing low order harmonics.

Particle Swarm Optimization (PSO) belongs to a relatively new family of algorithms and it may be used to find optimal solutions to numerical and qualitative problems. Inspired by social behaviour of birds flocking or fish schooling, this method was introduced by Russell Eberhart and James Kennedy in 1995. A very effective procedure for PSO implementation is presented in for MLIs [13-15].

Recently introduced optimization method is gravitational search algorithm GSA (2009) is work on the principle of Newton's law of motion is also incorporate for comparison [16, 17]. In this paper, a comparison has been made between various soft computing techniques (GA, PSO and GSA) and conventional NR method for single phase multilevel inverter. Curve fitting has been used for determination of approximate mathematical equation representing the output voltage of the stepped waveform in a Cascaded Multilevel Inverter. Another approach in the determination of switching angles for SHE technique to reduction of harmonics is the use of Curve Fitting technique has been reported by Azli and Yatim [18]. GA with least square curve fitting is studied in detail in [19], in which manipulation of binary coded strings is done by a GA to produce near-optimal solutions to least squares curve-fitting problems after viewing only a small portion of the search space. Another method of curve fitting technique in calculation of switching angles is given by authors [20]. The approach of curve fitting is applied on soft computing algorithms namely GA, PSO, GSA and conventional NR and are compared for obtaining THD of output voltage of cascaded 7-level multilevel inverter. Curve fitting tool is being utilized to obtain best fitting expression. Expressions in form of polynomial in which SPLINE method is tried out to obtain best fit for data with minimum THD. MATLAB/SIMULINK is used for simulation purpose which is then verified by experimental results.

The paper is organized as follows: Brief discussion on the approach used for calculating optimum angles using NR, GA, PSO and GSA methods in section 2. In section 3 and 4 techniques for switching IGBTS switches of inverter, hybrid modulation and polynomial curve fitting to reduce %THD with relevance to this paper are stated. Section 5 gives the simulation results of soft computing methods used in the paper. Hardware results without curve fit and with curve fit are shown and discussed in Sections 6 and 7 respectively and the last section is conclusion.

2. Optimization Techniques in Multilevel Inverter

Space vector modulation and selective harmonic elimination are normally used for low switching frequency modulation methods [21]. Whereas sinusoidal pulse width modulation (SPWM) technique has disadvantage that it cannot completely eliminate the low order harmonics results in switching loss and high filter requirement is needed. Space Vector Modulation technique cannot be applied for unbalanced DC voltages. SHE- PWM technique uses many mathematical methods to eliminate specific harmonics such as 5th, 7th, 11th, and 13th harmonics. SHE techniques are preferred, as it eliminates the pre-selected lower order harmonics which are more dominant into the nature while satisfying the fundamental component of output voltage. The “fitness function” is considered to be the most important term in SHE based optimization method discussed in this paper. The THD is a measure of quantity of distortion of the signal. Here the total harmonic distortion (THD) is considered as fitness function *f(θ)* to be minimized by evaluating optimal switching angles defined for a quarter range.

2.1. Newton Raphson method (NR)

The conventional SHE method has the merit of eliminating the required harmonic component but it has problems in solving nonlinear transcendental equations. NR is multi-dimensional, directional and one of the fast enough iterative numerical methods that finds solution of SHE equations. The solution is based on good initial guess for which solution exists for given values of modulation index. Here in this paper NR technique is developed in MATLAB in such a manner that it provides all possible solution of switching angles by taking a random guess for a range of modulation index. The best possible angle that yields the minimum THD is applied to get simulation and hardware results for given modulation index.

For solving multiple equations with multiple variables, matrix *f* comprising of expressions of all nonlinear equations and a matrix *f* 'comprising of derivatives of expressions of all nonlinear equations are defined, subsequently the iterations for each switching angle (variable here) is performed by using equation:

```
alpha1=alpha1+dalpha
until error is within acceptable limits
if dalpha>-1e-15 & dalpha<1e-15
break;
```

2.2. Genetic algorithm (GA)

In NR the switching angles are obtained by means of an off-line calculation to minimize the harmonics for each modulation index, which leads to increased use of look-up tables. Genetic algorithm is a computational model that solves optimization problems by imitating genetic processes and the theory of evolution by using genetic operators like reproduction, crossover, mutation etc. Amounts of applications have benefited from the utilization of genetic algorithm [10]. Genetic algorithm is still popular for PWM-SHE technique. The algorithm gives the steps for calculating optimized values of switching angles to minimize THD (fitness function) of the output voltage of cascaded MLI. The THD can be calculated by using Eq. (1).

$$\text{Minimize, } f(\theta)=\frac{\sqrt{\sum_{n=3,5,7}(b_n)^2}}{b_1}*100 \tag{1}$$

$$\text{Constraints} \quad 0\leq \theta_1 \leq \theta_2 \dots . \leq \theta_s \leq \frac{\pi}{2}$$

Step1. Randomly initialize switching angles θ_1, θ_2, θ_3

Step2. Compute generation and population index.

Step3. If $\theta <= \pi/2$ continue else go to step2.

Step4. Compute fitness function f (θ_1, θ_2, θ_3)

Optimal switching angles are obtained by minimizing fitness function subjected to $0 < \theta_1 < \theta_2 < \theta_3 < \pi/2$.

