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DEVELOPING MATHEMATICS INSTRUCTIONAL MATERIALS TO ENHANCE STUDENTS' ABILITY OF MAKING GENERALIZATION

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Abstract

As ulama cadres (as Islamic scholars), students of *pesantren* (Islamic boarding schools) are expected to demonstrate reasoning skills. One of the reasoning skills that is necessary to be developed is mathematical generalization. Through a design-based research procedure, this study attempted to develop mathematics instructional materials specialized to enhance *pesantren* students' ability to make mathematical generalization. The instructional material development procedure consisted of three stages: self-evaluation, prototyping, and field test. The results revealed that the developed instructional materials enhanced the subjects' ability in making generalization. They managed to recognize a rule/a scheme and to able to perceive or identify models. In addition, they became aware that the problems presented can be solved using rules/patterns. They also acquired an ability to describe a rule/pattern both numerically and verbally and to use the results of generalizations to solve the given problems. However, many students failed to produce a rule and a general scheme. Based on our results, four processes of mathematical generalization process that must be considered are perception of generality, expression of generality, symbolic expression of generality, and manipulation of generality.

Keywords: Generalization, Islamic boarding school students, Mathematics instructional development.

1. Introduction

Islamic boarding schools (in Indonesian commonly referred to as *pesantren*) are there to prepare students to become the cadres of ulamas (Islamic scholars) or mubaligh (preachers) [1], one of whose responsibilities is to issue fatwas on various issues [2]. Therefore, they are required to demonstrate reasoning skills as they play a very important role in the process of ijtihad, which is a process of making a legal decision by independent interpretation of the legal sources such as Quran and hadiths [3, 4]. These reasoning skills include making deductions, analogies, inductions, and conclusions important to make argumentation in the Islamic-making law process [5-8]. Students' reasoning skills can be enhanced through learning mathematics [9].

Mathematical reasoning is making conclusions based on mathematical characteristics [10]. It is the ability to formulate and represent a given mathematical problem, then explain and justify a solution or an argument [11]. Mathematical reasoning is classified into two types, namely inductive reasoning and deductive reasoning [12]. Inductive reasonings (drawing conclusions based on observed data) include (i) transductive, (ii) analogy, (iii) generalization, (iv) estimating answers, solutions or trends, interpolation, and extrapolation [8].

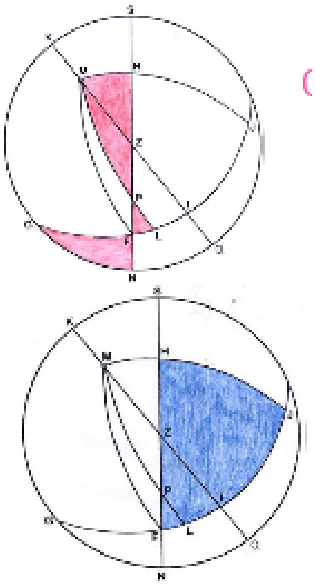
Generalization is among the most important reasonings, which is a type of argumentation known in Islamic law as istiqra, an activity of a process of analysis of particular things in order to draw a general and universal conclusion [13]. Generalization encourages students to look for situations that they can apply in various contexts [14]. If students are not trained to express generalizations, mathematical reasonings will not be enhanced [15]. Based on the authors' observation, mathematics instruction at Islamic boarding schools has not helped students develop the ability to make mathematical generalization. Therefore, this study attempts to develop mathematics instructional materials to address this issue.

2. Literature Review

2.1. The role of mathematics in muslim life

Mathematics has a very important stance in Islam. Quran considers the rules of mathematics as a way to find God's will [16]. For example, mathematics is required for determining the likes of prayer schedules, Qibla direction, the first day of Ramadan, Eid al-Fitr, Eid al-Adha, and so on [17]. As a case in point, al-Biruni used a formula illustrated in Fig. 1 to determine Qibla direction.

Mathematics is not alien to Islam. Muslim scientists played a very crucial role in the development of mathematics in the Middle Ages. Fibonacci even acknowledged that he was influenced by Islamic scholars, especially al-Khwarizmi [18, 19]. Among Muslim scholars who contributed to the development of mathematics were al-Khawarizmi, al-Kashi, Kayyam, and al-Kahzin. Al-Khawarizmi discovered that a unit was a number, a root was x, and a square was x^2. Al-Kashi found the value of the constant pi (π) = 3 ° 8 " 29 "44". Converting the value of the constant to the decimal system, he arrived at a conclusion that π = 3.141593. The work of al-Khazin is believed to have been motivated by the work of Al-Khujandi who claimed to prove that $x^3 + y^3 = z^3$ is impossible for all numbers x, y, z which of course is $n = 3$ cases of Fermat's Last Theorem [16].



$$(1)\ \sin\theta_1 = \sin(\Delta\lambda)\cos\varphi_M$$

$$(2)\ \theta_2 = \varphi_z - \sin^{-1}\left(\frac{\sin\varphi_M}{\cos\theta_1}\right)$$

$$(3)\ \cos\theta_3 = (\cos\theta_1)(\cos\theta_2)$$

$$(4)\ qibla = 90^0 - \left[\frac{\cos\theta_1\sin\theta_2}{\sin\theta_3}\right]$$

Fig. 1. Three uses of law of sinus and one use of rule of four quantities. Picture from Schwartz [17] p.13.

