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EXPLORATION OF THE EFFECT OF THE PENDULUM AND PERPENDICULAR MASS MOVEMENT ON THE SPRING-MASS IN STATIC AND DYNAMIC CONDITIONS USING MEMS

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Abstract

Pendulum oscillation and instability in electrostatic structures supported by micro springs influence the reliability of electromechanical systems. The imbalance resulting from the MEMS voltage change was identified and rebalanced. In the case of instability of the draw through the regulated operating systems, the pendulum's movement was prevented and rebalanced using the mems system and the stability of the draw for the dynamic and static drivers in the system. The current study aims to model and analyse the electromechanical mass stability in the state of rest and motion with the electrostatic current's effect. The phenomenon of withdrawal and threshold were examined systematically for the dynamic and static systems and the theories of nonlinear differential equations. Through the equations, the unstable and critical voltage is determined through the values resulting from the critical voltages, which have a major role in raising and lowering the amount of voltage. Thus, the effect on other system parameters is modelled and examined mathematically. The results show enhanced MEMS accelerometer simulation with variable voltage, mass, spring constant, and pendulum length. The displacement diagram shows that the displacement reached -0.2 meters. At the same time, the proportionality in it decreased from -0.05 to -0.15 in a gradual manner, the change in mass in a sin wave manner, it reached 0.15 kg, where the displacement was irregular, as it began to increase from -0.2 m and end with -0.03 m gradually. The displacement gradient decreased to -0.01 m due to a variable spring constant from 5 to 15 N/m. The final displacement value was -0.007 m by changing the pendulum length from 0.7 mm to 1 mm.

Keywords: Dynamic testing, Mechanical modelling, MEMS (Micro-Electro-Mechanical Systems), Pendulum effect, Static testing.

1. Introduction

Studying the effect of a pendulum and perpendicular mass movement on a spring-mass system using Micro-Electro-Mechanical Systems (MEMS) can provide valuable insights into mechanical systems' behaviour under static and dynamic conditions. MEMS technology allows miniature sensors and actuators to be integrated into a small-scale experimental setup. The spring-mass system is a typical and simple experiment frequently used to initiate the study of simple harmonic movements during the first academic year of graduation in physics [1]. Although the experimental apparatus is simple, an actual spring-mass system behaves as a simple harmonic oscillator only under certain conditions:

i. The spring mass must be negligible in comparison to the attached mass.

ii. The spring elongation caused by the attached mass must exceed the spring equilibrium length by one-fourth [2], and

iii. The initial spring stretch must be strictly vertical.

If these requirements are unmet, the associated mass's motion will not be simple harmonic. It will oscillate from vertical to horizontal in a random way, demonstrating parametric rather than harmonic oscillations [3-5]. A parametric oscillator is a harmonic oscillator with at least one parameter oscillating over time. This modulated component of gravitational force, combined with the elastic force exerted by the spring, causes parametric instability [6, 7] and multimode operation, which drives the system at specific frequencies and generates energy transfer between the two modes [8,] namely the spring-bouncing mode and the pendulum-swinging mode, both of which can be active at the same time. Spring oscillation amplitude decreases when pendular oscillation amplitude increases, and vice versa. The coupling between spring and pendulum oscillations is strongest when the spring oscillation motion frequency is double that of the pendular swinging motion [9-13].

Accelerometers track acceleration or tilt using MEMS pendulums, and by examining their behaviour, one may learn about dynamic systems and damping effects. Adding perpendicular mass movement can have a major influence on MEMS device behaviour. Researchers may change resonance frequencies, vibration modes, and energy transmission by modifying mass distribution inside a microstructure. This is particularly important for applications such as energy harvesters and inertial sensors. Many MEMS devices are built around spring-mass systems. These devices allow for studying oscillatory motion since they consist of a mass coupled to a spring. Adding a pendulum or perpendicular mass can significantly alter the system's resonance behaviour and energy transfer properties. MEMS devices are employed for sensing and actuation in static situations. Designing precise sensors and effective actuators requires an understanding of how pendulum motion and perpendicular mass movement affect static equilibrium. For instance, MEMS accelerometers rely on exact static calibration. MEMS devices are subject to various stresses and vibrations in dynamic environments. The interplay between the pendulum's motion, perpendicular mass displacement, and the spring-mass system can lead to complex dynamic responses. Designing MEMS-based devices that can resist and adapt to dynamic settings requires a thorough understanding of this concept. There are numerous uses for the knowledge discovered while researching the impacts of pendulum motion and perpendicular mass movement in MEMS.

