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CONJUGATE NATURAL CONVECTION IN A TWO-DIMENSIONAL ENCLOSURE WITH TOP HEATED VERTICAL WALL

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Abstract

In this paper, a two-dimensional numerical study of laminar conjugate natural convection in a square enclosure was analysed. The horizontal heating is considered. The results are presented to show the effect of the governing parameters on the heat transfer and fluid flow characteristics. It is found that for a given thickness of the bounded wall, either increasing the Rayleigh number and the thermal conductivity ratio, can increase the average Nusselt number in the fluid part of the enclosure, the interface solid/fluid temperature and the flow velocity. Generally it is observed that heat transfer rate in the fluid part decreases with the increase of the wall thickness, except at high conductive wall >1 and low Rayleigh number $< 10^4$, the value of Nusselt number increases with the increase of the wall thickness.

Keywords: Conjugate natural convection, Square enclosure, Top heated vertical wall, Finite volumes method, Thermal conductivity ratio.

1. Introduction

Natural convection in enclosures is a topic of considerable engineering interest. Applications range from thermal design of buildings, to cryogenic storage, furnace design, nuclear reactor design, and others. In many studies, the walls of the enclosure are assumed to be of zero thickness and conduction in the walls is not accounted for. In addition convection heat transfer is due to the imposed temperature gradient between the opposing walls of the enclosure taking the entire vertical wall to be thermally active. However, in many practical situations, especially those concerned with the design of thermal insulation. It is only a part

Nomenclatures

D	Dimensionless wall thickness
K_r	Thermal conductivity ratio, $Kr = k_w/k_f$
L	Cavity length, m
H	Wall height, m
h	Height of the active part of the wall, m
Nu	Local Nusselt number, Eq. (8)
$\overline{\text{Nu}}$	Average Nusselt number, Eq. (8)
P	Dimensionless pressure, $p/(a/H)^2$
Pr	Prandtl number of the fluid, ν/α
Ra	Rayleigh number, $g\beta H^3(T_h - T_c)/ \nu\alpha$
t	Dimensionless time, $t^*/H^2/\alpha)$
U, V	Dimensionless velocity components, $u/(\alpha/H)$, $v/(\alpha/H)$
X, Y	Non-dimensional Cartesian coordinates x/H, y/H

Greek Symbols

α	Thermal diffusivity, m^2/s
$\alpha*$	Thermal diffusivity ratio, α_w/α_f
θ	Non-dimensional temperature $(T - T_C)/(T_h - T_C)$
ψ	Non-dimensional stream function, $U=\partial\psi / \partial Y$

Subscripts

f	Fluid
w	Wall
wf	Solid-fluid interface

of the wall which is thermally active and conduction in the walls can have an important effect on the natural convection flow in the enclosure [1-11].

Laminar natural convection flow in a square enclosure with the effect of conduction in one of the vertical walls is studied by Kaminski and Prakash [1]. Three separate models are investigated: (i) two-dimensional conduction in the thick wall; (ii) one dimensional horizontal wall conduction and (iii) a uniform solid-fluid interface temperature. The three models predict nearly the same value for the overall heat transfer. Kim and Viskanta [2, 3] performed experimental and numerical studies on natural convection in a square cavity having four walls with finite thickness.

Mobedi [4] has numerically investigated the effect of conduction of horizontal walls on natural convection heat transfer in a square cavity. It is found that although the horizontal walls do not directly reduce temperature difference between the vertical walls of cavity, but they decrease heat transfer rate across the cavity particularly for high values or Rayleigh number and thermal conductivity ratio. Saeid [5] studied numerically conjugate natural convection-conduction heat transfer in a two-dimensional porous enclosure with finite wall thickness. The Darcy model is used in the mathematical formulation for the porous layer. It is found, in most of the cases that either increasing the Rayleigh number and the thermal conductivity ratio or decreasing the thickness of the bounded wall can increase the average Nusselt number for the porous enclosure.