Step5. Pick the best individuals correspond to minimized fitness function.

Step6. Create offspring using crossover and mutation operators.

Step7. Obtain final output if solution converges else go to step2 and repeat the process.

Through this algorithm optimized values of switching angles θ_1, θ_2, θ_3 for 7-level MLI can be obtained.

2.3. Particle swarm optimization (PSO)

Unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. Another reason that PSO is attractive is that there are few parameters to adjust. Algorithm for obtaining switching angles of multi-level inverter is as follows:

Step1. Initialize the position and velocity of each particle.

Step2. Evaluate the fitness value of each particle.

Step3. Set global best fitness to min (local best fitness).

Step4. Update fitness and velocity of each particle &evaluate the fitness value of each particle.

Step5. If current fitness< local minimum fitness, proceed else go to step 4.

Step6. If local best fitness = current fitness, proceed else go to step 4.

Step7. If criterion met, proceed else go to step 5.

In case of derived nonlinear equations for SHE, an objective function comprising all the expressions is obtained and it gives best result with population 20 and generation 50. Upper and lower bound for the switching angles are obtained define a function named PSO to perform optimization. Optimization time increases with increase in number of iterations and population.

2.4. Gravitational search algorithm (GSA)

In GSA, an agent is characterized by 4 parameters which are to be calculated and updated until the stopping criterion is reached. These four parameters are-position, Inertial Mass, Active Gravitational Mass and Passive Mass [16, 17]. GSA algorithm for optimizing switching angles is formulated as below.

Step1. Initialize population size.

Step2. For each agent calculate fitness function.

Step3. Update parameters such as (gravitational constant, best solution)

Step4. Calculate mass and acceleration of agents.

Step5. Update velocity and position of each agent.

Step6. If criterion met, proceed, else go to step 2.

Step7. Return best solution, select agent corresponds.

Step8. Stop.

The value of gravitational constant (G = 20).

3. Switching Technique

After applying soft computing techniques next step is to change these switching angles ($\alpha 1$, $\alpha 2$, $\alpha 3$) into time domain for PWM switching to get experimental results. Figure 1 shows the flow chart for converting optimized angles into PWM switching.

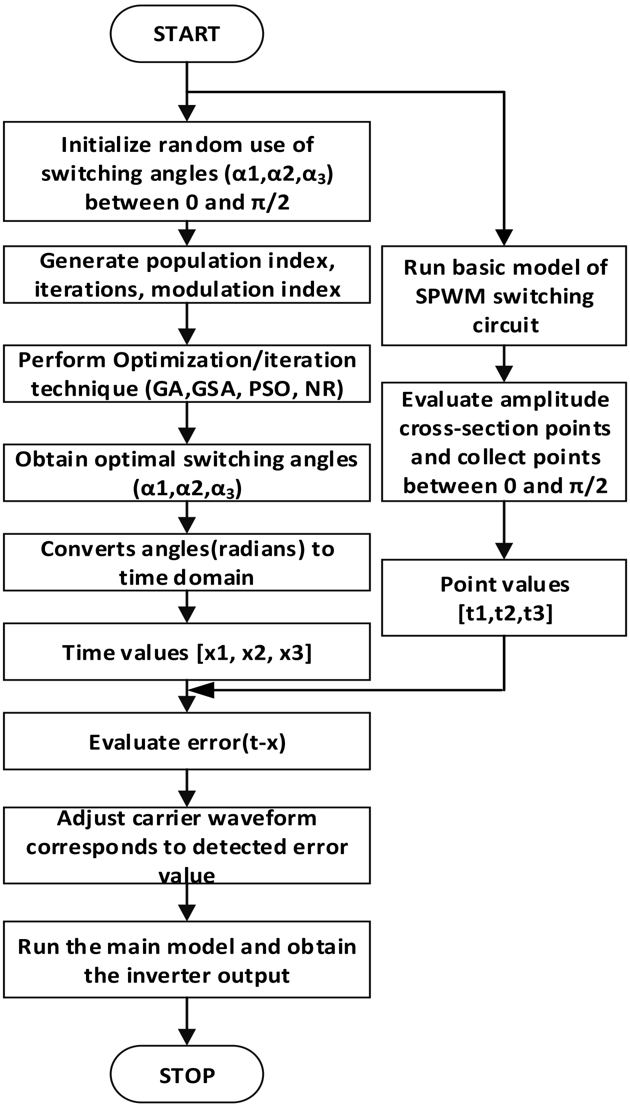


Fig. 1. Flow chart for PWM switching calculation using optimized values of switching angles.

4. Hybrid Modulation using Curve Fitting Technique

The flow chart (Fig. 1) shows the inverter output in improved stepped wave form for use in PWM switching calculations. This stepped wave form is arrived at, by use of a hybrid modulation technique. The Hybrid Modulation technique is an amalgamation of soft computing technique to find optimized switching angles and carrier based SPWM modulation method to get the stepped wave form. This Hybrid Modulation outputs stepped waveform of lower THD, which is further fitted into defined sine curve using "cftool" of curve fitting technique. To implement this modulation method the values of optimized switching angles obtained by using SC technique are first converted into time domain. The intersection points (t) of stepped waveform obtained by SPWM method for quarter cycle is evaluated and the carrier waveform adjusted up or down along Y axis according the calculated error (t-x). Finally, the interface program is used to run the Simulink model with appropriate calculation.The improved output waveform is then fitted into defined sine curve by adopting a suitable "curve fit" technique.