The study mathematically examines the effect on the spring vertical spring-mass system parameters. The instability of the clouds by systematized operating systems, and to be parallel in the mems system, of the clouds stability of the dynamic and static drivers of the system. For that, a mathematical model is developed and coded in MATLAB software to produce graphical results that enhance the understanding of the MEMS.

2. Materials and Methods

Micro-electro-mechanical systems (MEMS) have revolutionized various industries by enabling the development of compact and highly sensitive sensors, actuators, and mechanical devices at the microscale. Among the diverse applications of MEMS technology, understanding the behaviour of spring-mass systems at the microscale is paramount due to its relevance in fields such as microelectronics, biomedical devices, and robotics.

The present work consists of an integrated electromechanical system using a MEMS system as a part and actuators in a minimalist manner. In a different way from traditional mechanical work, semiconductors or MEMS are integrated circuits that work in complete harmony with each other and consist of a microprocessor for the surface, employing modern processing techniques. This is one of the advanced micro-structural elements that exist today, and an improved work has been developed with high features of the electrostatic device, characterized by fast response, low power, and a balanced standard currency.

Its good and accurate design and ease of operation with integrated circuits consist of a single-chip system used by static electricity in electromechanical systems and small sensors. It consists of two parts: a fixed part that acts as a node and a moving part that acts as a cathode and is connected to an electric current. MEMS systems consist of a source of Electric energy, and its principle of operation is that when the current passes, it deforms the moving piece by concave or bending towards the fixed part [5]. Because of this force, the potential difference between the two parties, and these parties reach the highest value between the two, so the system is in the process of critical balance, and this situation is called the drawing mode. The part of the opposite is called the point of withdrawal when touching the moving part, so the voltage value is limiting. When the structure moves on the operating pole, the microscopic configuration is maintained away from the reality it needs.

Moreover, this phenomenon is called the instability of withdrawal, and it is an important nonlinear phenomenon used in electrostatic mems systems and depends on several factors. The draw-down threshold voltage is less than or equal to the voltage used in the electric field of the microstructure used in the mems system and is often unstable in the Cloud system. Linear equations were used.

2.1. Mathematical modelling and methodology

Despite the widespread applications, a significant gap exists in understanding the dynamic behaviour of MEMS-based spring-mass systems, particularly in scenarios involving external perturbations. This research addresses this gap in the effects of pendulum length and perpendicular mass movement on such systems in static and dynamic conditions. To gain insights into the system's dynamics, mathematical models are developed. The general spring-mass motion equation is given in eq. (1). These models incorporate equations of motion, spring-mass relationships, shown schematically in Fig. 1, and damping factors to predict the system's behaviour under different conditions.

$$m\ddot{x} + c\dot{x} + kx = \frac{\varepsilon AV^{\wedge}2}{(d-x)^2} \quad (1)$$

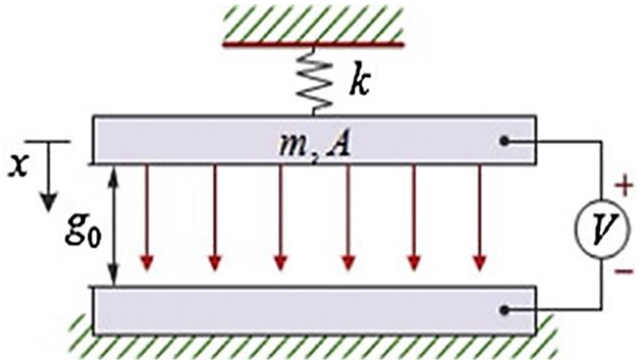

Fig. 1. Diagram of the vertical spring-mass system.

Without damping unit, Fig. 2, the equation could be reduced to:

$$mx + kx = \frac{k\varepsilon AV^{\wedge}2}{(go-x)^2} \quad (2)$$

Where *x* is the displacement of the mass, *m*; *c* and *k* are the system damping and stiffness, respectively. The dielectric constant of the gap domain is denoted, *ε*, and d is the initial capacitor gap, *A* is the cross-section area, and V is the electric load.