2.2. Generalization

Generalization is a type of reasoning that attempts to broaden the denotation from a concept [14]. Generalization also attempts to find general characteristics of a problem as well as general solutions to the problem in question. Generalization is a process of reasoning to draw a conclusion [19]. Generalization is a reasoning process to make a general conclusion of premises in the form of empirical proportions [20]. Generalization is also said to be the heartbeat of mathematics and appears in various forms. If the teacher is not accustomed to making students work to express generalizations then mathematical reasoning will not be developed. Therefore, students must be initiated to make generalizations from the very beginning [15].

The process of mathematical generalization consists of four steps; they are as follows [15]:

- Perception of generality: At this stage, students recognize a rule/ a scheme. At this stage, students are also able to perceive or identify models. Students already know that the problems presented can be solved using rules/ patterns.
- Expression of generality: At this stage, students could use the results of the model of identification to determine the next structure/ data/ image. At this point, students are also able to describe a rule/ pattern, both numerically and verbally.
- Symbolic expression of generality: At this stage, the students could produce a rule and a general scheme.
- Manipulation of generality: At this stage, students are able to use the results of generalizations to solve problems and are able to apply the rules/ models they found to solving various problems.

3. Research Method

The purpose of this study was to develop mathematics instructional materials specialized to enhance students' ability to make generalization. To this end, this study used a formative design-based research method. The research participants were 36 fifth grade students at one of the Islamic boarding schools in Indonesia (in the context of this study, fifth grade is equivalent to 11th grade in public schools).

The instructional material development procedure in this research consisted of three stages: self-evaluation, prototyping, and field test [20]. The self-evaluation stage included two steps: analysis and design. At this point, the researchers first analysed students' ability in making generalization and mathematics teaching syllabus at the research site. To figure out their ability in making generalization, students were assigned to solve mathematical problems regarding generalization. Having completed the generalization problems, some of them were interviewed to see the obstacles they faced in the process of solving the mathematical generalization problems in question. The interviews were based on students' responses to the given problems. The researchers then designed lesson plans, student activity sheets, and mathematical generalization test items. The designs were based on students' obstacles and the results of mathematics syllabus analysis.

The prototyping stage included three steps: validation, evaluation, and revision. At this stage, the researchers developed the first prototype, which was then validated by experts. The results of expert validation and evaluation were used as the basis for revision and designing the second prototype. The second prototype was then re-validated. The results stated that this prototype was valid, practical, and effective. Therefore, this product was ready for the try-out that will be done at the field test stage.

The second prototype was tried out on the participants. The mathematics instructional process (see Fig. 2) was based on the developed lesson plans and student worksheets. Upon completion of the instructional process, mathematical generalization test was administered.

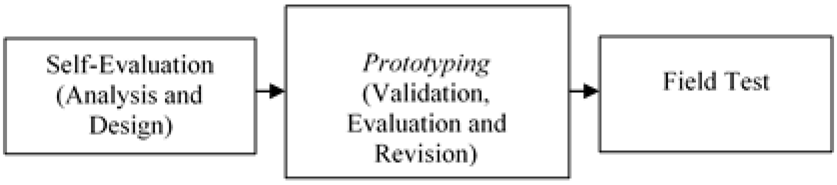


Fig. 2. Instructional materials development procedure.

4. Results and Discussion

Prior to instructional material process, a mathematical generalization test was administered to figure out students' ability in making generalizations. The results of this test showed that 88% of participants failed to understand the concept of mathematical generalization. This failure was due to the fact that, as revealed in the interviews, during the instructional process, students were not trained to look for patterns and to express patterns, especially making general formulas symbolically. Referring to this finding, the researcher developed lesson plans, student worksheets and a mathematical generalization test.

The developed instructional materials were associated with Islamic teaching topics, such as zakat, savings, prayer schedules, etc. The topic selection was based on suggestions from several experts, which is to bring the mathematics instructional materials to the context of Islamic teaching.