Curve fitting technique has been used to determine the approximate mathematical expression representing the output voltage of the stepped waveform in a Cascaded Multilevel Inverter. In curve fitting method the stepped waveform is adjusted into sine wave by using different curve fitting methods available in MATLAB. Hence the value of THD is reduced to a third of that obtained by SPWM method as shown in Figs. 9 to 12.

Curve Fitting signifies the process by which the trend in the data is captured by assigning a single function for entire range of the data. The straight line is represented by the Eq. (2).

$$y = f(x) = ax + b \tag{2}$$

If the order of the equation is increased to a second degree polynomial, the following results shown in Eq. (3).

$$y = ax^2 + bx + c \tag{3}$$

This will exactly fit a simple curve to three points.

Here, the values of the coefficients 'a' and 'b' need to be determined, such that the equations 'fits' the data pretty well. Interpolation and curve fitting techniques are popular methods to fit the graph in given format [18].The curve fitting methods are following types: (i) Polynomial curve fitting (ii) Sum of sine curve fitting. Here "SPLINE "method of polynomial curve fitting technique is applied to obtain reduced THD.

4.1. Problem formulation

Output of the multilevel inverter obtained using soft computing methods are then applied to polynomial curve fitting technique.

4.2. Newton Raphson based polynomial curve fitting technique

The optimized switching angles which are so obtained by using NR method are then fed to the multilevel inverter Simulink model.

The corresponding output voltage waveform is then obtained. This is split into sub waves per cycle. These waveforms are then fitted into quadratic polynomial equations using 'cftool' of MATLAB. The equations are fitted back and the corresponding THD is calculated. Figure 2 shows flow chart with the steps of fitting the output waveform into required format.

The optimized values of angles obtained by using the soft computing methods (GA, PSO and GSA) are fitted into curve fitting technique as per the flow chart shown in Fig. 2. This flow chart applies to all soft computing techniques.

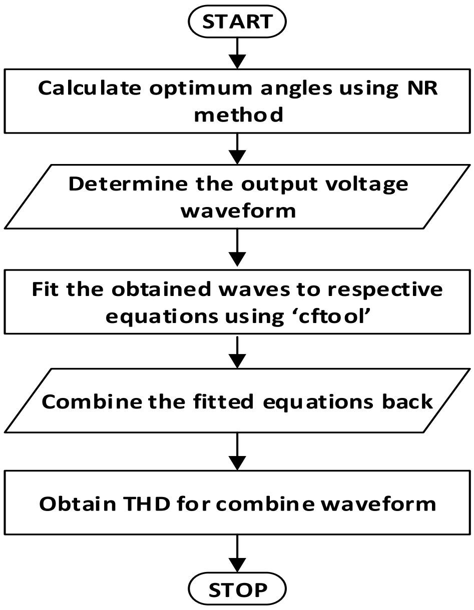


Fig. 2. Flowchart-NR based piecewise curve fitting.

5. Simulink Results and Switching Process

IGBTs receive gating signals from switching circuitry. Switching angles are obtained using soft computing techniques and then interfaced with model. By changing load, appropriate data set can be obtained. To perform simulation a resistive load of 100 Ω is considered. Three numbers of DC sources of 10V each are used in 7 level cascaded inverter.

In Fig. 3 values of "z" are used for interfacing of the model with programs of soft computing techniques for adjustment of switching angles. The reference signals shifts up or down as per the value of z.

Table 1 gives the information about the values of swithching angles used in aforementioned soft computing techniques and corresponding THD values that can be seen in Fig. 4(a), (b), (c) and (d) respectivly. It is found that through NR technique, a minimum THD of 18.06% is obtained, GA gives 15.1% and PSO gives 13.07% and GSA gives 17.07% at modulation index m = 0.85. Among these optimization SC methods PSO gives best result. Table 2 gives the comparison of the soft computing methods on different used parameters. Figure 5(a) gives the graphical approch of diffent SC techniques at different switching insant of MLI. Figure 5(b) shows the approch of curve fitting over stepped waveform obtained by GSA(applicable for all mentioned methods).

Table 3 shows the comparison of THD obtained by the Piecewise curve fitting with NR, GSA, PSO and GA for m = 0.8 and m = 0.85.

FFT waveform of given curve fitting methods is shwon in Fig. 6. It is concluded from above study that the lower value of THD is obtained with m = 0.85, lowest value being 3.44% for PSO.