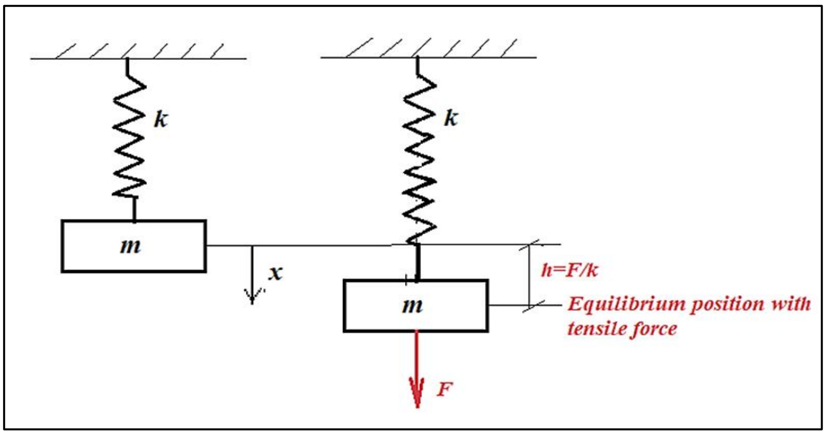

Fig. 2. The power diagram and the vertical movement of the spring.

The characteristics of the drag system and the effect of pendulum motion on it through applying linear equations and kinetic energy equations are investigated through numerical and interlocking analysis, which is a desire for a distinctive and good approach to studying equation solutions [11, 12]. This is used to solve a nonlinear and linear traveling wave and convert it to a stable one. The solutions show that there is a different voltage drop and deviation in the system in particular, which interferes with the voltage drop that defines the corresponding point of the system and many different places of the boundaries that are known to affect the vibrations and the linear behaviour that changes the energy path that converts it into a pendulum oscillatory and to recombine to Linear using electrostatic signals to return the system to its vertical position [13].

The electrostatic actuation mechanism is a starting point. To solve the common problem in this regard, consider a Spring-mass system that is actuated with electrostatic voltage [14, 15].

The methodology sequence to achieve the objectives is,

- Find the equilibrium points of the system.
- Specify the stability of equilibrium points.
- Find the static instability of the system.
- Find the dynamic instability voltage of the system.
- Plot the phase plane of the system for the voltage lower and higher than the pall-in voltage.

V has been changed every time and calculates the amount of change in the surface of the plate that contains (MEMS), which in turn corrects the path of energy from pendulum to linear on the spring and examines the change in *x*, g_0, and *k* every time it installs one of these parameters. It knows the effect of movement on it. This time is fixed, and the other mobile for the system [14, 15]. m is the mass of the moving plate, k is the stiffness of the spring, and *F* is given by [15, 16].

The system, denoted in Figs. 1 and 2, is structured mathematically in the following governing equations.

$$\mathrm{F} = \frac{d}{dx}(\frac{1}{2}CV^2) \quad (3)$$

$$C = \varepsilon\omega(\frac{\omega}{go-x} + \frac{2}{\pi} + \frac{2}{\pi}m(\frac{2\pi\omega}{go-x}) \quad (4)$$

$$mx + kx = \frac{\pi\varepsilon\omega V^2 + 2\varepsilon\omega V\text{^}2(go-x)}{2\pi(go-x)\text{^}2} \quad (5)$$

$g_o = 9.81$, $\mathrm{A} = \frac{\varepsilon\omega\text{^}2}{2m}$, $\mathrm{B} = \frac{\varepsilon\omega}{\pi m}$, $\beta = \frac{k}{m}$, $\mathrm{A} > 0, B > 0$, $\beta > 0$

$$x + \beta x = \frac{A}{(1-x)\text{^}2}V^2 + \frac{B}{1-x}V\text{^}2 \quad (6)$$

$$\frac{dy}{dx} = y, \quad \frac{dy}{dt} = \frac{A}{(1-x)^2}V^2 + \frac{B}{1-x}V^2 - \beta x \quad (7)$$

$$\frac{dy}{dx} = y(1-x)\text{^}2 , \frac{dy}{d\tau} = -\beta x(1-x)^2 + B(1-x)V\text{^}2 \quad (8)$$

$$\mathrm{H(x, y)} = \frac{1}{2}\mathrm{y\text{^}2} + \frac{B}{d\tau}x^2 - \frac{A}{1-x}V^2 + BV^2 \quad (9)$$

$$\mathrm{x} = \frac{2}{3} \pm \frac{1}{3}\sqrt{1} - \frac{3B}{\beta}V\text{^}2 \quad (10)$$

$$\mathrm{V(x)} = -\beta x(1-x)^2 + B(1-x)V^2 + AV\text{^}2 \quad (11)$$

where, $\mathrm{x1} < x2 < x < x3$

$$\mathrm{H(x)} = \frac{2}{3}BV\left(1 + \sqrt{1} - \frac{3B}{\beta}V^2\right) + AV \quad (12)$$

Nonlinear dynamic:

(x > go)_($\varepsilon = gap\ constant$)

$$kx = \frac{\varepsilon AV^\wedge 2}{2(go-x)^\wedge 2} \quad (13)$$

$$F = \frac{\varepsilon AV^\wedge 2}{2(go-x)^2}, \quad (14)$$

$$Vdc = \sqrt{\frac{2kx(go-x)^\wedge 2}{\varepsilon A}} \quad (15)$$

Usually, the nonlinear movement of the movable plate towards the stationary is nonlinear. It depends on the dynamic motion of the plate's behaviour, resulting in the equations and from the work of the moving plate under the influence of the electric voltage. It consists of instability in the clouds depending on the linear equations and the results obtained [17-19].

