Journal of Engineering Science and Technology
Vol. 11, No. 12 (2016) 1736 - 1743
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A STUDY ON HARMONIOUS COLORING OF SNAKE DERIVED ARCHITECTURE

FRANKLIN THAMIL SELVI M. S., AMUTHA A.*

Department of Mathematics, Sathyabama University, Chennai-600119, India
*Corresponding Author: amuthajo@gmail.com

Abstract

There are only few graphs which gives the precise value of the harmonious chromatic number. There are only few results concerning about this problem. We give the precise value of harmonious chromatic number of central graph of snake derived architecture. In this article, we find the harmonious chromatic number of central graph of snake derived architecture using the maximum degree of this architecture and the detailed proof of this is given by induction process. Further we have observed the Maximum matching number of this architecture and it is denoted by $\alpha'(G)$. Using these observations and theorem we have characterized the maximum matching number and edge covering problem of these architecture with harmonious chromatic number $\chi_h(G)$.

Keywords: Harmonious colouring, Snake graph, Matching, Perfect matching, Edge covering, Central graph.

1. Introduction

Graph colouring problem is an important concept in graph theory and it has potential applications in computer sciences. Vertex colouring is nothing but colouring the vertices of a graph such that each vertex receives exactly one colour. A colouring is called proper if the adjacent vertices does not have the same colour. In 1983, a new type of vertex colouring called harmonious colouring was defined by Hopcroft and Krishnamoorthy [1]. Harmonious colouring is a special type of graph colouring in which each edge is assigned a distinct colour pair, i.e. if one edge has the colours red and blue then there should not be any edge coloured with {red, blue}. Harmonious colouring flourished from a well-known colouring called line-distinguishing colouring which was introduced by Frank et al. [2].

The harmonious chromatic number of some graph were determined by different

Nomenclatures

C	Central graph of a graph (Fig. 3)
C_n	Cyclic graph with n vertices
E	Edges in a graph
G	Graph
kC_n	Snake graph with k blocks and n vertices (Fig. 2)
V	Vertices in a graph

Greek Symbols

α'	Maximum Matching number in a graph
β'	Minimum matching number in a graph
Δ	Maximum degree in a graph
χ_h	Harmonious chromatic number

authors. In that the lower bound of harmonious chromatic number is determined as $|E(G)| \leq kC_2$ where k is the number of colours, was obtained by Campbell and Edwards [3]. In the beginning the harmonious chromatic numbers of paths and cycles were determined. Later it was determined that harmonious chromatic number of a graph is NP-hard. In 1995 it was observed that finding the harmonious chromatic number remains harder even when restricted to the class of trees by Edwards and Diarmid [4].

Thus determining the harmonious chromatic number was given in the form of simple bounds and heuristic algorithms [5]. Even though there are numerous results and discussions on our problem, most of them deals only with approximation results [6]. In 2011, an approximate algorithm to obtain the harmonious chromatic number of honeycomb networks was determined by Bharathi et al. [7]. So authors started working on central, line and total graphs of some known graphs like vivin et al. [8]. We can apply this methodology to a colour scheme for use in the home, garden or business or for a ceremony. Thus our problem has potential application in communication networks, radio navigation system etc. Especially in radio navigation system it plays a vital role in controlling the system in bad weather condition or when distance objects are not able to see. It is also used hash function while calculating address of the blocks.

In radio navigation, the system is based on a network of very high frequency Omni directional range radio beacons. To identify the current position of a plane one has to measure the signals of two radio beacons. Assume that state authorities decided to modernize the existing network of radio beacons. In order to reduce the cost of this enterprise they decided to install as few types of beacons as possible. Moreover every country in the world has formal networks of airway. Two adjacent node of the network determine precisely one airway. By associating a vertex of graph G with each such node and modelling each airway by an edge of G, we get a graph corresponding to the network of airways. Now if we find χ_h of G then the harmonious chromatic number will be exactly the number of various types of beacons requested by the state authorities. Hence, the main significance of this paper is that if the guiding system can be converted to a graph which takes the form of central graph of snake derived networks and then $\chi_h(G)$ gives the minimum number of radio beacons used for these guiding networks.

2. Main Result

In this section we have discussed mainly about the central graph of snake derived architecture and their behaviour in harmonious colouring and matching number.

Definition 2.1

A colouring is said to be harmonious if its colouring is proper and each colour pair appears together on at most one edge. $\chi_h(G)$ denotes the harmonious chromatic number and it is the least number of colours used in colouring a graph G.