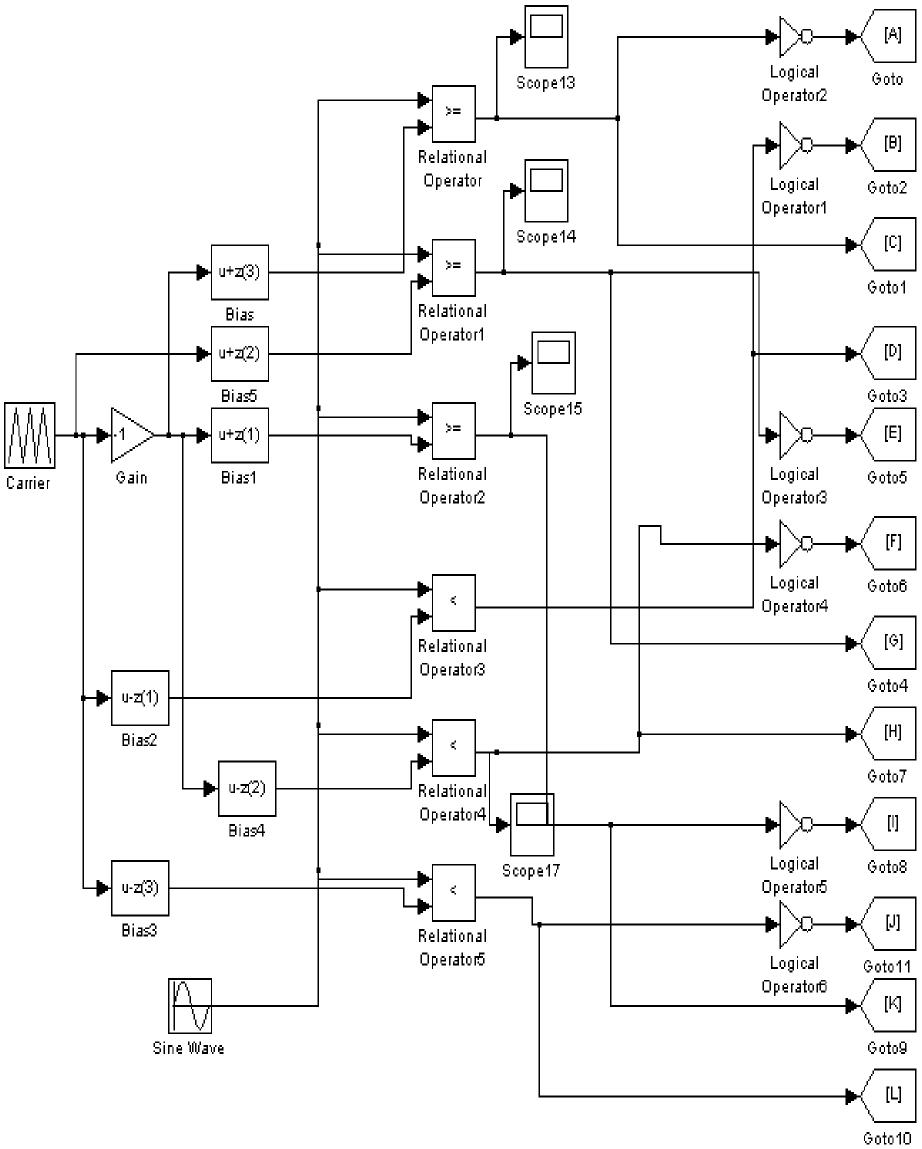


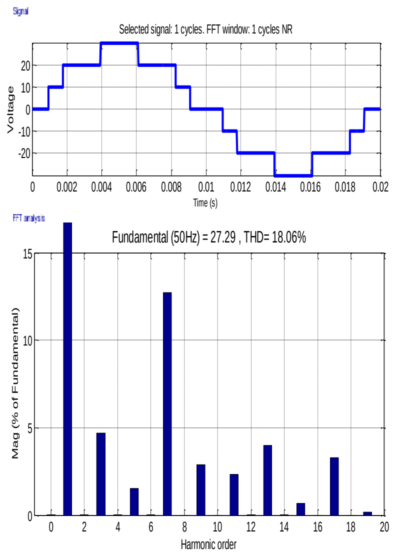
Fig. 3. Interfacing of the model with programs of soft computing techniques.

Table 1. Comparison of NR, GA, GSA and PSO at modulation index *m* = 0.85.

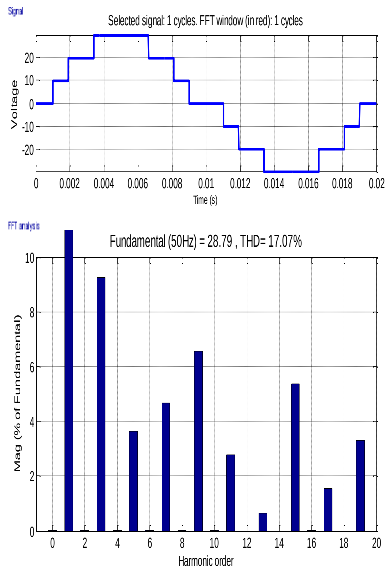
Methods used	M	α1	α2	α3	THD (%)
NR	0.85	0.2944	0.551	1.2307	18.06
GSA	0.85	0.3142	0.5969	1.0681	17.07
PSO	0.85	0.2112	0.4921	1.0223	13.07
GA	0.85	0.25	0.570	1.003	15.1

Table 2. Comparison of NR, GA, PSO and GSA for different parameters using modified PWM method.