Dynamic pall-in:

$$kx = \frac{\varepsilon AV^\wedge 2}{2(go-x)^\wedge 2} \quad (16)$$

$$H = \frac{kx^\wedge 2}{2} - \frac{\varepsilon AV^2}{2(go-x)} \quad (17)$$

$$x = \sqrt{\frac{2H}{m} - \frac{kx^\wedge 2}{m}} + \frac{\varepsilon AV^\wedge 2}{m(go-x)} \quad (18)$$

The electromechanical system determines the voltage according to the electrostatic systems that divide the voltage into several branches. Therefore, the system is excellent in terms of dynamics and stability in drag, depending on the results obtained from other parameters, which are m, k, g_0 and V. It is often one of the parameters that are constantly changed and checked. These parameters are constantly changing values, so it installs the rest each time and analyses the numerical values in the dynamic control of the system, which helps to stabilize the traction and stability more. Table 1 shows the MEMS accelerometer parameters.

Table 1. MEMS accelerometer parameters.

Variable	Unit	Value
Mass of the spring-mass system	(kg)	0.1
Spring constant	(N/m)	10
Acceleration due to gravity	(m/s^2)	9.81
Length of pendulum	(mm)	1
Mass of pendulum bob	(kg)	0.05
Amplitude of perpendicular mass movement	(mm)	0.05
Frequency of perpendicular mass movement	(Hz)	1
Maximum simulation time	(s)	10
Time step	(s)	0.01
Damping coefficient	-	0.1
Mass of MEMS-proof mass	(kg)	0.01
Beam stiffness	(N/m)	5
Beam damping coefficient	(N*s/m)	0.02
MEMS accelerometer sensitivity	(V/m/s^2)	0.1
The quality factor of the cantilever beam	-	100
The standard deviation of thermal noise	(V)	1e-5

3. Results and Discussion

The developed MATLAB code generates numerous charts to visualize several facets of the improved MEMS accelerometer simulation. The study of mechanical systems, particularly the effects of pendulum motion and perpendicular mass movement on spring-mass systems in static and dynamic settings, is an interesting use of MEMS technology. This study illuminates essential concepts and useful applications by investigating the complex interaction between mechanical and microelectronic components. Microfabrication techniques are used to build MEMS devices, enabling the development of minuscule but extremely effective mechanical systems. Numerous benefits of this miniaturization include decreased power usage, quicker reaction times, and the capacity to examine intricate processes at the microscale.

A common mechanical phenomenon known as "pendulum motion" is characterized by regular oscillations. This idea is used in MEMS to describe tiny pendulum structures, which have a variety of uses. These include energy harvesting technologies, environmental monitoring equipment, healthcare devices, and precise navigation systems. Solutions based on MEMS are being integrated more and more into daily life.

3.1. Accelerometer simulation results with variable voltage

Accelerometers made of Micro-Electro-Mechanical Systems (MEMS) have become more important in a range of applications, including consumer electronics, medical equipment, and vehicle safety systems. This research provides an upgraded simulation model using variable voltage as a critical component to enhance the precision and adaptability of MEMS accelerometer simulations. This model offers a more accurate portrayal of MEMS accelerometer behaviour under various operating situations by allowing for voltage modifications. Variable voltage's effects on accelerometer performance are examined in the study, and the results are shown in Fig. 3. The results underline the possible consequences of improving sensor calibration and design. Modern electronic systems must use MEMS accelerometers to measure acceleration forces accurately. These devices' design, testing, and calibration depend on accurate simulation.

Traditional accelerometer models normally only consider constant voltage scenarios, while dynamic voltage conditions are frequently present in real-world applications, which affect sensor function. Most earlier models of MEMS accelerometers concentrated on constant voltage levels, ignoring the dynamic nature of many applications. Variable voltage must be used to capture real-world operating situations' nuances and enhance sensor performance. The model enables the analysis of accelerometer behaviour under various voltage situations by allowing for the changing of the input voltage.