A control volume study on conjugate natural convection in square enclosure containing volumetric sources was analysed by Liaqat and Baytas [6]. Their results show a significant change in the buoyant flow parameters as compared to conventional non conjugate investigations. Dong and Li [7] studied conjugate natural convection inside a complex cavity. The authors show that the flow and heat transfer increase with the increase of the thermal conductivity in solid region; both geometric shape and Rayleigh number affect the overall flow and heat transfer greatly. Das and Reddy [8] presented a study of natural convection flow in a square enclosure with a centered internal conducting square block both of which are given an inclination angle.

Ben-Nakhi and Mahmoud [9] studied conjugate natural convection inside a building attic in the shape of rectangular enclosure bounded by realistic walls made from composite construction materials under summer day boundary conditions. Their results show that the values of *Ra* and the aspect ratio have significant effect on the temperature and stream function contours within the enclosure. The increase of these parameters leads to increase heat flux into the room. Laminar natural convection in inclined enclosure filled with different fluids was numerically studied by Varol et al. [10]. The enclosure was divided by a solid impermeable divider. Their results show that both heat transfer and flow strength strongly depend on thermal conductivity ratio of the solid material, inclination angle and Grashof numbers. Kuznetsov and Sheremet [11] studied numerically the transient thermosolutal convection in a cubical enclosure having finite thickness walls filled with air, submitted to temperature and concentration gradients. The influence of Rayleigh number on fluid motion and heat and mass transfer is analysed. The effect of the conductivity ratio on heat and mass transfer is also investigated.

In the above studies, the walls of the enclosures are assumed to be totally heated. However it is essential to consider a square cavity with partially heated vertical wall. This article presents a numerical study of steady laminar conjugate natural convection in a square enclosure with conductive vertical wall. The thick left vertical wall is partially heated at the top and the right one if fully cooled. The main focus is on examining the effect of the governing parameter (conduction in the wall, Rayleigh number, and wall thickness) on heat transfer rate and fluid flow.

2. Problem Geometry

The physical model and coordinates system are shown in Fig. 1. The geometry under consideration is a two-dimensional square enclosure ($L=H$) filled with air Pr = 0.70. The left vertical wall has a thickness d while the other walls are assumed to be of zero thickness. The horizontal heating is considered, where the right wall is maintained at cooled temperature T_c, while the outer surface of the left vertical wall is partially heated at the top $T_{h,}$ such that $T_h > T_c$ and $h=H/2$. The remaining left portion of the vertical wall and the two horizontal walls are considered adiabatic.

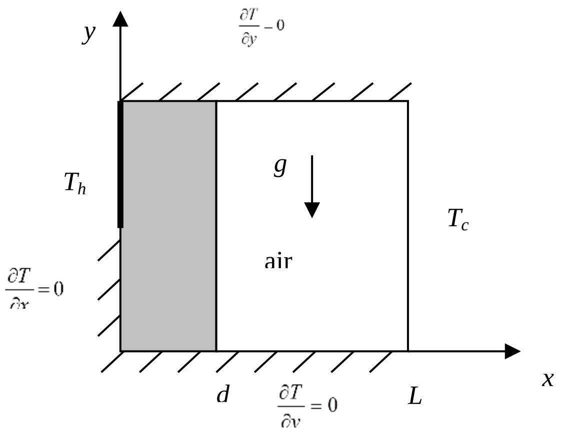

Fig. 1. Physical Configuration.

3. Governing Equations

In order to simplify the analysis, some assumptions are made, namely:

- The fluid is considered to be Newtonian and incompressible.
- The flow in the enclosure is laminar and two-dimensional.
- Viscous dissipation, heat generation and radiation effects are neglected.
- The thermo-physical properties of the fluid are assumed to be constant. Except density variation in the buoyancy term, which depends linearly on local temperature, i.e., Boussinesq approximation is valid:

$$\rho(T)=\rho_0\left[1-\beta_T\left(T-T_0\right)\right] \quad (1)$$

where

$\beta_T=-\frac{1}{\rho_0}\left[\frac{\partial \rho}{\partial T}\right]$ is the thermal expansion coefficient.

