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DYNAMICS OF FLUID IN OSCILLATORY FLOW: THE $\overline{Z}$ COMPONENT

V. C. -C. LEE[1,]*, Y. A. ABAKR[2], K. -C. WOO[2]

[1]School of Engineering and Science, Curtin University Sarawak Malaysia
CDT 250, 98009, Miri, Sarawak, Malaysia
[2]University of Nottingham Malaysia Campus, Department of Mechanical, Materials and Manufacturing, Jalan Broga, 43500 Semenyih, Selangor Darul Ehsan, Malaysia.
*Corresponding Author: vincent@curtin.edu.my

Abstract

In an oscillatory flow, the resistance to flow, more appropriately defined as the impedance to flow, is a function of oscillating frequency, which refers to the harmonic composition of the driving pressure wave. Flow in an elastic tube may be resisted in numerous ways such as the fluid viscosity, fluid inertia and tube elasticity. The concept of impedance arises in the dynamics of the Resistance-Inductance-Capacitance. In oscillating flow, these represent the fluid viscosity, inertia and tube elasticity. This paper describes the effects of impedance, or the $\overline{z}$ component as described in-text of an oscillating flow in a valveless impedance pump using numerical simulation. A one-dimensional lumped-system model is chosen to perform the analysis in this study. The simulation domain is a mimic to known experimental model previously conducted by Lee et.al. [18-21]. Impedance-induced flow has shown to be combined effects of fluid viscosity, inertia and tube elasticity. Results presented are in reasonable agreement with experimental results presented in Ref [21] with an estimate of 16% variance. This simple model has shown to predict results with significant values, using simple approximations; and further the understanding of fluid impedance's role in a valveless impedance pump.

Keywords: Oscillating flow, Valveless impedance pump, One-dimensional, Lumped-system model.

1. Introduction

Impedance pump is a type of valveless pump, formed by joining a flexible tube to a rigid one, which does not require vanes or blades in its operation [1-6]. By inducing asymmetrical compression, say x/L of 0.1, at a single location of the

Nomenclatures

$\overline{C}$	Tube elasticity
d/D	Normalized tube diameter
$\overline{L}$	Fluid inertia
l/L	Normalized tube length
m	Fluid mass
Q	Net flow rate
$\overline{R}$	Fluid viscosity
$\overline{S}$	Fluid reactance
$\dot{u}$	Acceleration
V	Fluid volume
x/L	Normalized compression location
$\overline{Z}$	Fluid impedance

Greek Symbols

ΔP	Total driving pressure difference
ω	Angular excitation frequency
ϑ	Phase difference
μ	Coefficient of fluid viscosity
ρ	Fluid density

fluid-filled elastic tube would result in a unidirectional flow due to the mismatch in fluid impedance, for x is the instantaneous compression location and L is the tube's total length.

The pumping mechanism is highly sensitive towards the impedance of the tube, the location of compression and frequency of excitation [1-10] which in turns a representation of the magnitude of the pressure waveform. Flow in an elastic tube may be resisted in numerous ways such that fluid viscosity ($\overline{R}$) opposes the flow by means of viscous shear at the wall, fluid inertia ($\overline{L}$) resists the pressure driving the flow and tube elasticity ($\overline{C}$) opposes the driving pressure as it expands the wall [11]. Studies [1-3, 11, 13-15] had been conducted in the past decades to investigate the effects of these parameters in reference to phenomenon of valveless impedance-induced flow. The effect of fluid impedance on flow generation is however not well defined in most literature.

For a unidirectional fluid flow in a rigid tube, the resistance to flow is defined as fluid viscosity ($\overline{R}$). Resistance to oscillating flow in an elastic tube on the other hand is defined as impedance ($\overline{Z}$) which consists of complex parameters of fluid viscosity and reactance ($\overline{S}$). Fluid viscosity exists due to the viscous shear at the wall, whereas reactance is the resistance to wave propagation caused by the combined effect to tube elasticity and fluid inertia in the tube. These complex parameters are essential in a valveless impedance pump as they govern the net flow and the unique flow characteristics of the pump. These include the unidirectional pulsatile flow, retrograde flow, non-linear responses of flow and wave dissipations

[4]. A simple correlation of these complex parameters is summarised in Fig. 1. The propagation and reflection of wave are effects found in elastic tube.

