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CHARACTERISTICS OF STUDENTS' ANSWER IN SOLVING ABSOLUTE VALUE INEQUALITY PROBLEMS BASED ON MATHEMATICAL UNDERSTANDING

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Abstract

The purposes of this study were to describe students' answers in resolving absolute value inequality in one variable based on mathematical understanding and to describe problems faced by students in the problem. Data were collected by tests, interviews, and documentation to four 10th graders. The results showed that at the level of instrumental understanding, the characteristics of participants' answers were: 1) using the correct concept at the beginning but unable to complete it thoroughly, 2) conducting algebraic operations, 3) less able to associate prior knowledge, and 4) less able to use mathematical symbols correctly. In addition, some difficulties faced by students were difficulties in finding solutions to problems because they do not have a strong understanding, difficulties in finding the relationship of absolute value with the concept of inequality, and difficulties in symbols in absolute values consistently. At the level of relational understanding, characteristics of participants' answers were: 1) using concepts to solve problems thoroughly, 2) conducting algebraic operations, 3) able to associate previous knowledge, 4) using assistance such as number lines or graphs to solve problems, 5) less able in using mathematical symbols consistently. The difficulty experienced by participants is to use symbol values consistently. The recommendation for further study is that researchers can carry out similar studies with more participants and different methods.

Keywords: Characteristics of answer, Instrumental, Relational understanding.

1. Introduction

Students must have a good understanding of concepts in order to be able to use mathematics as a tool to communicate their ideas. Understanding is a process that connects the components of ideas that are understood by individuals [1]. Students can understand a concept if the students know the concept and its relation to other concepts. For example, in mathematics learning, students who have an understanding of the concept of numbers will easily master the concept of number sets because they are able to make relations between the two concepts.

Mathematical understanding is an important ability that must be developed in classroom learning. It is defined as the internal relationship of conception of mathematical ideas, facts, and procedures. Most learning theories state that the purpose of the learning process is understanding the material or concept being taught. It should be noted that the accuracy of the teacher in providing an assessment of students' conceptual understanding of mathematics is also important [2]. Mathematics learning in classes should be directed at the purpose of understanding concepts, mathematical principles, and not just memorizing. Important aspects in the learning process of mathematics are: understanding, using and manipulating abstract symbols meaningfully [3]. The ability of mathematical understanding will always be solving mathematical problems and other sciences in everyday life. Some examples of problems in everyday life that require mathematical understanding are; a project leader who must be able to manage so that there are not many unemployed workers, construction workers who must understand the concepts of parallel and perpendicular so that the building will not be tilted and easily collapsed, bankers who must be able to understand statistical concepts, and many others. It is clear that the ability of mathematical understanding is needed not only in mathematics lessons in the classroom but also in everyday life.

Facts show that the ability of students' mathematical understanding is not optimal. In the observations on junior high school students in Semarang, it was found that several students have not been able to determine the elements which are recognized in the problem if the problem is in the form of stories. They have not been able to choose and apply concepts to solve problems and find it difficult if the questions given are slightly different from the previous example. The result of Trends in International Mathematics and Science Study (TIMSS) in 2015 shows that Indonesian students' understanding concepts and the ability to solve math problems were still very weak, but it is good enough in solving facts and procedures [4]. In addition, empirical evidence of TIMSS shows that several students in the world do not have optimal mathematical understanding [5-7]. Thus, the topic of mathematical understanding needs to get even more attention.

Research on mathematical understanding skills has been carried out. However, there is still little or no focus on the material of absolute value inequalities. In addition, the material of absolute value inequalities is considered difficult by most students [8]. Existing research on absolute value inequalities is associated with difficulties faced by students. Some studies focusing on this area were about analysis of student difficulties in solving problems of absolute value inequality [8, 9]. The discoveries about this difficulty attract researchers to concentrate more on the mathematical understanding of students. Based on this, the researcher conduct research that focuses on the characteristics of students' answers including mistakes made in solving problems of absolute value inequality in terms of students' mathematical understanding. The abilities of mathematical understanding which are used in this study are instrumental and relational understandings [10].

2. Theory of Absolute Value Inequality

In everyday life, the concept of absolute value plays a role in solving problems related to distance. For example, we want to calculate the distance between one city and another city, or the distance between two specific places, we can apply the concept of absolute value. Distance always has positive values. In other words, the distance measurement between two places is never negative. In particular, in English to give things whose value is always positive is given an understanding that we often call "price". So, the price which really matches the prevailing concept is always positive. Mathematically, the meaning of the price of each real number *x* is written with symbol *x*.

