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NONLINEAR INELASTIC MODEL OF STEEL COLUMN WITH SEMI-RIGID FRAME CONNECTIONS

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Abstract

Formulation of a nonlinear inelastic model of steel column with semi-rigid frame connections is outlined in this paper. Mechanical based equations were incorporated into the formulation of the model. Steel stress-strain curve with strain hardening was adopted as the material model. A pair of translation springs in vertical and lateral directions and a rotational spring was attached to each end of the column to represent a semi-rigid connection. Results obtained from the proposed model were compared with the corresponding results from finite element analysis. Good agreement was shown between the proposed model and the numerical results. It is evident from the results that the proposed model could be used to analyse the behaviour of a steel column in a semi-rigid frame.

Keywords: Elastic restraint, Nonlinear inelastic analysis, Semi-rigid connections, Steel column, Steel frame.

1. Introduction

In conventional steel frameworks design, the connections were usually simplified to be either completely rigid or flexible. However, in reality, a rigid connection does have some degree of flexibility while flexible connections do inherit some rotational rigidity. Experimental investigations done by previous researchers demonstrate that the actual joint behaviour lies somewhere between fully fixed and perfectly pinned. This indicates that semi-rigid connection is an accurate representation of the actual connection behaviour of a structure.

Testing of a semi-rigid steel frame is costly and most researchers opt to use computational model either by using analytical or numerical method such as energy method, finite element method and fiber element method. Foley and Vinnakota [1] performed an inelastic analysis of partially restrained unbraced steel frames using a finite element analysis. They concluded that the tendency of yielding to spread out along the column's length increases, the ultimate load decreases and the lateral displacement increases as the connection stiffness decreases. Li et al. [2] proposed a hybrid finite element consists of beam element with two spring restraints at the ends, which is based on the theory of finite deformation, the first-order and the second-order stiffness matrix.

Chiorean [3] developed a second-order inelastic flexibility-based element for an efficient analysis of large deflection analysis of 3D semi-rigid steel frame. The element was developed with semi-rigid connections and the effect of the initial geometric imperfections and the residual stresses were also included. Even though the analysis of the proposed method was done only by using one element per physical member, the output from the analysis was proved to be accurate with minimal computational effort required.

A numerical procedure using fiber force-based elements proposed by Thai and Kim [4] was used to accomplish second-order distributed plasticity analysis of semi-rigid steel frames. The nonlinear semi-rigid connection was simulated using a zero-length connection element with six degrees of freedom per node. Some researchers [5-8] prefer to use rotational spring stiffness to simulate behaviour of semi-rigid connection.

The aim of this study is to present a nonlinear inelastic model of a steel column with semi-rigid connections. The proposed model is derived from mechanical based equations. Stress-strain behaviour with strain hardening is used to represent material model of steel. Translational and rotational springs are attached to the ends of the columns to act as a semi-rigid connection. The proposed model is then verified by means of comparison with the ones obtained through finite element method.

2. Steel Stress-Strain Curve

Stress-strain behaviour with strain hardening is adopted in the formulation to simulate the material behaviour of steel and can be expressed using the equations shown below:

$$\sigma_s = \begin{cases} E_s \cdot \varepsilon_s, & \varepsilon_s \leq \varepsilon_{sy} \\ f_{sy}, & \varepsilon_{sy} < \varepsilon_s \leq 10 \cdot \varepsilon_{sy} \\ E_{st}\left(\varepsilon_s - 10 \cdot \varepsilon_{sy}\right) + f_{sy}, & 10 \cdot \varepsilon_{sy} < \varepsilon_s \leq 0.2 \end{cases} \tag{1}$$

in which E_s, E_{st}, f_{sy}, ε_s and ε_{sy} are the modulus of elasticity, tangent modulus at the onset of strain hardening, yield stress, strain of steel when it is loaded and yield strain respectively.

