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ERROR CONVERGENCE ANALYSIS FOR LOCAL HYPERTHERMIA APPLICATIONS

NEERU MALHOTRA[1,*], ANUPMA MARWAHA[2], AJAY KUMAR[3]

[1]Punjab Technical University, Jalandhar, Punjab, India
[2]Department of Electronics & Communication Engineering, S.L.I.E.T., Longowal, Punjab, India
[3]Department of Electronics & Communication Engineering, B.C.E.T., Gurdaspur, Punjab, India
*Corresponding Author: neeru_ins@yahoo.com

Abstract

The accuracy of numerical solution for electromagnetic problem is greatly influenced by the convergence of the solution obtained. In order to quantify the correctness of the numerical solution the errors produced on solving the partial differential equations are required to be analyzed. Mesh quality is another parameter that affects convergence. The various quality metrics are dependent on the type of solver used for numerical simulation. The paper focuses on comparing the performance of iterative solvers used in COMSOL Multiphysics software. The modeling of coaxial coupled waveguide applicator operating at 485MHz has been done for local hyperthermia applications using adaptive finite element method. 3D heat distribution within the muscle phantom depicting spherical leison and localized heating pattern confirms the proper selection of the solver. The convergence plots are obtained during simulation of the problem using GMRES (generalized minimal residual) and geometric multigrid linear iterative solvers. The best error convergence is achieved by using nonlinearity multigrid solver and further introducing adaptivity in nonlinear solver.

Keywords: FEM, Antenna, Solver, COMSOL.

1. Introduction

In finite element method the PDE problem approximates the unknown variables and corresponding system of equations is formulated. The solvers are then used to solve large systems of linear algebraic equations using different methods [1]. One particular method cannot be judged as the best for all conditions. These methods should be determined according to speed and accuracy of the system. In this

Nomenclatures

A	Magnetic vector potential
B	Magnetic flux density
E	RMS value of the time harmonic electric field strength inside the tissue
f_{c10}	TE10 mode cut-off frequency
V	Electric scalar potential

Greek Symbols

∇	Divergence operator
λ	Free space wavelength
σ	Electric conductivity
ρ	Mass density of the tissue
ε_{ω}'	Real part of the de-ionized (DI) water complex permittivity
$\varepsilon_{10'}$	Permittivity of dielectric medium

Abbreviations

DI	De-Ionized
FEM	Finite Element Method
GMRES	Generalized Minimum Residual Error Solver
SAR	Specific Absorption Rate

context, speed is an important factor in solving large systems of equations because the volume of computational domain involved is vast. Another important issue is the accuracy problem for the solutions rounding off errors included in executing these computations. The solvers for linear systems of equations can be categorized as direct solvers and iterative solvers. For solving large number of equations in a system, direct methods [2] are not appropriate for of solving particularly when the coefficient matrix is sparse, i.e., when the majority of elements in a matrix are zero. When the number of equations in a system is very large then iterative methods are suitable for solving linear equations. Iterative methods are very efficient regarding computer storage and time requirements. One of the major advantages of using iterative methods is that they need fewer multiplications for large systems [3]. They can be implemented in smaller programmes than direct methods. They are simple and fast to use when coefficient matrix is sparse. Advantageously they have fewer rounds off errors as compared to other direct methods. Contrary, the aim of direct methods is to calculate an exact solution in a finite number of operations. While iterative methods reproduce usually improved approximations in an infinite sequence whose limit is the exact solution and starts with an initial approximation. Direct methods work for such kind of systems in which most of the entries are non-zero whereas iterative methods are appropriate for large sparse systems which contains most zeros.

The iterative solver approaches the solution gradually by starting from a suitable initial guess and then depending on the error estimates successive updates are computed that finally converge to a solution. The accuracy in a model solving an equation for electromagnetic problem is often limited by how well the mesh resolves the domain. Time-dependent wave equations also put constraints on the time steps used by the solver. Therefore, the solution error due to poor or incomplete convergence cannot be ignored. Iterative solvers are more difficult to use because they require high-quality pre conditioners. The suitable choice of pre conditioner however reduces the number of iterations required hence saving computing time.

The coaxial coupled waveguide applicator for local hyperthermia applications has been numerically simulated here using COMSOL Multiphysics software. The convergence error for obtaining the solution of implemented model is compared for most popular GMRES (generalized minimal residual) method and multigrid iterative solvers [4]. Depending on whether the problem is linear or nonlinear, the solvers decompose it into one or several linear systems of equations. The choice of solver for linear systems thus affects the solution time and memory requirements also for nonlinear models. The accuracy in a model solving an equation for electromagnetic problem is often limited by how well the mesh resolves the system equation. Time-dependent wave equations also put constraints on the time steps used by the solver. The meshing is controlled by adaptive refinement and the appropriate solver settings are selected to obtain an accurate solution [5].