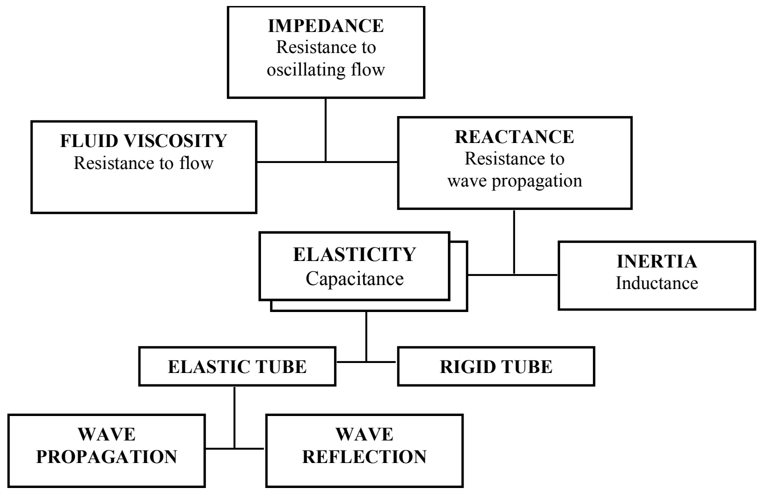

Fig. 1. Relationship of impedance with important contributing parameters.

The relation between the pressure and flow in an elastic tube depends on properties of the tube such as its diameter, length and elasticity. It also depends on the form of the driving pressure, in particular whether the pressure is steady or pulsatile. The complexity of this problem gives rise to the concept of lumped-system model. The focus of this paper is to study numerically the contribution of fluid impedance in generating a flow in a valveless impedance pump using a one-dimensional lumped-system model.

2. Theoretical Background and Modelling

In order to solve a fluid flow problem and fully determine the dynamics of the flow, including mapping the velocity field and the relation between prevailing pressure and flow fields, is possible only in the most simply constructed cases and mostly in the physical sciences [16,17]. Figure 2 shows the physical model of a two-stage impedance pump [18-21] illustrating how the flow is channelled in a two-stage system.

Typically, the system will be filled with certain volume of water (V_1 to V_5) where each subscript denotes different part of the tube. As the excitation pressure is applied on the elastic tube, momentum is created in the system denoted as fluid inertia ($m\dot{u}_1$ to $m\dot{u}_5$) with respect to the impedance difference created within the system. The fluid inertias hence induce flows (Q_1 to Q_5) in the system and caused a change in volumes ($\dot{V}_1$ to $\dot{V}_5$) creating pressure head difference in each individual reservoir. Maximum pressure head difference will be reached at zero flow when the system is under steady-state condition.

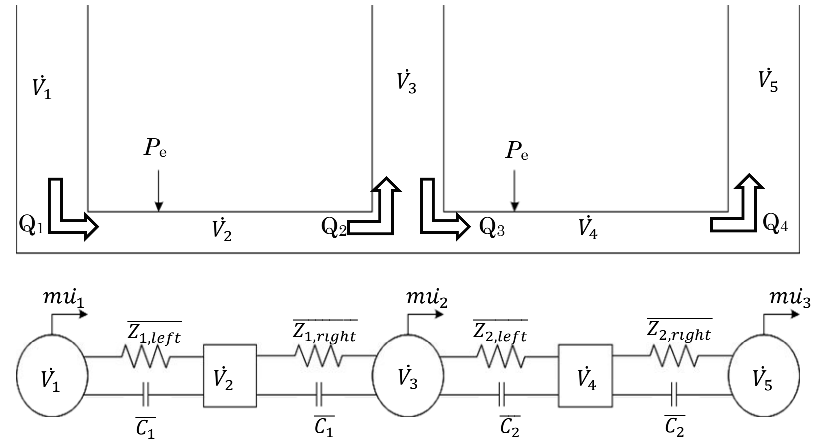

Fig. 2. Physical model of a two-stage system.