The absolute value of a real number x has to do with the concept of distance geometrically from x to 0. Now we consider the explanation for the distance in the number line as shown in Fig. 1.

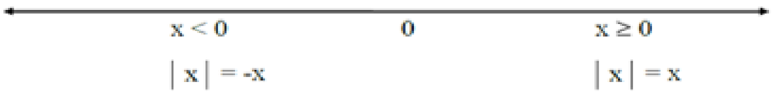

Fig. 1. Illustration of absolute value.

Conclusions obtained:

The distance x to 0 = x, if x ≥0 and will be become -x, if x<0.

For example: the concept of absolute value described on Fig. 2,

Fig. 2. The example of absolute value.

Inequality is a very important topic both in theory and in working on mathematical questions, including its relation to absolute values. An absolute inequality is an always true inequality for each variable substitute value. Absolute value inequality is often also called inequality and of course this inequality is a closed mathematical sentence.

3. Method

This research was a qualitative study with a case study approach. In this study, the researchers described the characters of students' answers in solving problems of absolute value inequality in terms of students' mathematical understanding. The research was conducted in one of the high schools in Indonesia. The study was conducted in one class of students in 10th grade who had learned material of absolute value inequality. The class consisted of 30 students (12 males and 18 females).

All students were given a problem-solving test then four students were selected for interviews. The selection of research subjects to be interviewed was based on the level of mathematical understanding of students and also based on recommendations from the mathematics teacher who taught at the school. The selected subjects were subjects with instrumental and relational mathematical understanding, two students for each level of understanding. Selected subjects consisted of a male and female for each type of understanding. Selected subjects have early mathematical abilities that are in the middle of class achievements.

The data in this study were collected through problem solving tests, interviews, and documentation. Data analysis was carried out by using a componential analysis technique. There were two components that will be analysed in this study, namely the instrumental and relational understanding. Each component has its own characteristics. Students who had instrumental understanding will have different characteristics with students who had relational understanding in mathematics problem solving. This was suitable for using a componential analysis technique that produced characteristics of each component rather than looking for similarities.

The research step was a problem solving test for students consisting of two items. The material chosen was the absolute value inequality. An interview was held to the participants and teacher to check whether the subject in question presented their abilities well in the tests given while also digging in depth information about the understanding of the subject in question. The interview process was recorded and documented the results.

4. Results and Discussion

The data obtained comes from material tests of absolute value inequality, interviews, and documentation. Participants involved in this study were four students (S1, S2, S3, and S4). The results of the study are presented in the form of drawings and an explanation of how the characteristics of students' answers in solving problems of absolute value inequality are seen from their understanding. As explained earlier, the ability of mathematical understanding is divided into two namely instrumental and relational understanding [11]. So as to facilitate the presentation of the results and the discussion it will be divided into two sub, namely the sub section of instrumental and sub-section understanding of relational understanding.

4.1. Instrumental understanding

S3 answers the inequalities of the absolute value given to find a solution. However, the solution given by S3 was still incorrect (see Fig.3). Based on S3 answers to the number one of absolute value inequality, it can be noted that S3 has completed the problem until he got the final result. In the problem solving process, S3 tended not to pay attention to the rules of writing in mathematics, even though this was very significant on the subsequent settlement process. In the red box in Fig. 3, S3 should write an absolute value sign, not just brackets. This clearly means different meanings between absolute value symbols and parentheses. Through symbols, someone is able to explain and communicate the idea [10].

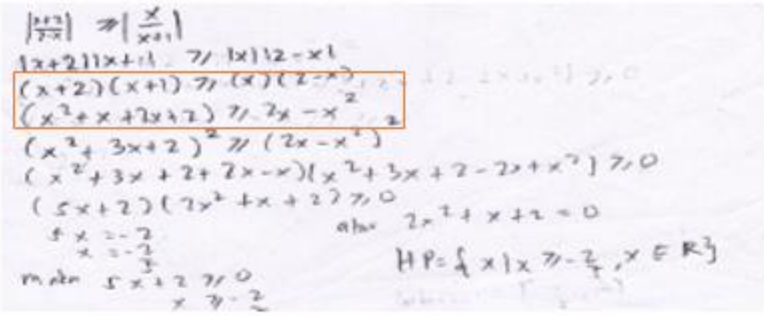

Fig. 3. Answer of S3 number 1.