2. Coaxial to Waveguide Coupled Applicator Design

Figure 1 depicts the perspective view of the developed coaxial coupled waveguide applicator. The energy is fed into the cross-sectional area a×b, (a = 2.4 cm, b = 4.8 cm) of the waveguide through a coaxial-to-waveguide coupled waveguide transition by utilizing a coaxial probe-type antenna inside the waveguide. The probe conductor, soldered to the central conductor of an N-type coaxial connector, was extended near to the opposite wall of the waveguide.

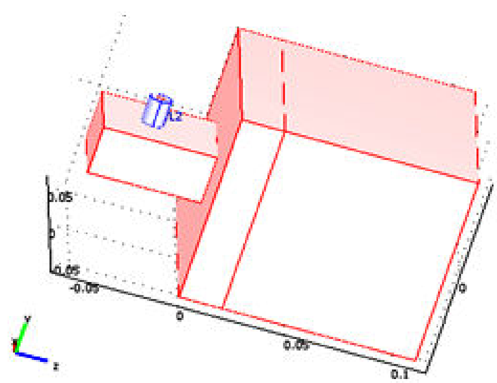

Fig. 1. Perspective view of the waveguide applicator.

The length *l* of one side of the short-circuited waveguide section was chosen equal to λg/2, where λg is the wavelength inside the waveguide. The λg of TE10 mode is given by the relation

$$\lambda_g = \frac{\lambda}{\sqrt{\epsilon_w{}'}} \frac{1}{1 - \left(\frac{f_{c10}}{f}\right)} \tag{1}$$

where $\epsilon_w{}'$ is the real part of the de-ionized (DI) water complex permittivity, λ is the free-space wavelength, $f_{c10} = {}^{c}\!/\!\left(2a\sqrt{\epsilon_{10}{}'}\right)$ is the TE10 mode cut-off frequency (c = 3×10^8 m/s), $\epsilon_{10}{}'$ is the permittivity of dielectric medium and $f = c/\lambda$ is the radiation frequency of 485 MHz. The cooling chamber or water bolus is the portion between the waveguide and the muscle phantom which isolates the waveguide cavity from the phantom. By circulating DI water through the cooling chamber, the antenna temperature was kept constant without any problem up to 100-W transmitted power levels.

3. Methodology

FEM has been extensively used in simulation of microwave antenna applicators because the computer simulations determine the optimal antenna design for the purpose of optimal power application and effectively localized thermal lesion [6]. This method carries on the processing in three stages: pre-processing, processing and post-processing. In pre-processing the whole structure is divided into elementary sub domains, which are called finite elements and the field equations are applied to each of them. In processing the FE methods are essentially based on determination of the distribution of electric and magnetic fields in the structures under study, based on the solution of Maxwell’s equations. The post processing performs analysis of results obtained by determination of other parameters based on the field distributions.

To solve electromagnetic problems with boundaries values, it is essential to formulate the problem in terms of magnetic vector potential A and the electric scalar potential V.

$$\mathrm{B} = \nabla \times \mathrm{A} \tag{2}$$

$$\mathrm{E} = -\nabla V - \frac{\partial \mathrm{A}}{\partial \mathrm{t}} \tag{3}$$

The electromagnetic field radiated in given tissue by an appropriate antenna can be determined by solving the Maxwell's equation stated below with the knowledge of tissue electromagnetic properties.

The total fields obtained due to electric and magnetic vector potential are obtained as:

$$\begin{aligned} \mathrm{E} &= -j\omega\mathrm{A} - \frac{1}{\omega\mu\epsilon}\nabla(\nabla.\mathrm{A}) - \frac{1}{\epsilon}\nabla \times V \\ H &= \frac{1}{\mu}\nabla \times \mathrm{A} - j\omega V - \frac{j}{\omega\mu\epsilon\nabla(\nabla.\mathrm{V})} \end{aligned} \tag{4}$$

The antenna applicator is located for improved distribution of SAR (Specific Absorption Rate) in affected area. SAR relates to heat generation in tissues due to E- field as

$$SAR = \frac{\sigma E^2}{\rho} \qquad (\mathrm{W/kg}) \tag{5}$$

where σ (S/m) is electric conductivity, ρ (kg/m^3) represents the mass density of the tissue and E (V/m) is the RMS value of the time harmonic electric field strength inside the tissue [7].