2.1. The dynamics in an oscillatory flow

The total pressure difference ΔP required to drive the flow in an elastic tube in the presence of a change in flow rate is the sum of all pressure difference needed to overcome the force of resistance due to fluid inertia, pressure difference needed to overcome the force of resistance due to fluid viscosity, and pressure difference needed to overcome the force of capacitance due to tube elasticity, that is

$$\Delta P_{\overline{L}} + \Delta P_{\overline{R}} + \Delta P_{\overline{C}} = \Delta P \tag{1}$$

The equation of motion to such system is then established as proposed in Refs [11, 14] and shown in Eq. (2).

$$\overline{L}\frac{dQ}{dt} + \overline{R}Q + \frac{1}{\overline{C}}\int Q\ dt = \Delta P \tag{2}$$

where $\overline{L}$ is the fluid inertia, $\overline{R}$ is the fluid viscosity, $\overline{C}$ is the tube elasticity, Q is the net flow rate and ΔP is the total driving pressure difference. The left hand side of the equation is the parameters contributing to flow creation while the right hand side of the equation is the total driving pressure difference. With respect to the equation, it is clearly seen that there are three main parameters governing the motion of the fluid or in simpler word, the flow. The fluid inertia, fluid viscosity and tube elasticity are recognized as the governing parameters in accordance to Eq. (2) and can be characterized as shown in Eqs. (3) to (5)

$$Q_{\overline{L}} = \int \frac{\pi d^2}{4\rho l}\Delta P_{\overline{L}}\ dt \tag{3}$$

$$Q_{\overline{R}} = \frac{\pi d^4}{128\mu l}\Delta P_{\overline{R}} \tag{4}$$

$$Q_{\overline{C}} = \overline{C}\frac{d(\Delta P_{\overline{C}})}{dt} \tag{5}$$

These equations generally describe the theoretical background of an oscillatory flow in an elastic tube as a lumped-system which subsequently serves as the ground work of the development of the fluid dynamics in impedance pump. Reactance subsequently presents an obstruction to flow in terms of changing the amplitude and phase of flow wave. Using the electrical analogy approach, reactance can be expressed as,

$$\overline{S} = \omega\overline{L} - \frac{1}{\omega\overline{C}} \tag{6}$$

where $\omega = 2\pi f$ is the angular frequency of oscillation.

Impedance embodies both the fluid viscosity and reactance; viscous resistance dissipates energy which must be replaced constantly from a source of driving energy while reactance on the other hand presents an impediment to flow in the sense of affecting the amplitude and phase of the flow wave but does not dissipate the flow energy. Under certain reactive effects, flow energy is only exchanged between pressure and kinetic energy, as when the fluid is accelerated or decelerated within the elastic energy of the tube wall. Impedance may be expressed as

$$\overline{Z} = \overline{R} + i\overline{S} \tag{7}$$

and the phase of flow wave is expressed as

$$\tan\vartheta = \frac{\overline{S}}{\overline{R}} \tag{8}$$

In an oscillatory flow, the concept of impedance is introduced in order to be able to write the expression between flow and pressure drop rather than a simple resistance term as

$$Q_{\overline{Z}} = \frac{\Delta P_o}{\sqrt{\overline{R}^2 + \overline{S}^2}}\sin(\omega t + \vartheta) \tag{9}$$

where ΔP_o is the constant representation of amplitude of driving pressure difference.

2.2. Numerical simulations

The numerical method employed in this paper is a one-dimensional lumped-system model. There have been various methods discussed in literature on solving the phenomenon, with notable ones being immersed boundary method [12, 13] and lumped-system model [11, 14]. The lumped-system model is chosen in this study due to its simplicity and lower computational cost as compared to immersed boundary method, and at the same time able to produce a rather close approximation to the solution. Numerical simulations were performed in accordance to Eqs (3), (5) and (9) where the effects that each parameter onto $\overline{R}$-$\overline{L}$-$\overline{C}$ system were thoroughly studied and analysed. The $\overline{R}$-$\overline{L}$-$\overline{C}$ system is an electrical-based analogy of equations in series in accordance to the proposed experimental studies [18-21]. A summary of parameters used in this numerical study is tabulated and shown in Table 1.