For evaluating sensor response in dynamic situations, this capability is essential. Numerous accelerometer features are voltage-dependent, including sensitivity, bandwidth, and noise characteristics. These dependencies are considered in the model, which results in a more accurate depiction of sensor performance at various voltage levels, as in Fig. 3, which illustrates a displacement diagram, demonstrating how the displacement reached -0.2 mm while gradually decreasing proportionality from -0.15 to -0.05.

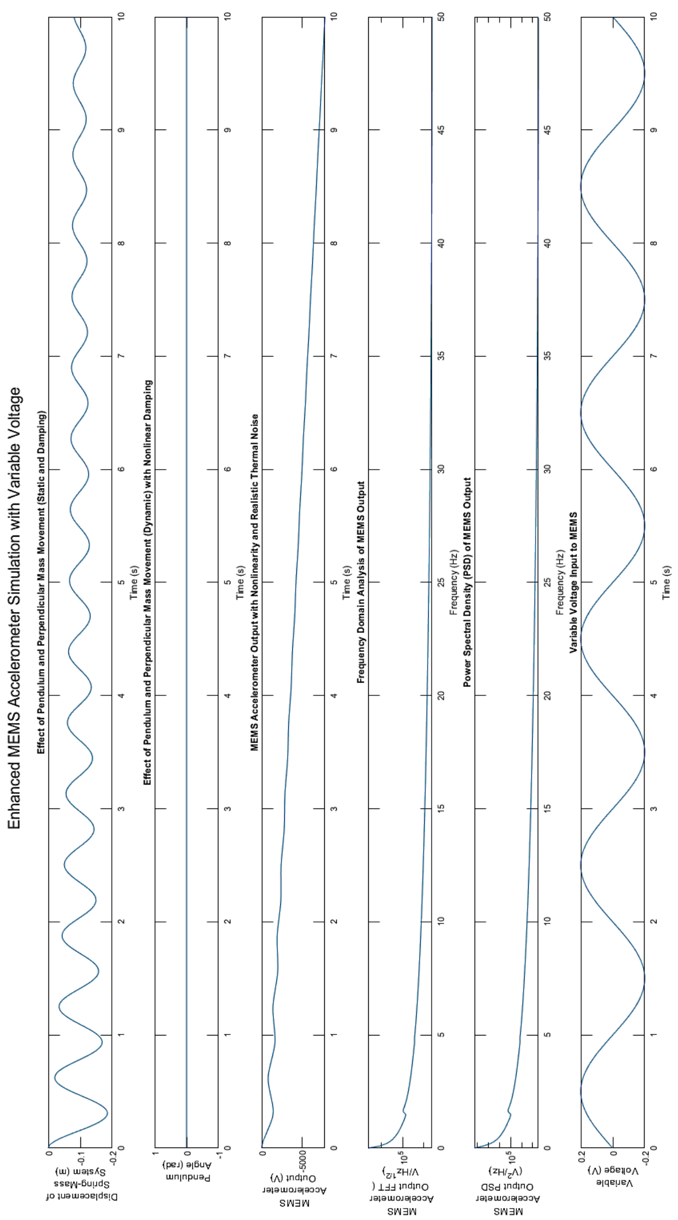

Fig. 3. Enhanced MEMS accelerometer simulation with variable voltage.

3.2. Accelerometer simulation results with variable mass

Variable mass MEMS accelerometers are a potential new sensor technology advancement. Mass fluctuation may be achieved in various methods, including adding or deleting microscale masses from the sensor structure. The primary purpose of the current analyses is to investigate the potential benefits of real-time variable mass adaption of the accelerometer, particularly in situations where traditional fixed-mass accelerometers may be insufficient. The study's findings might influence various disciplines and applications requiring precise acceleration measurements. Wearable technologies, robotics, structural health monitoring, and car safety systems are among the applications for enhanced MEMS accelerometers with variable mass designs. It can enhance the design and optimization of these sensors by better understanding their behaviour through simulation. This will drive sensor technology innovation and broaden the variety of uses for these sensors. These results of the variable mass accelerometer in MEMS are shown in Fig. 4. The results show sin-wave representation of the change in mass demonstrates that it increases steadily from -0.2 m to -0.03 m, reaching 0.15 kg, at which point the displacement was irregular.