4. Results and Discussion

The governing equations of the model are solved using different types of solvers for analysing the error convergence. The solution initially is evaluated for GMRES linear iterative solver. The error reaches the value of the order of 10^{-6} only after 10 iterations of the computation as shown further in Fig. 2.

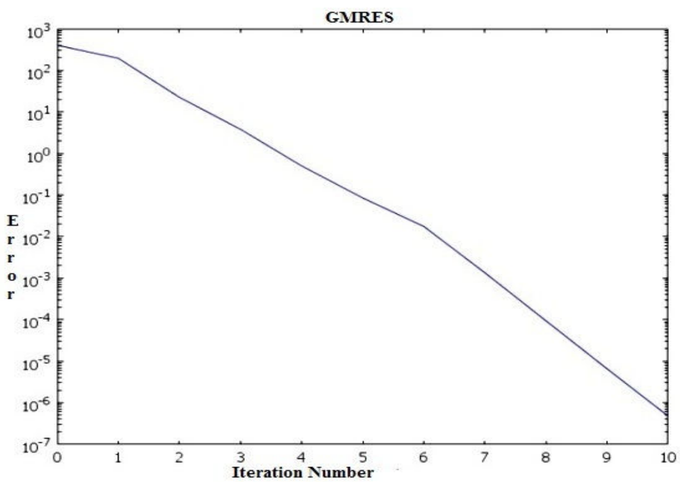

Fig. 2. Error convergence plot using GMRES solver.

The similar error convergence of the order of 10^{-6} is obtained with geometric multigrid linear solver but the solution converges only after more than 35 iterations in this case as depicted in Fig. 3.

For improving the error convergence nonlinearity is introduced in the multigrid solver and as can be observed from Fig. 4 that error value reduces to 10^{-10} in around 3 iterations.

To further enhance the performance, adaptivity is incorporated with nonlinear solver. The L2 norm is selected for the error estimate. The adaptive solver terminates when it has obtained solutions for the number of refined meshes specified by the maximum number of refinements. As the number of mesh elements exceeds the maximum value, the solver terminates even if it has not reached the specified maximum number of refinements. Figure 5 shows the convergence plot for the adaptive solver and it can be observed that the convergence error drops down to value of the order of 10^{-14} in just 3 iterations hence proving to be the best solver in terms of convergence error. The accuracy of results obtained with linear and linear solvers has also been evaluated in terms of the SAR plot for the waveguide applicator.

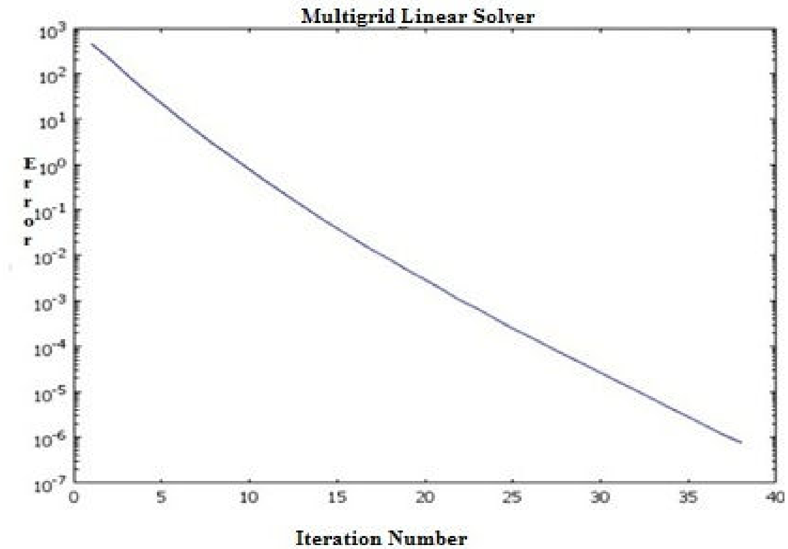

Fig. 3. Error convergence plot using geometric multigrid linear solver.