Figure 1 gives an example for colouring a graph *G* harmoniously.

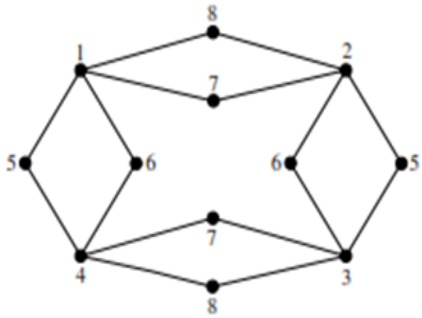

Fig. 1. $\chi_h(G)$ = 8 [Here numbers represent colours].

Definition 2.2

A kC_n snake is a connected graph with *k* blocks whose block-cut point graph is a path and each of the *k* blocks is isomorphic to C_n. The number of vertices and edges of a kC_n snake are $kn - (k - 1)$ and kn respectively.

The following Fig. 2 is a snake graph with k = 5 and n = 5, which is an example of definition 2.2.

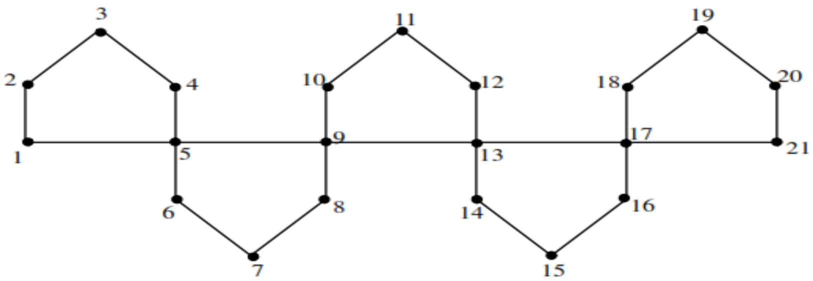

Fig. 2. 5C5 [Here numbers represent vertices].

Definition 2.3

The central graph of any graph *G* is got by separating each edge of *G* exactly once and joining all the non adjacent vertices of *G*.

Figure 3 is an example of central graph of snake graph $2C_3$

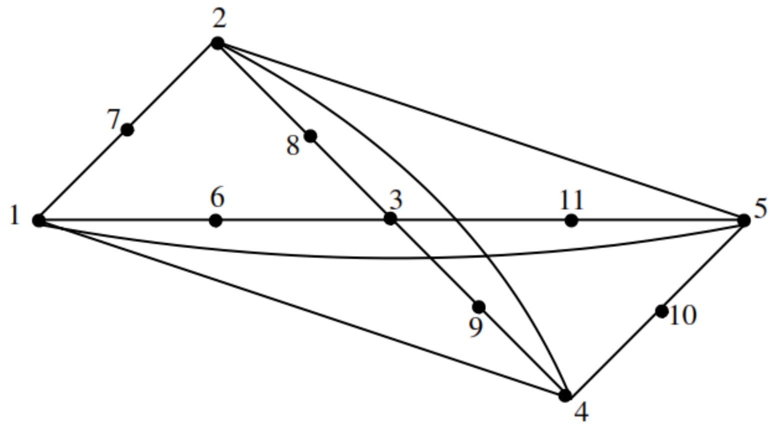

Fig. 3. Central graph of 2C3 snake graph [Here numbers represent vertices].

Theorem 2.4

Let G be a central graph of Snake Graph kC_n , then for $n = 3, k > 1$.
$\chi_h[C(kC_n)] = \Delta[C(kC_n)] + \Delta[kC_n] - 1 \; for \; n = 3, k > 1$. (Refer Figure 4)

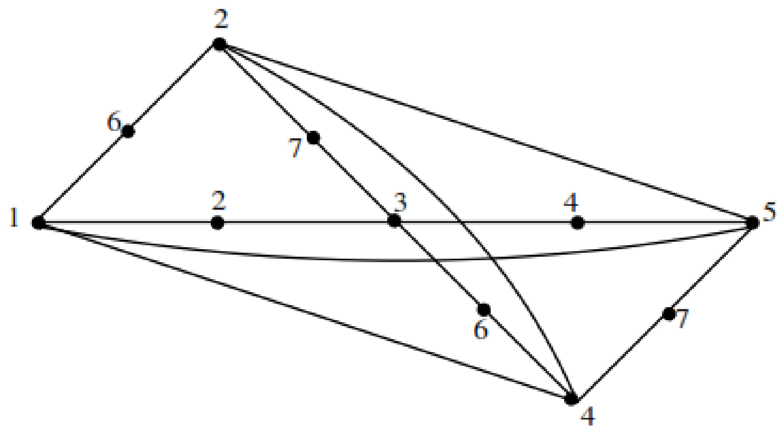

Fig. 4. Illustration of Theorem 2.4 ($\chi_h[C(2C_3)] = 7$)
[Here numbers represent colours].