Table 1. Parameters of investigation for $\bar{R}$ - $\bar{L}$ - $\bar{C}$ components.

Parameters	Range	Step size
Tube length, *l* (mm)	10-500	10
Tube inner diameter, *d* (mm)	10-50	10
Constants		
Driving pressure difference, $\Delta P_{\bar{L}} + \Delta P_{\bar{R}} + \Delta P_{\bar{C}}$ **(Pa)**		$101325 \sin(\omega t)$
Coefficient of fluid viscosity, μ (kg m^{-1} s^{-1})		0.001
Fluid density, ρ (kg m^{-3})		998
Angular frequency of oscillation, ω		30.16

As this study concentrates on the investigation of individual parameters, necessary assumptions are made. Amplitude of driving pressure difference of each parameter is made constant in this study and the angular frequency of oscillation is set at value of 30.16 to create an oscillatory motion to it. Angular frequency of 30.16 is chosen in this simulation with reference to [21] in which the designated frequency is observed to be at its resonance, where pumping is most efficient. The tube length and inner diameter is normalized with respect to their highest value in the range, in this case 500 mm for the characteristic length (L) and 50 mm for the characteristic diameter (D). This non-dimensional representation is for the ease of comparisons in the discussion. Similarly, flow rate is normalized with respect to the highest achievable flow rate obtained, giving the highest flow rate to be denoted as one. The presented results are simulated under steady-state conditions. Based on these simple assumptions, numerical simulations on the parameters are established. The y-axis motion of the elastic tubing is not considered in this simulation.

3. Results and Discussion

A set of one-dimensional numerical expressions were modelled based on the physical model of a valveless impedance pump. In accordance to Eq. (3), the fluid inertia term is dependent on two physical parameters, the tube characteristic length and tube diameter. Figure 3 shows the effects of fluid inertia towards a flow with respect to the characteristic length and diameter of the elastic tube.

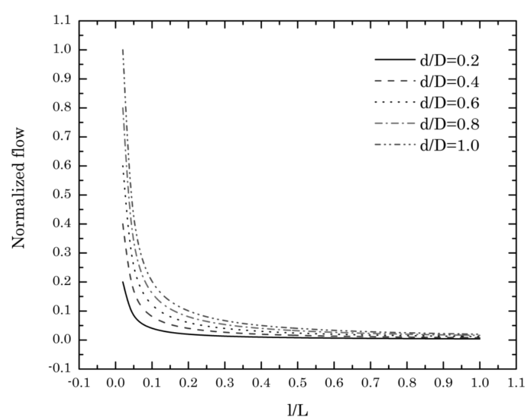

Fig. 3. Normalized flow due to fluid inertia with respect to tube characteristic length and diameter.

Based on Fig. 3, it is seen that as the characteristic length, *l*/*L* ratio increases, flow in the tube decreases. Such phenomenon is due to the increase of fluid volume in the tube that in turn creates a higher inertia. Longer length indicates larger potential energy stored and therefore higher driving pressure is required for the fluid to accelerate. As this study emphasizes on the parametric characteristics of the fluid, driving pressure difference was hence remained constant. Therefore, such a phenomenon is observed.

Consequently, as the characteristic diameter, *d*/*D* ratio increases, the flow increases due to the decrease in inertia. This is shown in Fig. 3 where a higher flow is initiated. Inertia, similar to fluid viscosity is a form of hindrance, but obstructs flow in a different manner, which resists the flow from accelerating or decelerating. Hence, with larger diameter, lower impediment will be imposed on the fluid motion and fluid may then accelerate with more free will. This explains the phenomenon shown in the curves in Fig. 3. It is also observed that as the diameter increases, the curve gradient of flow reduce, indicating the flow takes extra length for the fluid momentum to diminish. This shows that the diameter will be able to compensate for the loss in fluid momentum for a tube with longer length without having to change the driving pressure.