The dimensionless form of the governing equations can be written as

- at $t=0$; $U=V=0$ and $\theta_w=\theta_f=0$

- For $t>0$

● ***Fluid part***

Continuity equation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (2)$$

Momentum equations:

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (3)$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra.Pr.\theta \quad (4)$$

Energy equation:

$$\frac{\partial\theta_f}{\partial t}+U\frac{\partial\theta_f}{\partial X}+V\frac{\partial\theta_f}{\partial Y}=\left(\frac{\partial^2\theta_f}{\partial X^2}+\frac{\partial^2\theta_f}{\partial Y^2}\right) \quad (5)$$

● ***Solid part***

Energy equation:

$$\frac{\partial\theta_w}{\partial t}=\alpha^*\left(\frac{\partial^2\theta_w}{\partial X^2}+\frac{\partial^2\theta_w}{\partial Y^2}\right) \quad (6)$$

The following dimensionless variables and reference values are employed in the above dimensionless equations:

$$X=\frac{x}{H},\ Y=\frac{y}{H},\ U=\frac{u}{\alpha/H},\ V=\frac{v}{\alpha/H},\ t=\frac{t^*}{\alpha/H^2},\ P=\frac{p}{\rho(\alpha/H)^2}$$

The boundary conditions are (see Fig.1):

$$U(D,Y)=U(1,Y)=U(X,0)=U(X,1)=0 \quad (7a)$$

$$V(D,Y)=V(1,Y)=V(X,0)=V(X,1)=0$$
(7b)

$$\theta_w(0,Y\geq 0.5)=1,\ \frac{\partial\theta_w(0,Y<0.5)}{\partial X}=0,\ \frac{\partial\theta_w(X,0)}{\partial Y}=0,\ \frac{\partial\theta_w(X,1)}{\partial Y}=0 \quad (7c)$$

$$\theta_w(D,Y)=\theta_f(D,Y),\ \left.\frac{\partial\theta_f}{\partial X}\right|_{fluide}=Kr\left.\frac{\partial\theta_w}{\partial X}\right|_{solide} \quad (7d)$$

$$\theta_f(1,Y)=0,\ \frac{\partial\theta_f(X,0)}{\partial Y}=0,\ \frac{\partial\theta_f(X,1)}{\partial Y}=0 \quad (7e)$$

The dimensionless parameters that characterize the problem are the Prandtl number $Pr=\frac{\nu}{\alpha}$, the Rayleigh number $Ra=\frac{g\beta_T\Delta TH^3}{\nu\alpha}$, the thermal diffusivity ratio $\alpha^*=\alpha_w/\alpha_f$, the dimensionless wall thickness $D=d/H$ and the thermal conductivity ratio $Kr=k_w/k_f$.

The local and average Nusselt numbers are defined respectively by:

$$Nu=\left(-\frac{\partial\theta}{\partial X}\right)_{X=D,1};\ \overline{Nu}=\int_0^1 Nu\,dY \quad (8)$$

4. Numerical Method

Equations (2) to (6) subjected to the boundary conditions (7) are integrated numerically using the finite volume method described by Patankar [12]. A 2D, uniform and staggered grid is used. A hybrid scheme and first order implicit temporally discretization are employed. Pressure and velocity corrections are implemented in accordance with the SIMPLER algorithm [12] to achieve a converged solution. The discretized algebraic equations are solved by the tridiagonal matrix algorithm (TDMA). Seven sets of grids, 40x40, 50x50, 60x60, 70x70, 80x80, 90x90 and 100x100 are studied for Ra = 10^5; the grid 90x90 is selected and used in all computations. The iteration process is terminated under the following conditions:

$$\sum_{i,j}\left|\phi_{i,j}^{n}-\phi_{i,j}^{n-1}\right|/\sum_{i,j}\left|\phi_{i,j}^{n}\right|\leq 10^{-5} \quad (9)$$

For wall side $\overline{Nu}\big|_{X=0}=\overline{Nu}\big|_{X=D}$ (10a)

For fluid side $\overline{Nu}\big|_{X=D}=\overline{Nu}\big|_{X=1}$ (10b)

where ϕ represents: U, V and θ ; n denotes the iteration step.