According to Fig. 3, it turned out that S3 was still able to continue up to the next procedure which was eliminating absolute values by squaring two segments even though he had not been able to interpret the symbols. After squaring and counting, S3 was able to write the result to the simplest form: $(5x + 2)(2x^2 + x + 2) \geq 0$. This indicated that S3 was only mechanical in solving the problem given. It means that S3 was only able to solve problem in accordance with the procedure exemplified by the teacher without doing the manipulation process.

After obtaining the form $(5x + 2)(2x^2 + x + 2) \geq 0$, S3 continued the completions process by finding a zero maker in both equations. For the equation $(5x + 2)$, S3 was able to get it. But for the equation $(2x^2 + x + 2)$, S3 did not explain anything and immediately wrote the set of resolutions. This indicated that the scheme of the concept of equation owned by S3 was still weak, so S3 has not been able to explain in a whole step by step of problem solving. The set of solutions that have been written have not yet shown the correct solution to the problem. S3 did not pay attention to the conditions that must be met from the problem solution. The initial question was in the form of fractions, so the S3 should be able to use the scheme that it already has in relation to fraction resolution, namely by paying attention to the denominator of the fraction in order not to be zero. Fractions are often considered students as two separate numbers, not as a ratio between numerators and denominators [12]. This misinterpretation directs the error in solving inequalities in the form of fractions. The result of the observations by calculating the value of half of 1/8 are only 47% of 6^{th}, 8% of 4^{th}, and 63% of 8^{th} grade students were accuratelyable to answer these questions correctly [13]. This greatly affects the scheme that students build to deal with problems or subsequent concepts. Overall, it means that S3 had not been able to link the schemes that she had and even the scheme was still weak. This indicates the scheme that S3 had was weak and may be a memory that was only stored in short-term memory. The S3 completion process in the next question had the same characteristics.

Then, S4's answer in question number one was slightly different from S3's. On the S4's answer sheet, it appeared that S4 has not been able to complete the problem solving process until it finds a solution to the problem given. It can be seen in Fig. 4.

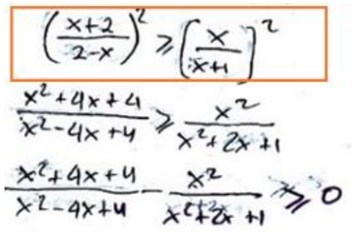

Fig. 4. Answer of S4 number 1.

Unlike S3, S4 did not use cross-multiplication first. But S4 immediately eliminated absolute value by squaring it. But this actually made S4 confused to determine the next step in solving the problem. S4 was only able to express ideas to manipulate absolute values into simple forms. Similar to S3, S4 has also not been able to link the scheme associated with the requirements for fraction formation, that the denominator cannot be zero. The scheme owned by S4 was so weak that she had not been able to be used for the basis of the next scheme. If students have formed abstract thought schemes since childhood in certain cultural environments, they will have a foundation for mathematical ideas [14]. This mathematical idea which later developed into mathematical knowledge. Mathematical knowledge is obtained based on a combination of deduction and mathematical intuition according to the orthodox concept [15]. Orthodox concept is a commitment or agreement on a true standard concept in mathematics, which applies in general.

The difficulties faced by subjects with instrumental understanding in solving problems of absolute value inequality are difficult subjects to find solutions to problems because they do not have a strong understanding, it is difficult to find absolute value relationships with the concept of inequality, and it is difficult to use absolute value symbols consistently. This finding is in accordance with previous research which found that students' difficulties in solving absolute value problems cannot fully apply the concept of absolute value so that they cannot solve problems thoroughly, misunderstand absolute value requirements, misunderstandings of absolute values are always positive, experience conceptual, procedural difficulties, and algorithmic [8, 9]. The relationship with the characteristics of students in answering the problem of absolute value inequality, namely the difficulties of students can be seen from the characteristics of students in answering problems.

S3 and S4 have an instrumental understanding. Students who have an instrumental understanding have the characteristics of being able to memorize things separately, being able to apply concepts to problems that are routine or simple, completing things procedurally, and often referred to as mechanical understanding [10]. From the results of the subject's answers to the questions given, it appears that the subject belongs to the level of instrumental understanding. Subjects were able to apply the concept separately, namely applying the concept of absolute value, but were less able to relate it to other concepts such as the concept of fractions, and inequalities. Therefore, the characteristics of the answers were writing according to the initial procedure of working on the concept of absolute value without using the scheme that was owned so that they were unable to solve the problem thoroughly. In addition, in the answer to the question the subject has performed algebraic correctly, but still not able to use mathematical procedures correctly.