3. Theoretical Model

3.1. General

Most of the researches done previously on semi-rigid connections are dealing with either steel frame, Fig. 1(a), or sub-assemblage of frame, Fig. 1(b). Idealized steel column with elastic connections as shown in Fig. 1(c) will be adopted in this paper. Steel column with length, L consists of arbitrary cross-section as shown in Fig. 2(b) is considered. Eccentricity, e is measured from the origin, O, Fig. 2(a). The cross-section is assumed to be fully elastic when stressed below the proportional limit. However, part of the cross-section will behave elastically and the other parts will behave inelastically i.e. yielding and strain hardening when it is stressed beyond the yield stress as shown in Fig. 2(c). It is assumed that plane sections before deformation remain plane after the deformation so that Euler-Bernoulli beam theory is applicable, Fig. 2(d). The steel column in Fig. 1(c) is restrained at both ends with lateral and vertical elastic restrained of stiffness k_h and k_v and also a rotational spring of elastic stiffness r which are attached to the elastic centroid of the cross-section to simulate semi-rigid connections. The elastic spring stiffness for lateral, vertical and rotational springs respectively, can be defined by

$$k_h = \beta_s\left(\frac{E_sA}{L}\right),\ k_v = \beta_s\left(\frac{E_sA}{L}\right), \text{and } r = \beta_s\left(\frac{E_sI}{L}\right) \tag{2}$$

where A, I and β_s are the cross-sectional area, moment of inertia and spring stiffness coefficient.

3.2. Equilibrium and compatibility

The total strain, ε_t at a typical cross-section of the steel column can be expressed in terms of the membrane strain, ε_m, first and second terms of Eq. (3) and the curvature, κ

$$\varepsilon_t = \frac{dw}{dz} + \frac{1}{2}\left(\frac{dv}{dz}\right)^2 + (\bar{y} - y)\kappa \tag{3}$$

in which $\bar{y}$ is the geometric centroid of the steel section which is coincides with the origin, O and y is the location of the steel fiber in y-axis, Fig. 2(b).

Since the Euler-Bernoulli beam theory is valid, it can be shown that the total strain has the alternate form

$$\varepsilon_t = (y_n - y)\kappa \tag{4}$$

where y_n defines the location of the neutral axis of the cross-section measured from the origin, O.

The axial force acting at an arbitrary cross-section is

$$N = \int_{A_e} \sigma_s\, dA_e + \int_{A_y} \sigma_s\, dA_y + \int_{A_{st}} \sigma_s\, dA_{st} \tag{5}$$

in which the first term of Eq. (5) is the axial force in the elastic portion of the cross-section, A_e and second and third term of Eq. (5) are the axial forces in the yielded portion of the cross-section, A_y and strain hardened portion of the cross-section, A_{st} respectively.

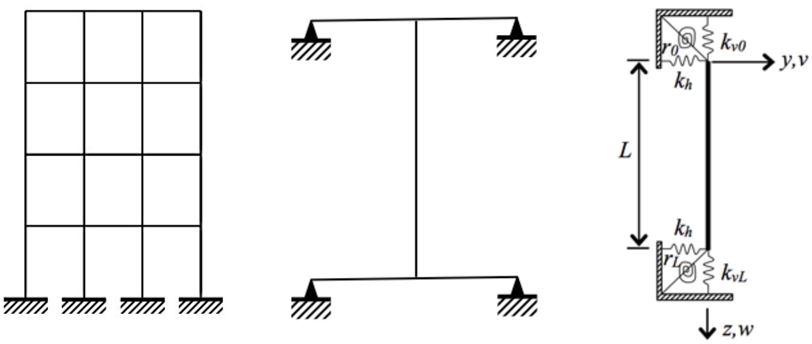


Fig. 1. (a) Typical steel frame, (b) Frame sub-assembly, (c) Idealized column with elastic restraints.

Substituting Eq. (1) and Eq. (4) into Eq. (5) leads to

$$N = [y_n(E_s A_e + E_{st} A_{st}) - (E_s Q_e + E_{st} Q_{st})]\kappa + \overline{SA} - \varepsilon_{st} E_{st} A_{st} \quad (6)$$

where

$$Q_e = \int_{A_e} y \, dA_e \quad Q_{st} = \int_{A_{st}} y \, dA_{st} \text{and } \overline{SA} = f_{sy}(A_y + A_{st}) \quad (7)$$