5. Numerical Validation

Our numerical solution is validated by comparing the average Nusselt numbers with those reported by Kaminski and Prakash [1]. The authors consider a similar configuration (see Table 1). Different values of Rayleigh number (Ra = 7×10^2 and 7×10^4) and thermal conductivity ratio (Kr= 1, 5, 10 and ∞) are considered. These comparisons show good agreement between the obtained and reported results can be observed.

Table 1. Comparison of $\overline{Nu}$ with Kaminski Solution [1].

Ra	*Kr*	Kaminski [1]	Present Study
7×10^2	1	0.87	0.867
	5	1.02	1.017
	10	1.04	1.040
	∞	1.06	1.063
7×10^4	1	2.08	2.084
	5	3.42	3.417
	10	3.72	3.719
	∞	4.08	4.076

6. Results and Discussion

The results are generated for different values of governing parameters: $\alpha^*=1$; $0.2\leq D\leq 0.5$, $500\leq Ra\leq 10^6$ and $0.1\leq Kr\leq 100$.

6.1. Streamlines plots

For Rayleigh number Ra = 10^5, Figs. 2(a)-(c) shows the effect of both wall thickness *D* and thermal conductivity ratio *Kr*, on fluid motion in the enclosure. The circulation pattern is in clockwise direction, with flow upward at the hot left solid-fluid interface and downward at the cold right wall. Panel (a) represents a poorly conducting wall (*Kr* = 0.1), Panel (b) equal solid/ fluid conductivity (*Kr* = 1) and panel (c) a highly conducting wall (*Kr* = 10). It is found that for all cases (a), (b) and (c) and for a fixed *D*, the flow intensity increases with *Kr* since the wall conductivity becomes more important. For example: at D=0.2, $\psi_{\max}$ increases from 3.64 at *Kr*=0.1 to 8.15 at *Kr*=10. In addition, we can note also that for cases (a) where *Kr*=0.1 and (b) where *Kr*=1 that the increase of wall thickness *D* leads to reduce the maximum values of the dimensionless stream function $\psi_{\max}$. While for case (c) where *Kr*=10 the opposite behavior is observed. This means that the flow intensity decreases with *D* for $Kr \leq 1$, i.e., convection mechanism becomes weaker because of the temperature drop in the wall region. Furthermore it can be seen also from Fig. 2 that the flow motion seems to be asymmetric especially for *D* = 0.2. Then it tends to become symmetric as *D* increases. This behavior in fluid motion can be explained as follow: since the left vertical wall is partially heated on the top, temperature distribution in the solid layer will not be uniform and so that for the interface solid/fluid temperature. This in consequence affects the shape of the fluid motion. As *Kr* increases, heat conduction becomes more significant and temperature distribution in the wall tends to become uniform especially for high thickness wall. In this case the symmetric fluid motion is obtained.