Figure 4 shows the normalized flow due to the elasticity of the tube. Based on the figure, it is shown that as *l*/*L* and *d*/*D* ratios increase, the flow increases as well. Zero flow is observed for *d*/*D* ratio of 0.2, such is due to the elastic tube is acting as a normal rigid tube due to the ratio of tube's inner and outer diameter. With small inner diameter and large tube thickness, the driving pressure required to expand the tube wall will be relatively large as well. For the inner diameter of 10 mm and a tube thickness of approximate 20 mm constitutes a relatively low elasticity, compression and expansion of tube is hardly achievable as the driving pressure is not sufficient for this action. This explains the zero flow observed for *d*/*D* ratio of 0.2, as for the applied driving pressure, it is not sufficient to expand the tube wall and hence unable to induce a flow. For *d*/*D* ratio of 0.4, however, increase in flow is observed although it is very minimal. With the increase in the inner diameter, flow is shown to increase drastically. Such phenomenon is due to the larger inner diameter, indicating larger volume for fluid motion to expand and at the same time less effort is required from the driving pressure. With larger *d*/*D* ratio, the wave can effectively propagate which is another factor for inducing a higher flow. Curve gradient of flow is observed to increase as *l*/*L* ratio increases in a very linear manner. It is therefore shown that the tube diameter and thickness ratio is a critical factor in inducing a flow.

Normalized flow due to fluid impedance with respect to tube characteristic length and diameter is shown in Fig. 5. Impedance is theoretically a complex parameter involving the $\overline{R}$ - $\overline{L}$ - $\overline{C}$ as it incorporates both fluid viscosity and reactance which is dependent on fluid inertia and elasticity of the tube. Different from the Fig. 3 and 4, the curves in Fig. 5 shows an optimum length where maximum flow can be achieved. Optimum length is observed to increase along *l*/*L* ratio as *d*/*D* ratio increases. Curve gradient is observed to increase drastically as well for *d*/*D* ratio of 0.8 and 1.0. Comparing Figs. 3 - 5, it may be deduced that the flow due to impedance is highly dependent of the elasticity of tube which in turns represents the reactance. Reactance is a measure of resistance to wave propagation. With higher *d*/*D* ratio, the elasticity of tube is higher as explained in the previous section, which in turn will give a lower reactance. It was however noted that as *l*/*L* ratio increases, the fluid viscosity increases as well, to a length where the resistance due to fluid viscosity is larger than the reactance. At this point, the main hindrance to flow is the fluid viscosity while reactance will only be of minor contribution. This explains the maximum flow with optimum *l*/*L* as presented in Fig. 5.

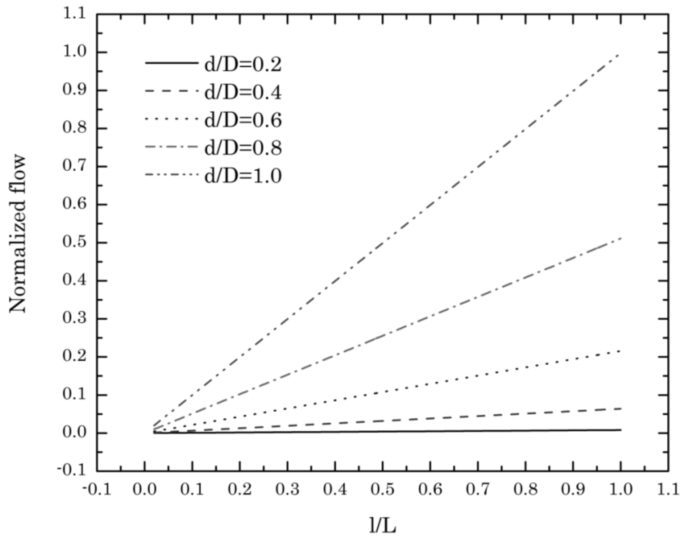

Fig. 4. Normalized flow due to elasticity of tube with respect to tube characteristic length and diameter.