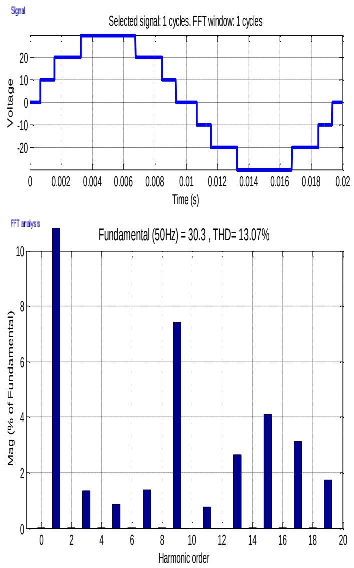
Parameters	NR	GA	PSO	GSA
Number of variables	3	3	3	3
Number of iterations	50	50	50	50
Population size	-	20	20	20
Modulation index	0.85	.85	0.85	0.85
Average Computational time	4.727880	.003067	2.91693	2.19139
Complexity	high	Moderate	moderate	moderate
Best THD(%) value	18.06	15.1	13.07	17.07
Search space Dimensions	Uni-dimensional	Uni-dimensional	Uni-dimensional	Uni-dimensional
Flexibility	low	Low	good	good



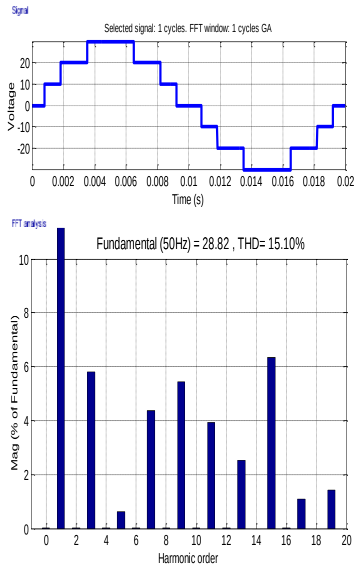
(a) FFT analysis of 7 level Inverter output voltage using NR



(b) FFT analysis of 7 level Inverter output voltage using GSA.



(c) FFT analysis of 7 level inverter output volage using PSO.



(d) FFT analysis of 7 level inverter output voltage using GA.

Fig. 4. FFT results by using SC techniques.

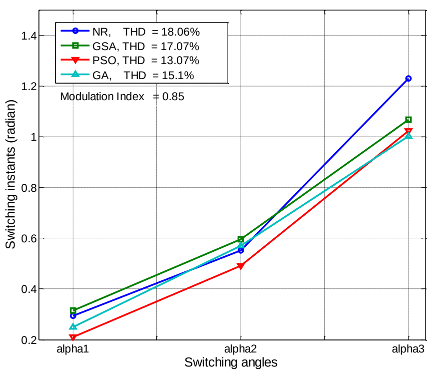


Fig. 5(a) switching inatants for SC techniques.

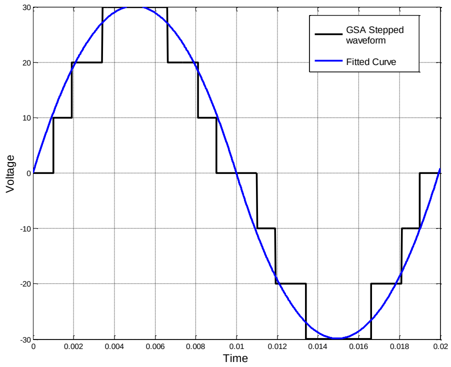
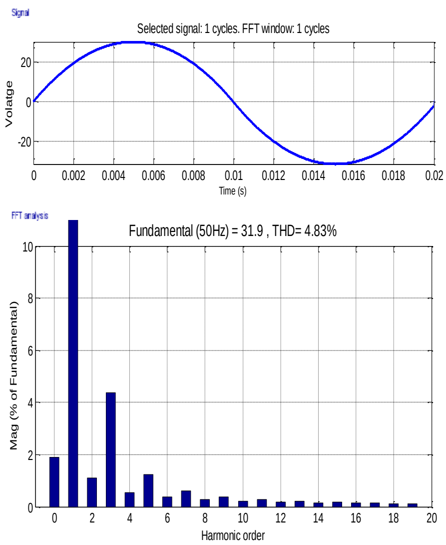


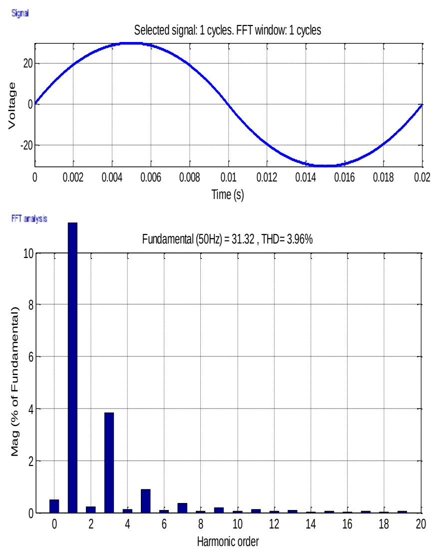
Fig. 5(b) Piecewise Curve fitted Voltage Waveform for GSA.

Table 3. Comparison of THD with and without curve fitting for different soft computing methods.