When interviewing the related subject why the subject did not solve the problem given, the subject explained that he did not remember how the next step. It has been explained that remembered knowledge is a result of learning [16]. Memory plays an important role in mathematical understanding. But keep in mind that the memory of someone who understands mathematics is different from those who do not. From the answer of the subject above, it can be seen that the subject does not fully have mathematical understanding. This indicates that the information or mathematical concepts he received were only in short-term memory only and were not transmitted to long-term memory. The concept of memory is very important in solving mathematical problems especially.

Many factors cause student difficulties in solving problems of absolute value inequality including factors from inside and outside [9]. Internal factors are related to learning motivation and lack of positive attitudes towards material inequalities in absolute values. External factors are learning processes that are less attractive and do not use props to facilitate the delivery of material concepts inequalities of absolute value. Meanwhile, the cause of the difficulty is that students only memorize without understanding the concept of absolute value and are accustomed to using fast formulas [8]. One of the difficulties of students in completing the absolute value inequality is caused by a lack of knowledge of the preconditions related to inequality of squares and rational inequality.

4.2. Relational understanding

Relational understanding is owned by S1 and S2. S1 answers for question number one can be seen in Fig. 5. It was very different from the answers of subjects who have instrumental understanding since S1 answers were more complete and correct. S1 had been able to use mathematical procedures correctly. S1 wrote absolute marks correctly as seen in Fig. 5.

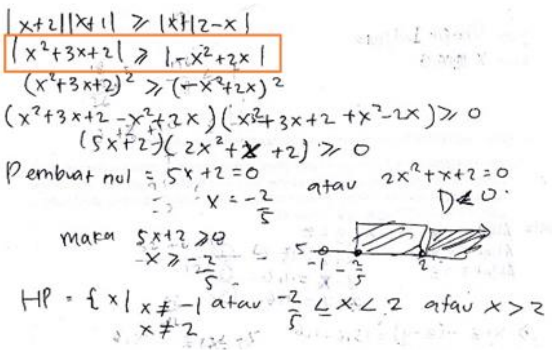

Fig. 5. Answer of S1 number 1.

Based on Fig. 5, S1 really understands the concept of absolute value that he has. S1 had been able to associate his knowledge of absolute value with the knowledge he had before. S1 associated the discriminant value to determine the root of the equation. S1 had also been able to determine whether the discriminant is positive or negative. S1 wrote D <0. This means that S1 has mastered the concept of how the formula looks for discriminant what is the use of the discriminant value.

Then further S1 used a number line to help him find a solution set of questions. In writing the settlement set, S1 was quite careful by paying attention to the requirements so that the inequality has a value. S1 wrote the conditions at the front, namely $x \neq -1$, $x \neq 2$. This indicates that S1 fully understands the usefulness of these conditions. Some understanding becomes formal in the process so that it can be accessed automatically [17]. From the answers that written by S1, he did not evaluate the set of solutions. S1 did not add the terms he had written at the beginning of the answer. This indicates that the S1 level of accuracy is still not optimal. After an interview with the subject, the subject acknowledged that he forgot to include the conditions that should be written down.

S2 characters in the problem solving process given were almost similar to S1. S2 had also solved the number 1 problem that he gave as appeared in Fig. 6.