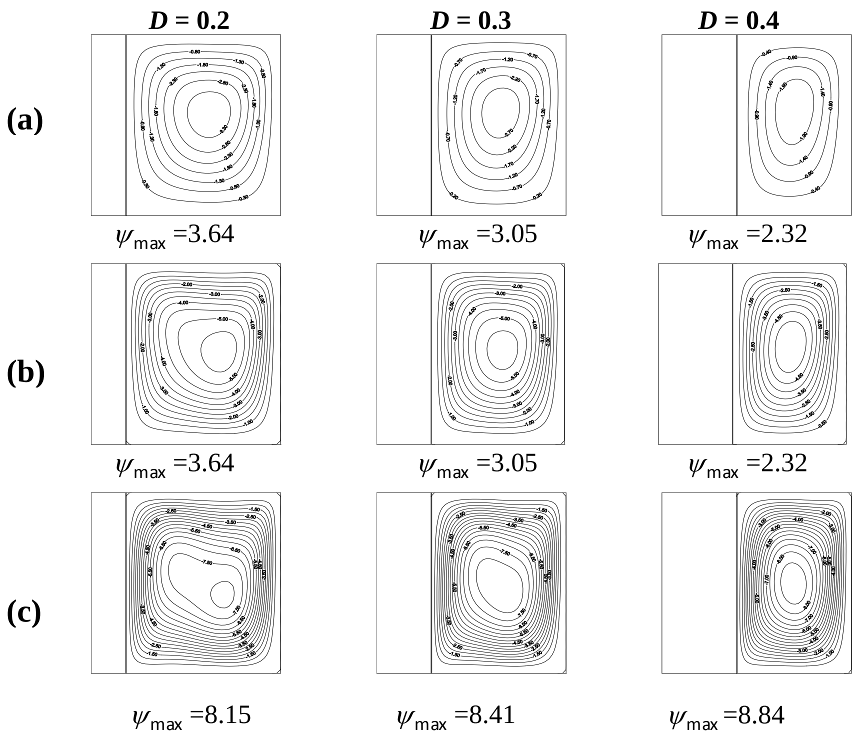

Fig. 2. Streamlines from left to right; *D* = 0.2, 0.3, 0.4.
(a) *Kr* =0.1, (b) *Kr* =1, (c) *Kr* =10.

6.2. Isotherms plots

In this section, the isotherms plots are presented (Fig. 3) in order to examine the effect of both wall thickness *D* (*D*=0.2, 0.3, 0.4) and thermal conductivity ratio *Kr* (0.1, 1, 10) on temperature field at a fixed Rayleigh number (Ra = 10^5).

For poor conductive wall in Panel (a) (*Kr* = 0.1), the average Nusselt number have low values comparing with those in panel (b) and (c). This is a logical result since reducing the thermal conductivity of the wall leads to increase the thermal resistance of the overall system and therefore reducing the Nusselt number. Heat transfer rate is very low in the fluid layer ($\overline{Nu}$ <<1) so the flow motion can be neglected in this region. In addition it is important to note that the isotherm lines in the solid part are quite parallel to the vertical walls especially in the top region. This means that conduction in the wall is not affected by convection in the fluid part since the fluid motion is very slow.

As *Kr* increases, Panel (b) (*Kr*=1), conduction in the solid wall becomes higher than that in case (a) since the thermal resistance is less important. In consequence isotherms lines are more significant in the core of the fluid part and so that for heat transfer rates ($\overline{Nu}$ >1). Furthermore isotherm lines in the solid region change their direction toward the solid/fluid interface which means that they are affected by the fluid motion.

The third case, Panel (c), represents a highly conductive wall (*Kr* = 10). Heat transfer rates in this case are more important ($\overline{Nu}$ >3) comparing with those in the previous cases (a) and (b). Moreover isotherms lines in the solid wall are strongly affected by the fluid motion since the thermal resistance is very low.