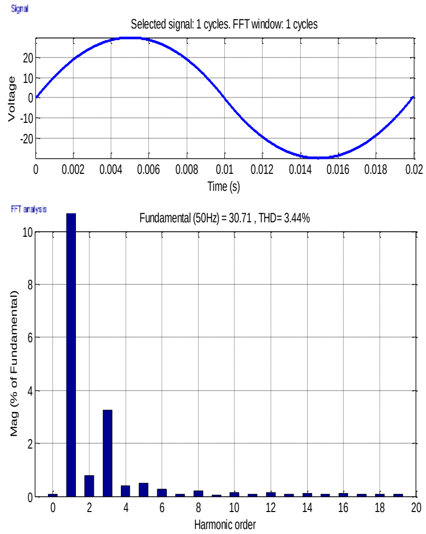
m	**With Curve Fitting(%THD)**				**Without Curve Fitting(%THD)**			
	NR	GSA	PSO	GA	NR	GSA	PSO	GA
***m* = 0.8**	4.99	4.87	4.11	3.99	18.10	18.48	13.18	15.13
***m* = 0.85**	4.83	3.96	3.44	3.52	18.06	17.07	13.07	15.10



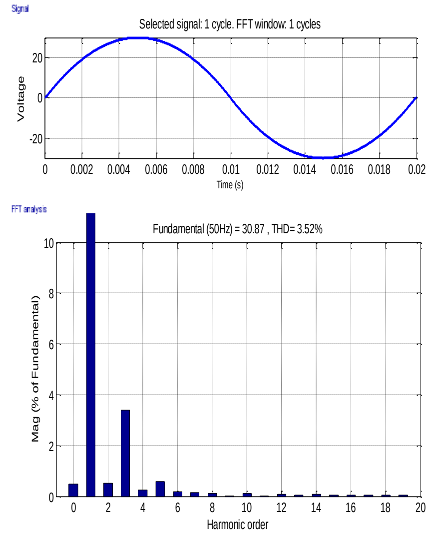
(a) Harmonic spectrum of fitted output voltage -NR.



(b) Harmonic spectrum of fitted output voltage -GSA.



(c) Harmonic spectrum of fitted output voltage -PSO.



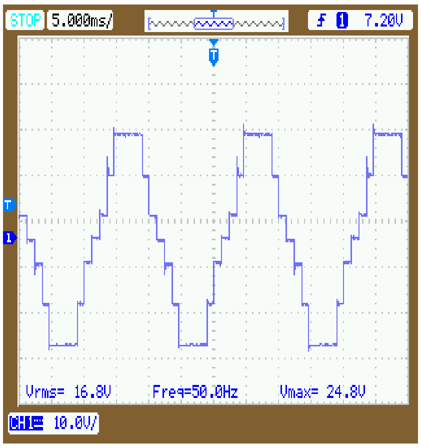
(d) Harmonic spectrum of fitted output voltage -GA.

Fig. 6 FFT results of SC techniques with curve fitting.

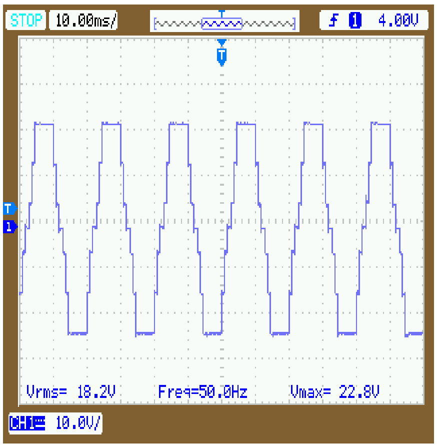
5.1. Hardware results (without curve fit) m = 0.85

The results obtained by using Simulink in MATLAB, are then validated using dSpace1104 and PWM modules with values of DC sources kept at 10 V each.This can be seen by their output waveform and corresponding harmonic spectrum of above mentioned SC methods.

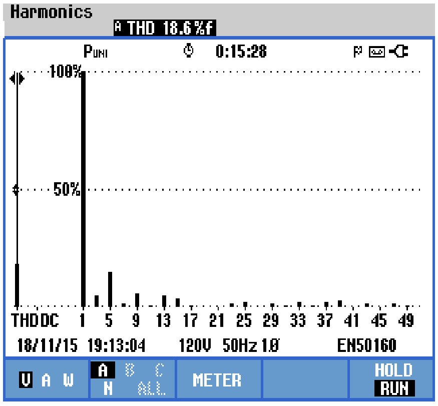
Figures 7(a) and (b) show the waveform and corresponding harmoic spectrum, respectively for cascade 7 level inverter using NR technique without curve fit technique. Similarly Fig. 8(a) and (b) gives the output waveform and corrsponding FFT spectrum for GSA technique which is 17.2% and very close to the value obtain commencing simulation result.



(a) Waveform obtained through NR technique.

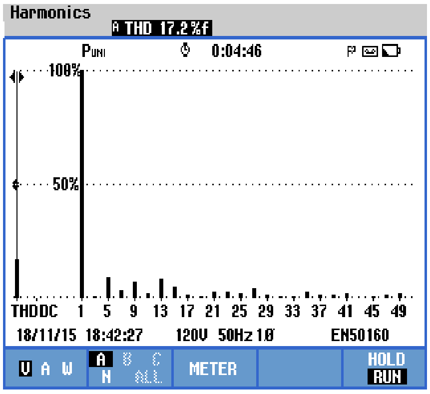


(a) Waveform obtained through GSA technique.