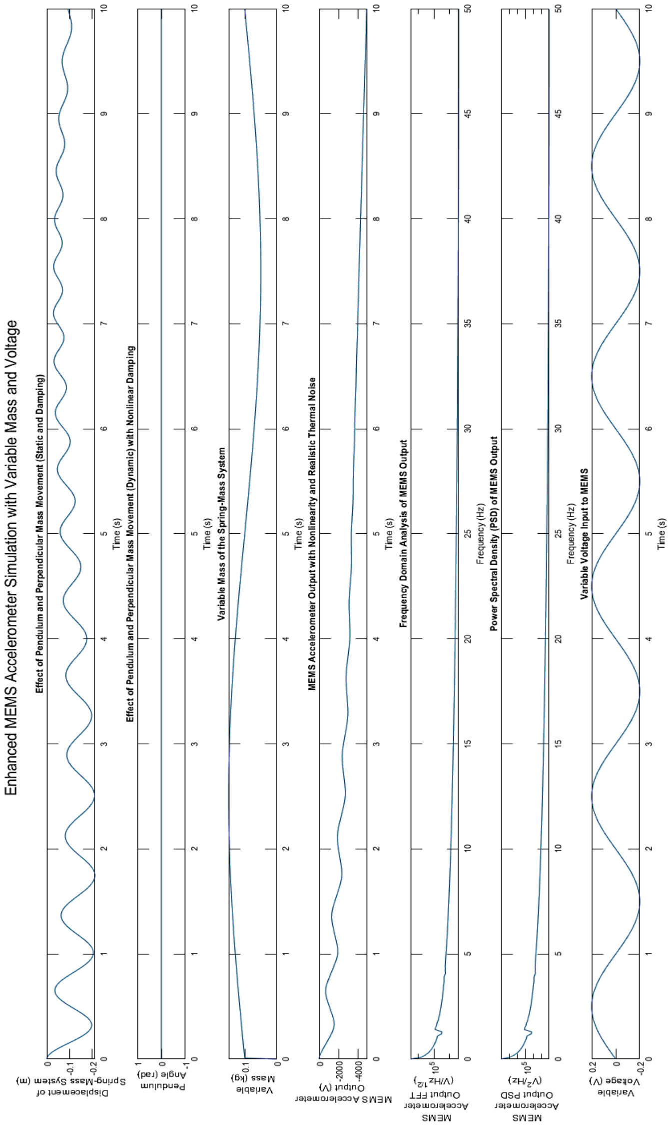

Fig. 4. Simulation results of MEMS accelerometer with variable mass.

3.3. Analyses of MEMS accelerometer with variable spring constant

Conventional MEMS accelerometers have fixed spring constants in their sensor elements which limits their ability to adapt and perform effectively in a variety of operating conditions. The concept of MEMS accelerometers with changeable spring constants is a creative and intriguing new avenue in sensor technology. By adding the capacity to dynamically adjust the spring constant inside the accelerometer architecture, these sensors may be tweaked to improve their performance and flexibility in various environments and applications.