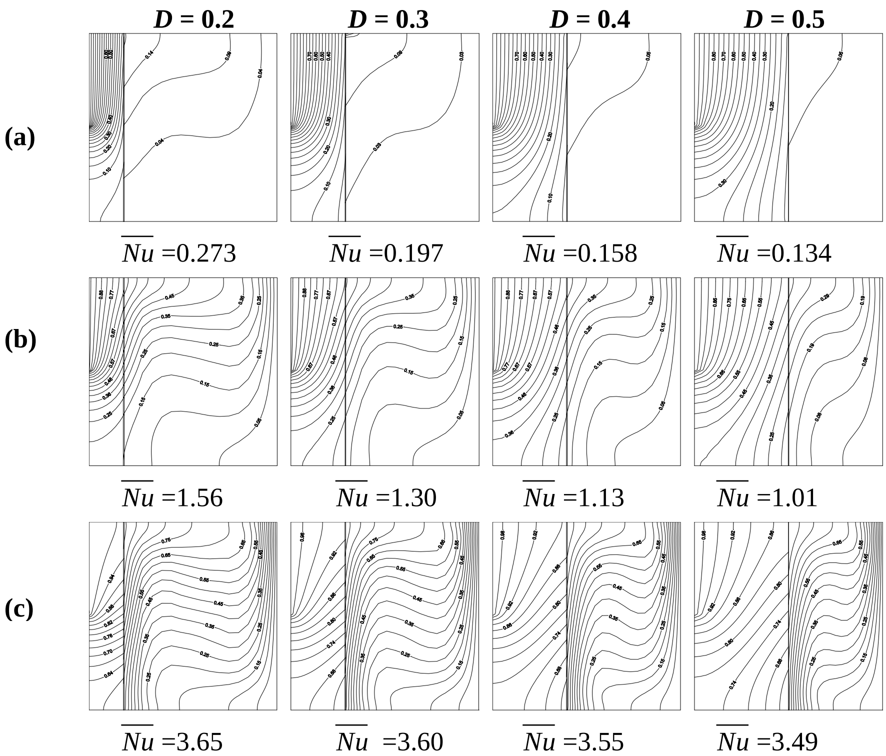

Fig. 3. Isotherms from left to right; *D* = 0.2, 0.3, 0.4, 0.5.
(a) *Kr* = 0.1, (b) *Kr* =1, (c) *Kr* =10, Ra=10^5.

6.3. Interface temperature

The variation of solid/fluid interface temperature is plotted in Fig. 4 against the cavity high (*H*) and thermal conductivity ratio (*Kr*). A fixed Rayleigh number (Ra = 10^7) and wall thickness (*D*=0.2) are selected. A comparison is made with the standard enclosure with (*D* = 0 and $Kr \to \infty$).

It appears in Fig. 4, that the interface temperature at a fixed conductivity ratio increases with the high (*H*) of the enclosure. It becomes more important with the increase of *Kr* and tend to reach the standard case of the enclosure ($Kr \to \infty$) with isothermal vertical wall (θ =1) and zero wall thickness (*D* =0). Furthermore, As in Fig. 4 shows that the temperature profile across the solid/fluid interface is quite non uniform since the solid wall is heated on the top. This non uniformity makes the flow structure asymmetric as shown in Fig. 2 and explained in section 6.1. As expected, it is also important to note that for poor conductive wall, the interface temperature is very low that makes convection in fluid part neglected.

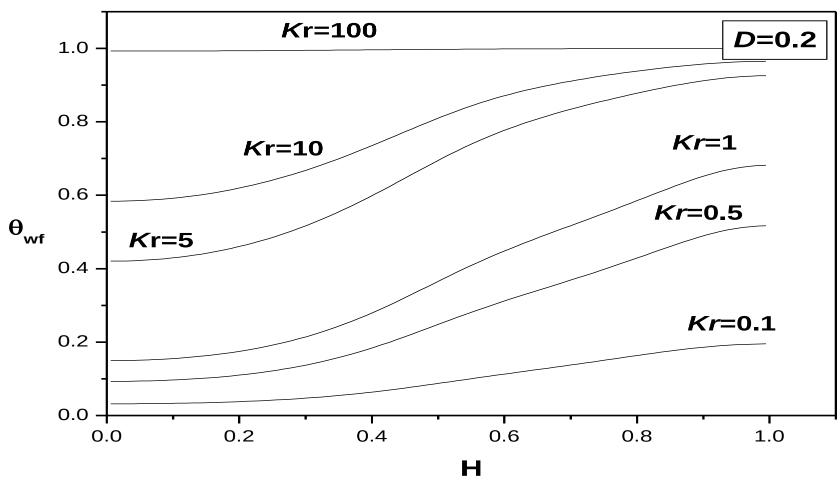

Fig. 4. Variation of wall-fluid interface temperature (Ra =10^5).