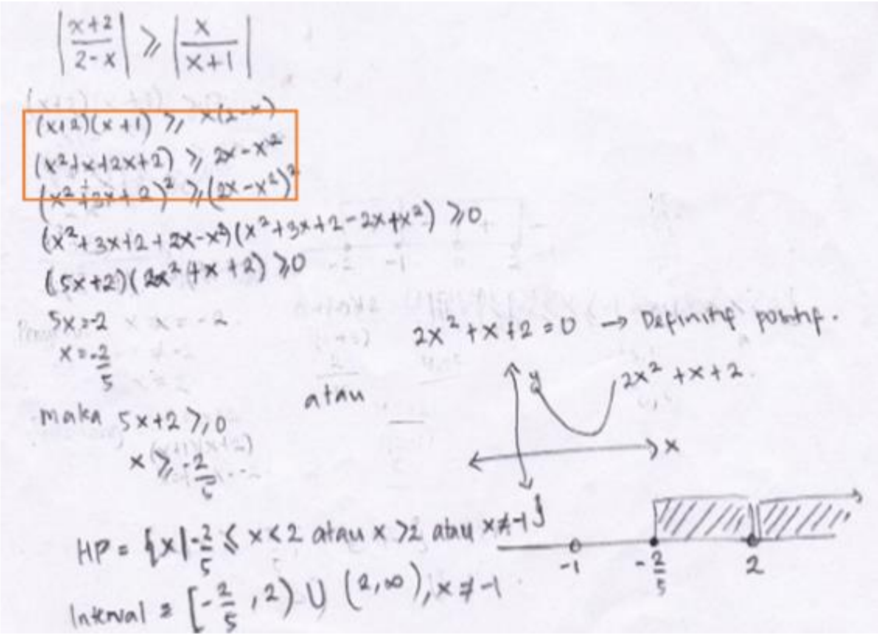

Fig. 6. Answer of S2 number 1.

The concept used to solve the problem was the concept of absolute value. S2 has been able to use the concept of absolute value correctly. However, S2 was still not disciplined in writing the symbol of absolute value, S2 was still able to solve the problem correctly. This indicates that S2 mathematics procedures are weak, but understanding the concept is quite high. Therefore, the problem was possibly solved. The language of mathematics is significantly related to numeracy skills [18]. After an interview with the subject, it turns out that the subject explained that there was indeed a slight mistake in writing the symbol of absolute value, but he also mentioned that he fully understood what he had written. Misunderstanding of mathematical symbols is one of the factors that influence the decline in students' mathematical performance [19].

S2 was very familiar with the discriminant concept and positive or negative definite. In the answer sheet, S2 explained that the equation he found was positive definite. S2 also used knowledge or concepts that he had before. S2 drew curves to make it easier to analysis the findings regarding positive definitions. This indicates that the S2 has carried out the process of recalling memory that has been in mind. This also indicates that S2 has a strong scheme. If the initial scheme owned by an individual is incorrect, it will be difficult to assimilate subsequent ideas and even the worst possibility is impossible to happen [10]. So, it can be concluded that strengthening the scheme is very important in learning for 1) helping in integrating knowledge and understanding, 2) helping to work with others, and 3) helping the individual's self-growth [10].

The scheme possessed by subjects with relational understanding is quite strong. In this case, the understanding is owned by S1 and S2. As for understanding and developing concepts related to rational inequality, students must understand rational numbers well [20]. Students with relational understanding are better in recognizing and using symbols in terms of the meaning of the symbol "=", construction of pre-algebraic problems, and using advanced problem solving strategies [21]. Difficulties experienced by subjects with relational understanding are difficult to use symbols of absolute values consistently. Based on the results of the interview, it can be concluded that the mistakes made because the subject felt that the teacher would immediately understand what they wrote without having to pay attention to the rules of writing symbols. The difficulties experienced by the subject with this relational understanding were due to the habitability factor. When the subject resolved a similar problem in the class, he was accustomed to ignoring absolute value symbols. The habit was carried onwards. Even though the teacher will be able to know the level of analysis, time of sequence, and basic level of detail as an effective builder of mathematical persuasive, argumentative and informative explanations based on students' mathematical writing [22].

5. Conclusions

Subjects who have instrumental understanding, in problem solving process related to absolute value inequality, tended to have particular characteristics, namely: 1) using the correct concept at the beginning but unable to complete it thoroughly, 2) conducting algebraic operations, 3) less able to associate prior knowledge, 4) less able to use mathematical symbols correctly. This is because some difficulties faced by the subjects, such as difficult to find solutions to problems because they do not have a strong understanding, difficult to find the relationship of absolute value with the concept of inequality, and symbols that are difficult to use absolute values consistently. Meanwhile, the subjects at the level of relational understanding, characteristics of participants' answers were: 1) using concepts to solve problems thoroughly, 2) conducting algebraic operations, 3) able to associate previous knowledge, 4) using assistance such as number lines or graphs to solve problems, 5) less able in using mathematical symbols consistently. The difficulty experienced by participants is to use symbol values consistently. For further studies, researchers are recommended to conduct similar studies with more participants and also create a learning design that makes it easier for students to learn the material in absolute value inequality so that the difficulties faced by students can be reduced or even disappeared.

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