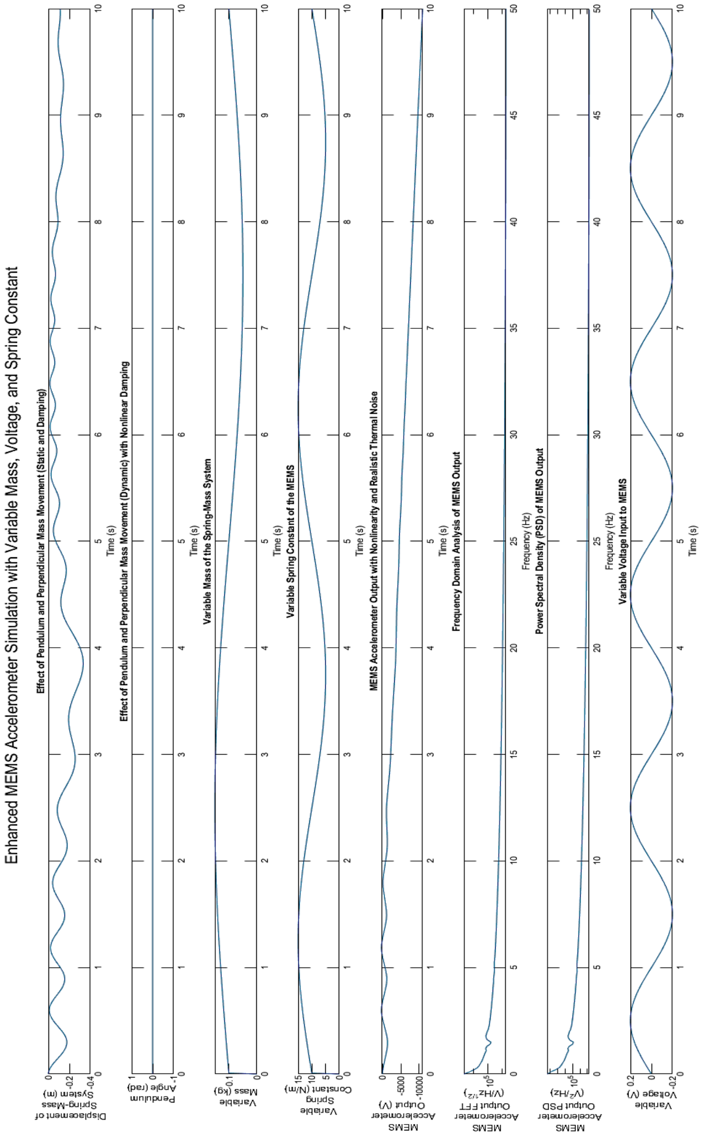

Fig. 5. Simulation results of MEMS accelerometer with variable spring constant.

Advanced microfabrication techniques can be used to obtain the spring constant variability, enabling real-time modifications that are suited to operational requirements. This study investigates the enhanced MEMS accelerometers fitted with variable spring constants through simulation and assessment. The results are shown in Fig. 5. The main goal is to look at the benefits and capacities of these flexible sensors, especially in situations where more conventional fixed-spring accelerometers may have constraints. The underlying physics guiding their behaviour, the fundamental operating principles, and the details of spring constant adjustment have been predicted and presented in Fig. 5. The simulation results illustrate how the presence of a varied spring constant ranging from 5 to 15 N/m caused the displacement gradient to drop to -0.01 m.

As a result, this research offers new opportunities for enhanced performance and adaptability in dynamic and demanding conditions, helping realize MEMS accelerometers' full potential with variable spring constants, as shown in Fig. 5.

3.4. Analysis of MEMS accelerometer with variable pendulum length

The effect of Spring-Mass system movement (Static and Damping) over time is depicted in Fig. 6. It shows how the system reacts to damping and the impacts of changing the mass, spring constant, and pendulum length. Dynamic Pendulum Angle with Nonlinear Damping This graph shows the pendulum's angle with time. It demonstrates the nonlinear damping effect and how variations in the pendulum's length affect its motion. This graph shows how the spring-mass system's mass fluctuates sinusoidal over time. It shows how the dynamics of the system are impacted by changing mass.

The variable Spring Constant of MEMS effect on the system's dynamic and static behaviour is provided in Fig. 6. This graph displays how the MEMS devices' spring constant changes over time. It emphasizes how the sinusoidal variation in the MEMS spring is constant over time.

Variable length of the pendulum. The variation in pendulum length is shown in this graph. It shows how the length of the pendulum varies sinusoidal over time. MEMS Accelerometer Output with Realistic Thermal Noise and Nonlinearity The output of the MEMS accelerometer is seen in this plot. It offers an understanding of the behaviour of the sensor by including nonlinearity in the MEMS model and accounting for realistic thermal noise. MEMS output frequency domain analysis, a frequency domain study of the MEMS accelerometer output, is shown in this plot. It displays the MEMS output signal's amplitude spectrum, which shows which frequencies are present. MEMS Output Power Spectral Density (PSD) The MEMS accelerometer output's power spectral density (PSD) is seen in this plot. It details how the signal's power is distributed among its frequency components.

The fluctuating voltage signal sent to the MEMS device is depicted in this plot. It stands for the source of sinusoidal voltage that impacts the MEMS output. These plots together offer a comprehensive view of the simulation, allowing us to analyse the effects of variable mass, variable spring constant, variable pendulum length, and other parameters on the MEMS accelerometer's behaviour. It can further modify the simulation parameters to investigate various scenarios and see how these elements affect the operation of the accelerometer. Figure 6 demonstrates how increasing the pendulum's length from 0.7 to 1 mm resulted in a final displacement measurement of -0.007 mm.