(b) Corresponding Harmonic spectrum through NR

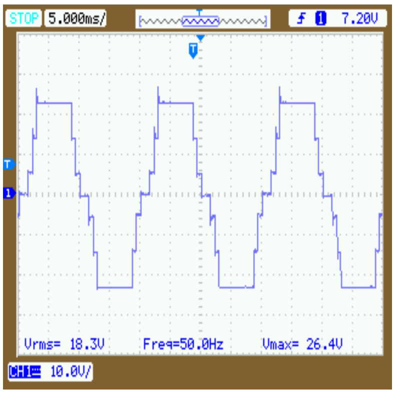
Fig. 7. Harmonic spectrum without curve fit using NR.



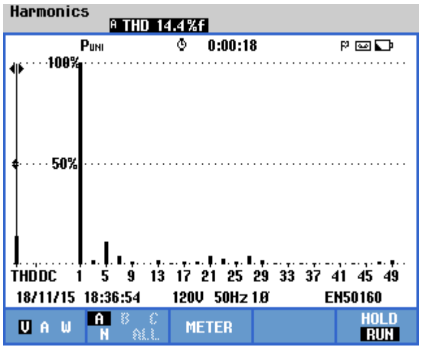
(b) Corresponding Harmonic spectrum through GSA.

Fig. 8. Harmonic spectrum without curve fit using GSA.

Similarly Figs. 9 and 10 show the waveform and respective FFT for PSO and GA methods. Thus Simulation and Hardware results both confirm that lowest %THD is obtained by use of PSO technique. By using PSO technique THD obtained from hardware is 14.4% which is quite near to the %THD obtained from the simulation i.e. 13.07% for modulation index of 0.85.

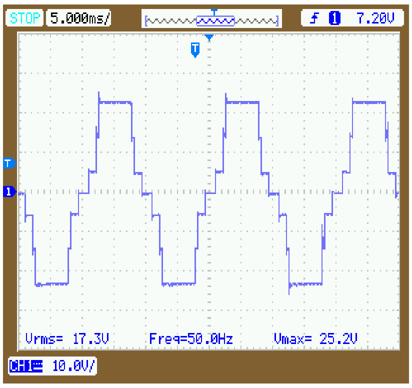


(a)Waveform obtained through PSO technique.

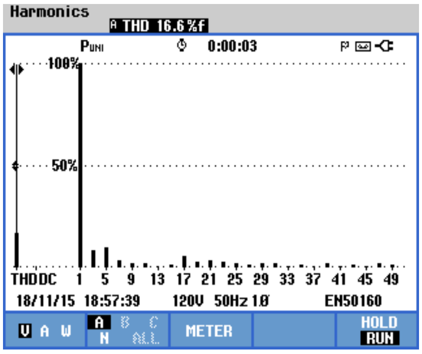


(b) Corresponding harmonic spectrum through PSO.

Fig. 9. Harmonic spectrum without curve fit using PSO.



(a) Waveform obtained through GA technique.



(b) Corresponding harmonic spectrum through GA.

Fig. 10. Harmonic spectrum without curvefit using GA.

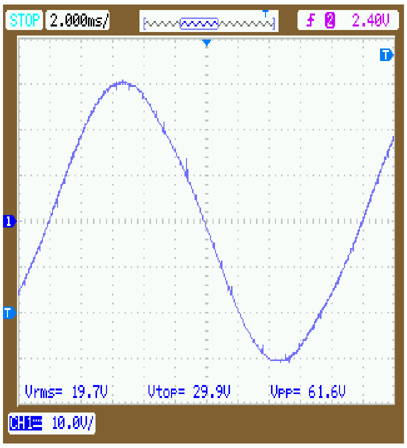
5.2. Hardware results (with curve fit) *m* = 0.85

The simulation results given in Table 3 for curve fitting technique is again validated experimentally using hardware setup. The values of the FFT harmonics achieved experimentally by using various soft computing technique discussed above are given in Table 4. Results are stated below.

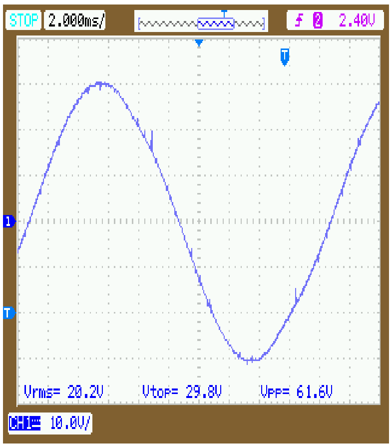
The curve fitting technique can be used for high power high voltage applications, if the number of levels of inverter is increased with high values of DC sources at different loads. Here only prototype model is disigned to validate the simulation results for resistive load only. so that value of DC supply voltage is taken as 30 V. Figures 11 and 12 show the waveform and respective harmonic spectrum for cascade 7 level inverter using NR and GSA technique which is 5.3% and 4.5% respectively at m = 0.85 using curve fit method , as the values of FFT values obtained from simulations are less at this value for one cycle at fundamental frequency.

Table 4. Experimnt results for comparison of THD with and without curve fitting for different soft computing methods.