6.4. Average Nusselt Number

Figure 5 shows the effect of Rayleigh number, thermal conductivity ratio and wall thickness on heat transfer rate across the enclosure. firstly we note that for the three cases (*Kr* =0.5, 1, 5), the average Nusselt number $\overline{Nu}$ increases with the increase of *Ra*. Natural convection inside the fluid part is driven by the temperature difference between the interface and the cold wall. For low conductive wall (*Kr*=0.5), where the solid part is an insulate material, $\overline{Nu}$ has small values comparing with those with *Kr*=1 and *Kr*=5. In this case temperature difference driving the flow is very small, so most of heat transfer is by heat conduction. As the conductivity ratio increases *Kr*=1, reducing the wall thickness leads to enhance heat transfer by natural convection and so that for $\overline{Nu}$. In addition for $Ra < 10^3$, $\overline{Nu}$ is almost constant so no effect of wall thickness in this case. For high conductive walls (*Kr*=10) and low values of *Ra* ($Ra < 10^4$) we found that $\overline{Nu}$ is decreasing with reducing the wall thickness. This means that the thermal resistance of the wall is less than that of fluid medium for highly conductive walls and $Ra < 10^4$.

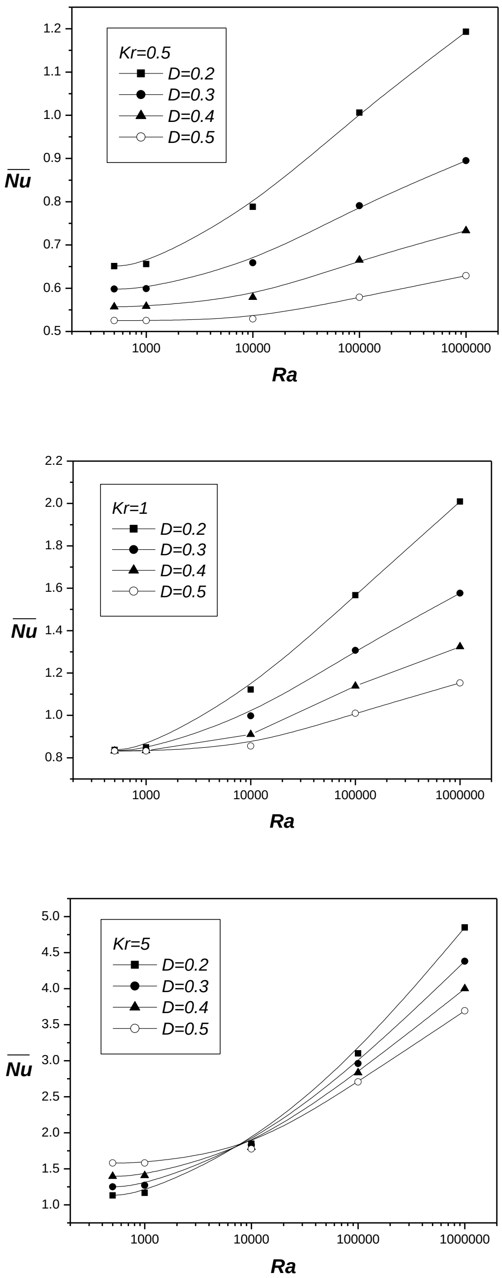

Fig. 5. Variation of $\overline{Nu}$ with Rayleigh number Ra for different values of *Kr*.

7. Conclusion

A numerical model was employed to analyse the flow and heat transfer of air filled in a square enclosure with partially heated and conductive vertical wall. The following conclusions are given below.

- Conduction and wall thickness make a strong effect on natural convection in the fluid part.
- It is found that the heat transfer rate increases and the fluid moves with greater velocity when the values of Rayleigh number and conductivity ratio increase.
- It is observed that the temperature difference between the interface and the cold boundary is reducing with increasing the wall thickness and therefore reducing the average Nusselt number. It is found that as the wall thickness increases the average Nusselt number decreases and the maximum value of the dimensionless stream function in the fluid part are higher with thin walls.
- It is observed for the special case at low *Ra* ($Ra<10^4$) and high conductive walls (Kr =10), the values of $\overline{Nu}$ are increasing with the increase of the wall thickness.
- For low values of *Kr* (poor conductive wall), where the solid wall is an insulation material, $\overline{Nu}$ has low values comparing with those at high *Kr* because of the increase in the thermal resistance of the overall system and vice versa.
- The heat is transferred mainly by conduction in both wall and fluid layer for small values of *Ra* and the average Nusselt number is very low.

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