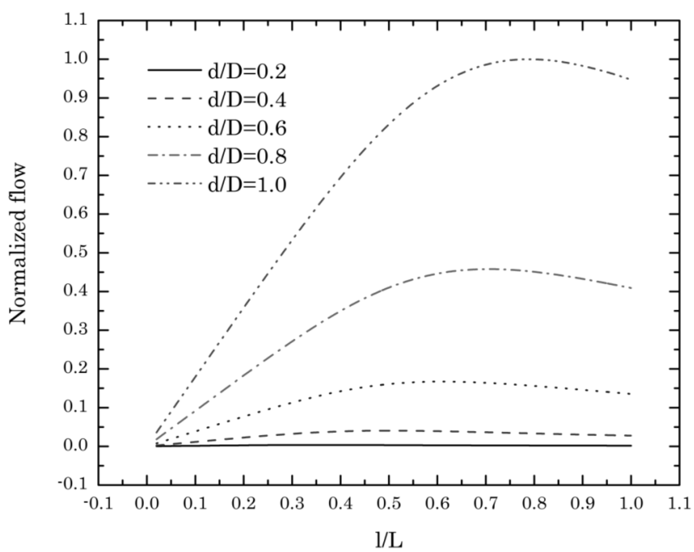

Fig. 5. Normalized flow due to fluid impedance with respect to tube characteristic length and diameter.

Phenomenon observed in Fig. 5 numerically shows an acceptable agreement with the experimental observations [21] of a valveless impedance pump where the optimum non-dimensional location of compression x/L of 0.1 as shown in Fig. 6. The red line indicates the point of comparison where $x/L = 0.1$.

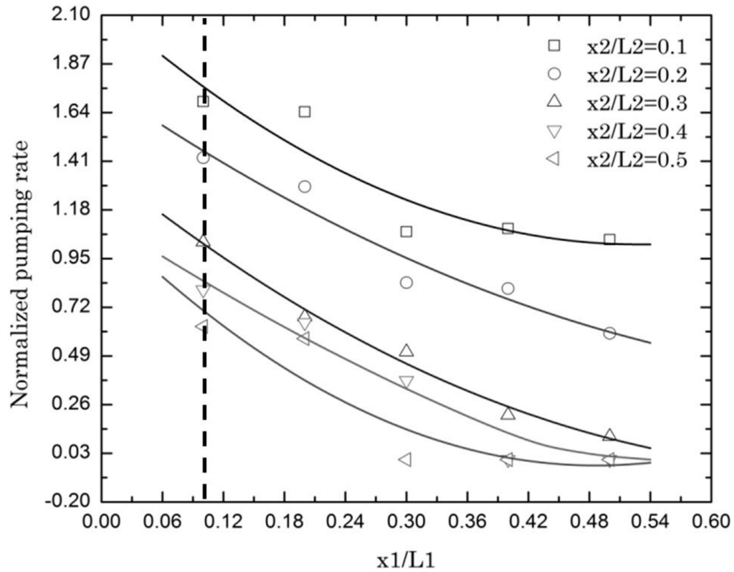

Fig. 6. Pumping rate at angular frequency of 30.16 [21].

With reference to Fig. 2 and the relevance of non-dimensional location of compression of 0.1, optimum location for compression of 0.1 effectively gives a l/L ratio of 0.9. The numerical simulations show a maximum flow induced at l/L ratio of 0.75, which constitutes a percentage difference of 16.7% in comparison to Ref [21]. The observed percentage difference is considered within acceptable range due to the one-dimensional approximation and assumptions made earlier. In order to represent the actual phenomenon, a coupling of the tube structural domain in the y-axis and the flow in the x-axis is required whereas in this paper, the structural domain of the elastic is not included in the simulation.

4. Conclusions

As concluding remark, it is shown numerically that a one-dimensional lumped-system model can be used to study the insight on the effects of fluid impedance onto an oscillating flow in a valveless impedance pump. Although the model is still in an infancy stage, this model gives a preliminary impression of the contribution of fluid impedance in a unidirectional oscillating flow generation. The effect of fluid inertia has shown to be less significant as the length and diameter of tube increases, whereas the effect of tube elasticity increases alongside the two parameters. The combining effects of both hence generated an impedance-driven flow which exhibits highest pumping at only one point along the length of tube. A percentage difference of 16.7% is obtained comparing the numerical results with the experimental findings in [21]. The proposed model can be further modelled with coupling of the structural and fluid domain for future work in better understanding of the fluid impedance in a unidirectional oscillating flow generation.