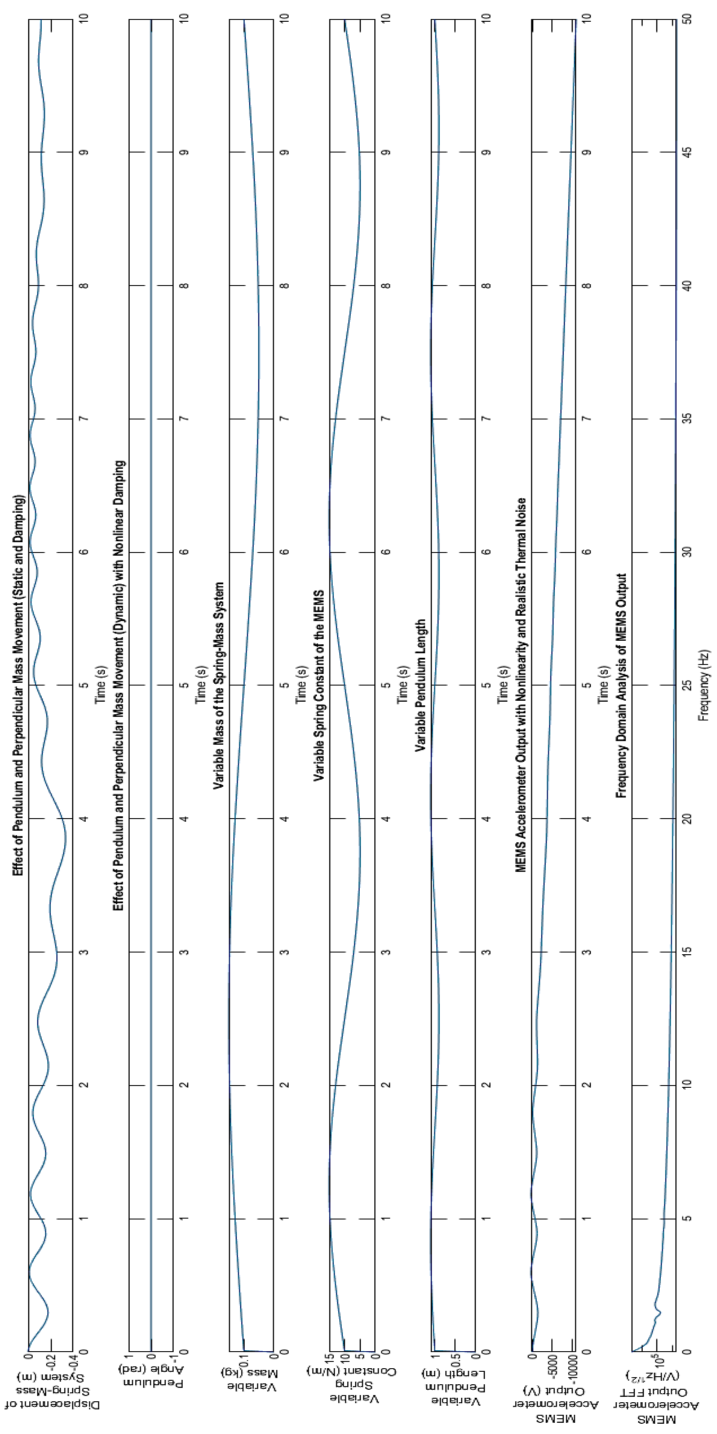

Fig. 6. Simulation results of MEMS accelerometer with variable pendulum length.

4. Conclusions

The investigations into the dynamics and stability of MEMS-based electromechanical systems under the influence of various parameters, including mass (m), spring constant (k), and other variables, have provided valuable insights and outcomes. It has been observed that the system exhibits commendable dynamics while maintaining stability, a vital attribute in practical applications where precise control and reliable operation are essential.

The system's excellent performance in handling voltage modulation underscores its potential utility in various electromechanical applications, particularly in scenarios where dynamic stability is paramount. The system's adaptability to different motion configurations, combined with the fact that pendulum motion causes a variation in spring elongation by the appended mass (i.e., energy is exchanged between the two modes independently of the spring and pendulum frequency ratio), lends support to the system's classification as a parametric converter.

The displacement reached -0.2 meters, while the proportionality gradually fell from -0.05 to -0.15. The mass change follows a sin wave pattern, reaching 0.15 kg, while the displacement follows an erratic pattern, gradually increasing from -0.2 mm to -0.03 mm. A variable spring constant ranging from 5 to 15 N/m lowered the displacement gradient to -0.01 mm. The final displacement measurement was -0.007 mm after increasing the pendulum length from 0.7 mm to 1 mm.

As it moves forward, further investigations into the intricacies of parameter manipulation and developing advanced control strategies will yield even more refined and optimized systems. It remains committed to pursuing innovative electromechanical solutions that can address the evolving needs of modern technology and industry.

Nomenclatures

A	1. Angular displacement, m
	2. Amplitude of pendulum oscillations, 1/s
a	Acceleration of the mass, m/s^2
F_{damp}	Damping force, N
F_{ext}	External force applied to the system, N
F_{fric}	Frictional force, N
F_{perp}	Force due to perpendicular mass movement, N
f	Frequency of pendulum oscillations, Hz
k	Spring constant, N/m
m	Mass of the system, kg
T	Period of pendulum oscillations, s

Greek Symbols

ω	Angular velocity of pendulum

Abbreviations

MEMS	Micro-Electro-Mechanical Systems
SHM	Simple Harmonic Motion

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