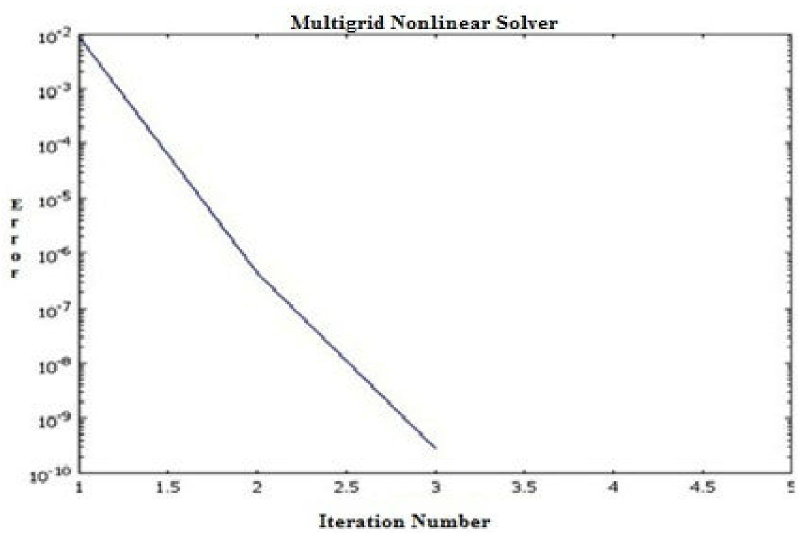

Fig. 4. Error convergence plot using multigrid nonlinear solver.

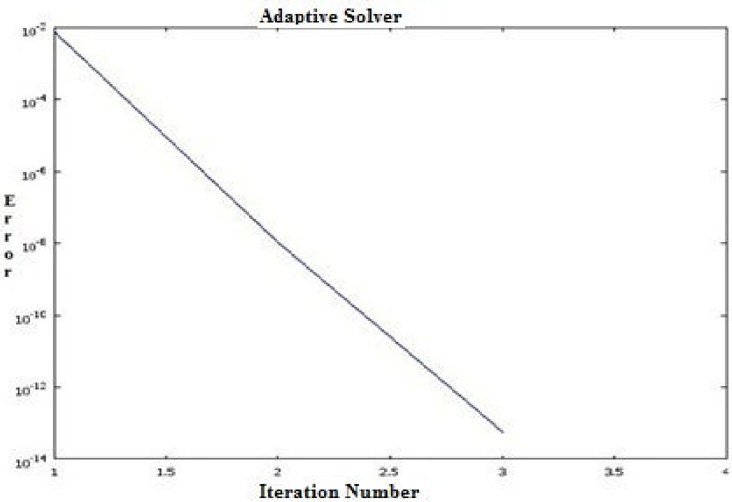

Fig. 5. Error convergence plot using nonlinear adaptive solver.

The COMSOL software produces reduced errors with good estimation for the linear system of equations however for nonlinear problems the accuracy of results is affected. This problem can be overcome by incorporating adaptivity in the nonlinear nature of the problems with reasonable and consistent errors hence improving the accuracy. Figure 6 shows SAR distribution with localized heating pattern when using nonlinear adaptive solvers hence proving to more useful for heat distribution problems.

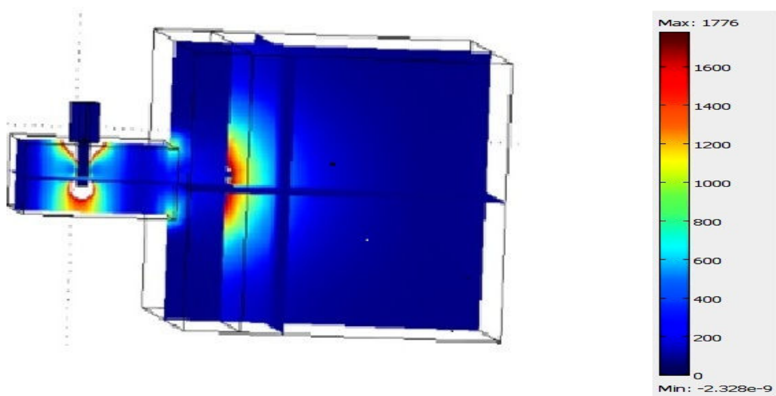

Fig. 6. SAR heat distribution in the muscle phantom.

5. Conclusions

In this paper, various iterative methods are studied and compared for solving system of linear equations. The term "iterative" refers to a wide range of techniques that use successive approximations to obtain more accurate solutions to a linear system at each step. The coaxial coupled waveguide antenna applicator is designed for hyperthermia applications. The error convergence plots are obtained during simulation of the problem using GMRES (generalized minimal residual) and multigrid linear and nonlinear solvers. Some concluding observations from the investigations are as below:

- GMRES method is suitable for solving linear systems of equations. Particularly, it is fitted to solve sparse matrices that arise in the application of computer science, engineering and computer graphics.
- The error convergence is successfully achieved with linear solvers but the number of iterations required is much higher.
- The applicator model requires heat distribution to be evaluated where the radiation terms depend on the resistive heating, the problem therefore becomes nonlinear.
- The error convergence is efficiently achieved using nonlinear adaptive solver with error values of the order of 10^{-14} in minimum number of 3 iterations hence reducing the computation time.

The accurate results with spherical lesion of SAR are also obtained for best localized heating pattern.

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