References

1. Liebau, G. (1954). Über ein ventilloses pumpprinzip. *Naturwissenschaften*, 41, 327. (in German).
2. Moser, M.; Huang, J. W.; Schwarz, G. S.; Kenner, T. and Noordergraaf, A. (1998). Impedance defined flow generalization of William Harvey's concept of the circulation-370 years later. *International Journal of Cardiovascular Medicine and Science*, 1(3-4), 205-211.
3. Borzi, A.; and Propst, G. (2003). Numerical investigation of the Liebau phenomenon. *Z. Angew Math. Physics*, 54, pp. 1050-1072.
4. Hickerson, A.I. (2005). *An experimental analysis of the characteristic behaviours of an impedance pump*. Ph.D. Thesis, California Institute of Technology.
5. Manopoulus, C, G.; Mathioulakis, D. S. and Tsangaris, S. G. (2006). One dimensional model of valveless pumping in a closed loop and a numerical simulation. *Physics of Fluids*, 18, 017106.
6. Loumes, L. (2007). *Multilayer impedance pump: a bio-inspired valveless pump with medical applications*. Ph.D. Thesis, California Institute of Technology.
7. Avrahami, I.; and Gharib, M. (2008). Computational studies of resonance wave pumping in compliant tubes. *Journal of Fluid Mechanics,* 608, 139-160.
8. Bringley, T.T.; Childress, S.; Vandemberghe, N.; and Zhang, J. (2008). An experimental investigation and a simple model of a valveless pump. *Physics of Fluids*, 20, 033602.
9. Ottesen, J.T. (2008). Valveless pumping in a fluid-filled closed elastic tube system: one dimensional theory with experimental validation. *J. Math. Biol*, 46, 309-332.
10. Timmermann, S.; and Ottesen, J.T. (2009). Novel characteristics of valveless pumping. *Physics of Fluids*, 21, 053601.
11. Zamir, M. (2005). *The Physics of Coronary Blood Flow*. Springer Science+Business Media.
12. Peskin, C.S. (2002) The immersed boundary method. Acta Numerica, 479-517.
13. Jung, E. and Peskin, C.S. (2001) Two-dimensional simulations of valveless pumping using the immersed boundary method. *SIAM J. Sci. Comput*, Vol 23(1), 19-45.
14. Jung, E. (2007) A mathematical model of valveless pumping: a lumped model with time-dependent compliance, resistance, and inertia. *Bulletin of Mathematical Biology*, 69, 2181-2198.
15. Lee, J.; Jung, E.; and Kim, D.-W. (2011) A multi-dimensional system of open valveless pumping. In: *8th European Conference of Mathematical and Theoretical Biology and Annual Meeting of The Society of Mathematical Biology*, Krakow, June 2011.

16. McCormack, P.D. and Crane, L. (1973). *Physical Fluid Dynamics*. Academic Press, New York.
17. Tritton, D.J. (1988). *Physical Fluid Dynamics*. Clarendon Press, Oxford.
18. Lee, V.C.-C.; Abakr, Y.A. and Woo, K.-C. (2011) Experimental investigation of multi-stage impedance pumping system. In: Wahid M. A., Samion S., Sheriff J.M., Sidik N.A.C. (Eds), *AIP Conf. Proc. "4th International Meeting of Advances in Thermofluids"*, 1440, 1206-1211. Springer
19. Lee, V.C.-C.; Abakr, Y.A.; Woo, K.-C. and Al-Atabi, M. (2011). Experimental investigation of open-loop multi-stage impedance pumping system. *Journal of Engineering Science and Technology*, 6(5), 551-557.
20. Lee, V.C.-C.; Abakr, Y.A. and Woo, K.-C. (2013). Valveless pumping using a two-stage impedance pump. *Frontiers in Mechanical Engineering*, 8(3), 311-318.
21. Lee, V.C.-C.; Gan, H.-S.; Abakr, Y. A. and Woo, K.-C. (2014) Bulk flow behaviour of a two-stage impedance pump. *Engineering Letters*, 22(2), 53-62.