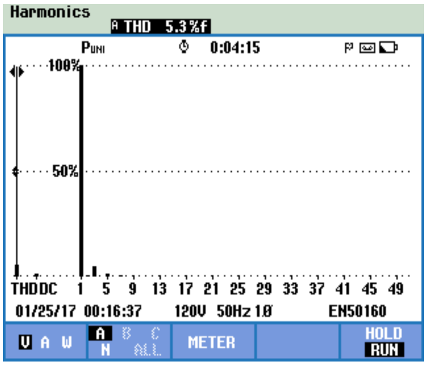
m = 0.85	With Curve Fitting				Without Curve Fitting			
	NR	GSA	PSO	GA	NR	GSA	PSO	GA
%THD	5.3	4.5	3.6	4.2	18.6	17.2	14.4	16.8
Vrms	19.8	20.2	20.2	20.2	16.4	18.2	18.3	17.8



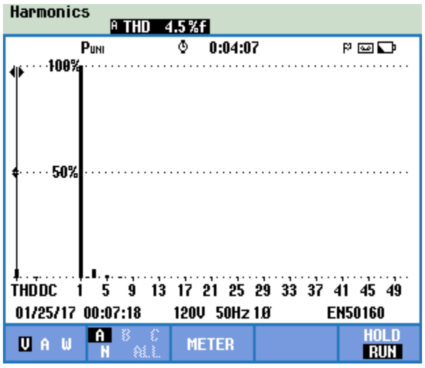
(a) Waveform obtained through NR technique.



a) Waveform obtained through GSA technique.



(b) Corresponding harmonic spectrum through PSO.

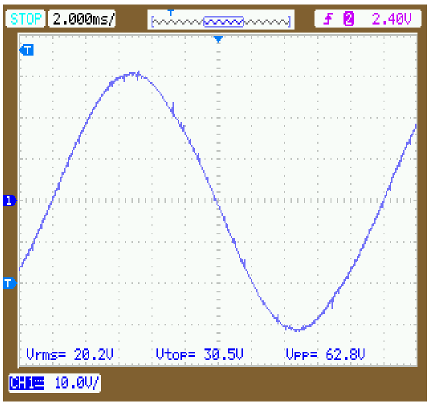


(b) Corresponding harmonic spectrum through GA.

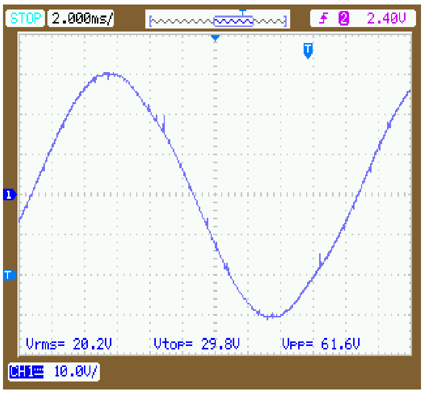
Fig. 11. Harmonic spectrum with curve fit using NR.

Fig. 12. Harmonic spectrum with curve fit using GSA.

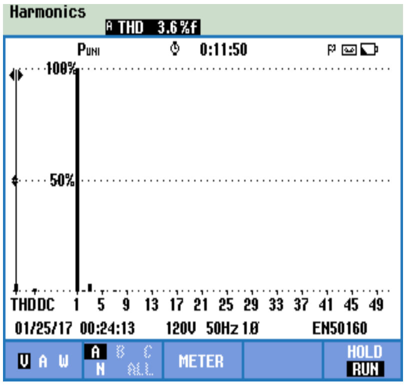
Figures 13 and 14 show the waveform and respective harmonic spectrum using PSO and GA optimization technique which is 3.6% and 4.2% respectively and within IEEE std. 519. From Table 4 and from Fig. 13 it is evident that the results obtain by simulation and hardware setup for PSO is better than other softcompting methods. Table 4 shows the Vrms values obtained by using various softcomputing methods with and without curve fitting technique.



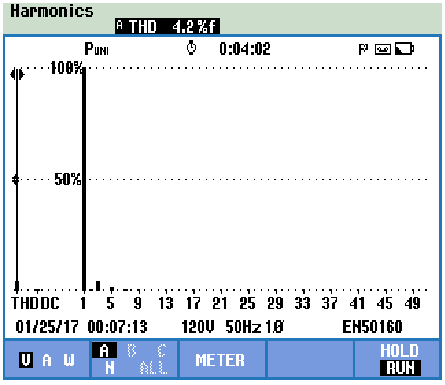
(a) Waveform obtained through PSO technique.



a) Waveform obtained through GSA technique.



(b) Corresponding harmonic spectrum through PSO.



(b) Corresponding harmonic spectrum through GA.

Fig. 13. Harmonic spectrum with curve fit using PSO.

Fig. 14. Harmonic spectrum with curve fit using GA.

6. Conclusion

Iteration based soft computing techniques such as Newton Raphson method and population based algorithms such as genetic algorithm, Particle swarm optimization and Gravitational Search algorithm for finding out optimized switching angles have been explored. It is found from the above simulation and experimental results that PSO gave minimum THD of 13.07% with best optimized switching angles. This value is less than multiple level PWM techniques as reported in the literature for 7-level MLI. The curve fitted to the corresponding obtained 7 level waveform too gives least THD of 3.44% by simulation and 3.6% through hardware for PSO optimization method. Thus different soft computing techniques may be applied to different MLI configuration for higher levels to get minimum possible THD and can be applied to high power applications.

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