Table 1 shows the instructional objectives outlined in the developed lesson plan. It shows that the mathematics instructional materials were developed to help students acquire the ability to recognize a number pattern, describe a numerical or verbal number pattern, produce a rule or a general number pattern, and apply number rules/ patterns to various problems such as zakat, wadi'ah (savings), prayer schedules, etc.). The lesson plan also outlines the basic competencies including analysing arithmetic and geometric sequences based on iterative and recursive patterns and using arithmetic or geometric sequence patterns to present and resolve contextual problems.

Table 1. Mathematics instructional objectives outlined in the developed lesson plan.

Basic Competencies	Indicators
3.8 analyse arithmetic and geometric sequences based on iterative and recursive patterns	3.8.1 recognize a number pattern (generalization) 3.8.2 describe a numerical or verbal number pattern (generalization) 3.8.3 produce a rule or a general number pattern (generalization)
4.8 use arithmetic or geometric sequence patterns to present and resolve contextual problems.	4.8.1 Apply number rules/ patterns to various problems (zakat, wadi'ah prayer schedules, etc.).

Table 2 exemplifies one of the student worksheets. Students were assigned to fill in the blanks in the livestock zakat table. The objective was to help students develop the ability to make generalization of mathematical models of number patterns.

Table 2. Example of student worksheet.

No.	Number of camels owned	Zakat
1	1-4	0
2	5-9	1 Goat
3	10-14	2 Goat
4	15-19	…
5	…	..
6	25-35	1 One-year-old she-camel
7	…	…
8	…	…
9	…	1 Four-year-old she-camel
10	76-90	…
11	…	…

Table 3 shows an example of generalization test problems related to savings in the Islamic bank. Students are instructed to figure out the general number patterns. They are provided with the following questions: 1) Can you find a number pattern? 2) Is the pattern explained? 3) Can you determine the general formula? If possible, write down the formula; 4) fill in the blanks in the table. The first question relates to the perception of generality, the second question relates to the expression of generality, the third question relates to the symbolic expression of generality, and the fourth question relates to manipulation of generality.

Table 3. Example of generalization test.

Ahmad				Ilyas			
Month	**Credit**	**Balance**	**Bonus**	**Month**	**Credit**	**Balance**	**Bonus**
Jan	1,000,000	1,000,000	849	**Jan**	2,000,000	2,000,000	1,699
Feb	1,000,000	2,000,849	1,535	**Feb**	2,000,000	…	3,070
Mar	1,000,000	3,002,384	2,550	**Mar**	2,000,000	6,004,768	…
Apr	1,000,000	4,004,934	3,292	**Apr**	2,000,000	…	6,583
May	1,000,000	5,008,226	4,254	**May**	2,000,000	10,016,452	…
Jun	1,000,000	6,012,479	4,942	**Jun**	2,000,000	…	9,884
Jul	1,000,000	7,917,421	5,960	**Jul**	2,000,000	…	…
Aug	1,000,000	8,023,381	6,814	**Aug**	2,000,000	…	…
Sep	1,000,000	9,030,196	7,422	**Sep**	2,000,000	…	…
Oct	1,000,000	10,037,618	8,525	**Oct**	2,000,000	…	…
Nov	1,000,000	11,046,143	9,079	**Nov**	2,000,000	…	…
Dec	1,000,000	12,055,222	10,239	**Dec**	2,000,000	…	…

Having completed the prototype process, the researcher conducted a field test to see the implementation of the developed lesson plans and student worksheets. During the instructional process, students were enthusiastic because the materials were associated with Islamic teaching topics such as zakat, prayer times, etc. After several classroom meetings, a mathematical generalization test was administered. The results are summarized in Table 4.

Table 4. The results of student generalization tests.

The Process of Mathematical Generalization	Number of Students	Percentage
Perception of generality	31	86%
Expression of generality.	27	75%
Symbolic expression of generality	13	36%
Manipulation of generality	27	75%

The results of the test revealed that students' ability of making generalization was enhanced [21]. Many students managed to recognize a rule/ a scheme. They were also able to perceive or identify models and already knew that the problems presented can be solved using rules/ patterns. They were also able to describe a rule/ pattern both numerically and verbally. They were also able to use the results of generalizations to solve the given problems. However, many students failed to produce a rule and a general scheme. In other words, the students were successful at the three out of four processes of mathematical generalization, perception of generality, expression of generality, and manipulation of generality, and they failed at the other one, symbolic expression of generality.

The results of this study indicate that students must be trained to express generalizations to help develop their mathematical reasoning skills [15] such as making deductions, analogies, inductions, and conclusions which are very important for students at Islamic boarding schools to demonstrate since these skills could help them with the process of Islamic law-making [6].

5. Conclusion

The results of this study lead to a conclusion that the developed mathematics instructional materials enhanced students' ability of making generalization despite unsatisfactory results at the process of symbolic expression of generality. Further studies are hence recommended to address this issue.

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