Theorem 2.5

Let G be a central graph of Snake Graph kC_n , then for $k = 1$, n is odd (except n = 3) $\chi_h[C(kC_n)] = \Delta[C(kC_n)] + \Delta[kC_n] + 2$.

Theorem 2.6

Let G be a central graph of Snake Graph kC_n , then for $k = 1$, n is even $\chi_h[C(kC_n)] = \Delta[C(kC_n)] + \Delta[kC_n] + 1$.

Theorem 2.7

Let G be a central graph of Snake Graph $\mathrm{kC_n}$, then for $\mathrm{k} > 1$, $\mathrm{n} > 3$ $\chi_h[\mathrm{C}(\mathrm{kC_n})] = \Delta[\mathrm{C}(\mathrm{kC_n})] + \Delta[\mathrm{kC_n}] + 1$.

Proof

Let kC_n be the snake graph with $(n - 1)k + 1$ vertices and kn edges. Let v_i where $1 \leq i \leq (n - 1)\, k + 1$ be the vertices of the Snake Graph kC_n for $k > 1\, , n > 3$. By the definition of central graph, each edge of the graph G is separated by a new vertex. There by the number of vertices and edges in central graph of Snake Graph kC_n for k > 1 , n > 3 is $(2n - 1)k + 1$ vertices and $\{[k(n - 1)(k(n - 1) + 1)]\, /\, 2\} + kn$ edges. (Refer Figure 5)

Now we assign colours to these vertices as follows. Since the vertices v_i where $1 \leq i \leq (n - 1)k + 1$ of $C(kC_n)$ has maximum degree $(n - 1)k$, these vertices are coloured with $(n - 1)k + 1$ colours. It is sufficient to colour the remaining nk vertices with 4 colours which is equal to the maximum degree of kC_n in a cyclic order.

Clearly we need $(n - 1)k + 4 + 1$ colours. That is $\Delta[C(kC_n)] + \Delta[kC_n] + 1$ colours. (Refer Figure 5)

Hence $\chi_h[C(kC_n)] = \Delta[C(kC_n)] + \Delta[kC_n] + 1\, for\; k > 1\, , n > 3$. We prove the result by induction on n and k for $k > 1$, $n > 3$. For $k = 2$ and $n = 4$ the result is obvious.

Assume that the theorem holds for $n = n$ and $k = k$, then $\chi_h[C(kC_n)] = \Delta[C(kC_n)] + \Delta[kC_n] + 1\; for\; k > 1\, , n > 3.$

Let us verify by induction for $n = n$ and $k = k + 1$. Consider $C[(k + 1)C_n]$ by introducing a copy of C_n. The vertices of $(k + 1)$ th copy will be $(n - 1)$. And by the definition of central graph, the maximum degree of $C[(k+1)C_n]$ is $\Delta[C(kC_n)] + (n - 1)$.

Therefore,

$$\begin{aligned}\chi_h\{C[(k+1)C_n]\} &= \Delta[C(kC_n)] + (n - 1) + \Delta[kC_n] + 1\, , k > 1\, , n > 3\\ &= (n - 1)k + (n-1) + \Delta[kC_n] + 1\\ &= nk - k + n - 1 + \Delta[kC_n] + 1\\ &= (n - 1)(k + 1) + \Delta[kC_n] + 1.\end{aligned}$$

Let us verify by induction for $n = n + 1$ and $k = k$. Consider $C[kC_{(n+1)}]$ by increasing one vertex in each component k hence by definition of central graph the maximum degree is increased by k.

Thus $\chi_h[C(kC_{(n+1)})] = \Delta[C(kC_n)] + k + \Delta[kC_n] + 1$ for $k > 1$, $n > 3$

$$\begin{aligned}&= (n - 1)k + k + \Delta[kC_n] + 1\\ &= nk - k + k + \Delta[kC_n] + 1\\ &= nk + \Delta[kC_n] + 1.\end{aligned}$$

Finally Let us verify by induction for $n = n + 1$ and $k = k + 1$.

By our previous discussion we have

$$\begin{aligned}\chi_h\{C\,[(k+1)C_{(n+1)}]\} &= \Delta[C(kC_n)] + (n + 1 - 1) + k + \Delta[(kC_n)] + 1 \\ &= (n - 1)k + n + k + \Delta[kC_n] + 1 \\ &= nk - k + n + k + \Delta[kC_n] + 1 \\ &= n(k + 1) + \Delta[kC_n] + 1.\end{aligned}$$

Hence the proof.

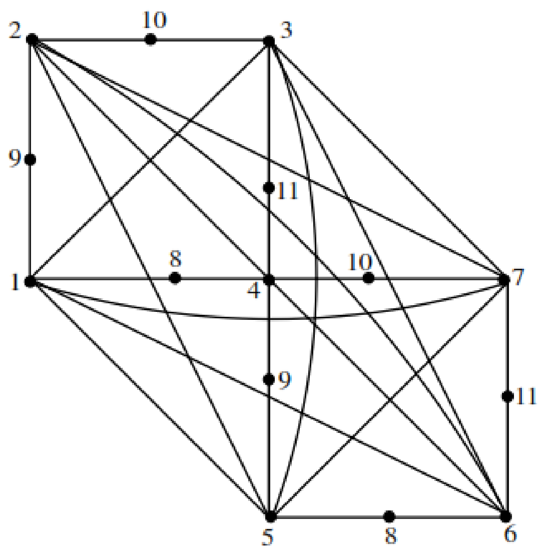

Fig. 5. Illustration of Theorem 3.7 (χh [C(2C4)] = 11)
[Here numbers represent colours].

Observation 2.8

Let *G* be a snake graph kC_n then,

(i) If k is even then kC_n does not have perfect matching for all n.

(ii) If k is odd then for $n = 2n$, kC_n snake graph have perfect matching and for $n = 2n + 1$, kC_n snake graph does not have perfect matching.

Observation 2.9

Let G be a central graph of snake graph kC_n then,

(i) For $k > 1$ and $n > 2$ $\alpha'(G) = k(n - 1) + 1$

(ii) For $k = 1$ and $n > 2$ $\alpha'(G) = nk$

Observation 2.10

Let G be a central graph of snake graph kC_n for $k = 1$ and $n > 3$ then,
$\chi_h[C(kC_n)] = \alpha'(G) + 3$ for n is odd and $\chi_h[C(kC_n)] = \alpha'(G) + 2$ for n is even if and only if G has a perfect matching.

Observation 2.11

Let G be a central graph of snake graph kC_n for $k > 1$ and $n \geq 3$,
then G does not have perfect matching and it has $(k - 1)$ unsaturated vertices.

Theorem 2.12

Let G be a central graph of snake graph kC_n for $k > 1$ and $n > 3$ then,
$\chi_h[C(kC_n)] = \alpha'(G) + \Delta(kC_n)$ if and only if G has $(k - 1)$ unsaturated vertices.

Proof

If the harmonious chromatic number of G is $\alpha'(G) + \Delta(kC_n)$, where $\alpha'(G) = k(n - 1) + 1$ and $\Delta(kC_n) = 4$. Then $\alpha'(G) = k(n - 1) + 1$ covers $2(k(n - 1) + 1)$ vertices, which is not equal to the number of vertices in G. Hence G does not have a perfect matching. Then there exist unsaturated vertices say $(k - 1)$. By Observation 2.11

Now if G has $(k - 1)$ unsaturated vertices the G does not have perfect matching and $\alpha'(G) = k(n - 1) + 1$. By observation 2.9 and by theorem 2.7 we know,

$$\begin{aligned} \chi_h[C(kC_n] &= \Delta[C(kC_n)] + \Delta[kC_n] + 1 \\ &= k(n - 1) + \Delta(kC_n) + 1 \\ &= \alpha'(G) + \Delta(kC_n) \end{aligned}$$

Hence $\chi_h[C(kC_n)] = \alpha'(G) + \Delta(kC_n)$ for $n > 3$ and $k > 1$.

Hence the proof.

Observation 2.13

Let G be a central graph of snake graph kC_n for $k > 1$ and $n > 3$ then,
$\chi_h[C(kC_n)] = \beta'(G) + \Delta(KC_n) - k + 1$ if and only if G has $(k - 1)$ unsaturated vertices.

3. Conclusion

Thus we have obtained the harmonious chromatic number of central graphs of snake derived architectures. Also we have characterized the maximum matching number and minimum edge cover of central graphs of snake derived architectures. Moreover this work can be extended for well- known architectures.

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