The location of the neutral axis can be obtained from rearrangement of Eq. (6) such that

$$y_n = \frac{E_s Q_e + E_{st} Q_{st}}{E_s A_e + E_{st} A_{st}} + \frac{1}{(E_s A_e + E_{st} A_{st})\kappa}(N - \overline{SA} + \varepsilon_{st} E_{st} A_{st}) \quad (8)$$

Similarly, the bending moment on the entire cross-section is a summation of the elastic moment and moment acting on the yielded and strain hardened portion of the cross-section as

$$M = \int_{A_e} \sigma_s y \, dA_e + \int_{A_y} \sigma_s y \, dA_y + \int_{A_{st}} \sigma_s y \, dA_{st} \quad (9)$$

Substituting Eq. (1) and Eq. (4) into the equation above, so that

$$M = [y_n(E_s Q_e + E_{st} Q_{st}) - (E_s I_e + E_{st} I_{st})]\kappa + \overline{SQ} - \varepsilon_{st} E_{st} Q_{st} \quad (10)$$

in which

$$I_e = \int_{A_e} y^2 \, dA_e, \overline{SQ} = f_{sy}(Q_y + Q_{st})$$

and

$$I_{st} = \int_{A_{st}} y^2 \, dA_{st}, Q_y = \int_{A_y} y \, dA_y \quad (11)$$

The curvature at across-section can be written as a function of the internal actions by substituting Eq. (8) into Eq. (10)

$$\kappa = \frac{(E_s A_e + E_{st} A_{st})(M - \overline{SQ} + \varepsilon_{st} E_{st} Q_{st}) - (E_s Q_e + E_{st} Q_{st})(N - \overline{SA} + \varepsilon_{st} E_{st} A_{st})}{(E_s Q_e + E_{st} Q_{st})^2 - (E_s A_e + E_{st} A_{st})(E_s I_e + E_{st} I_{st})} \quad (12)$$

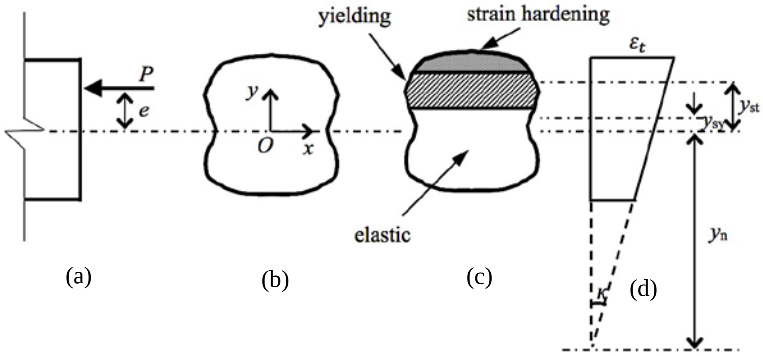


Fig. 2. (a) Profile view. (b) Arbitrary section. (c) Loaded beyond proportional limit. (d) Total strain over cross-section.

3.3. Solution procedure

The magnitude of the internal bending moment at each cross-section of the steel column shown in Fig. 3(a) can be expressed in terms of the bending moment, vertical and horizontal reactions at the top of the column using equilibrium of the member in its deformed configuration as

$$M_{int} = M - M_0 + (P - N_0)\hat{y} + (Q - R_0)\hat{z} \quad (13)$$

in which v and w are the horizontal and vertical deformation along the column respectively.

The internal shear force, V_{int} and internal axial force, N_{int} at each cross-section located at any z-axis of the column can be written in terms of the support reaction at $z = 0$ using the equations of equilibrium as

$$V_{int} = (P - N_0)\sin\theta + (Q - R_0)\cos\theta$$
$$N_{int} = (N_0 - P)\cos\theta + (Q - R_0)\sin\theta \quad (14)$$

In order to perform a nonlinear inelastic analysis, the column needs to be discretized as shown in Fig. 3(b). The difference in slope between two points along the column can be found by integrating Eq. (11) numerically with respect to z.

$$\theta_{i+1} - \theta_i = \frac{z_{i+1} - z_i}{2}\sum_{j=1}^{n_g} w_j \cdot \kappa_{i+1}(z_j) \qquad i = 0 \text{ to } L \quad (15)$$

in which z_i is the points within the column, z_j is the Gauss-Legendre integration points, n_g is the number of integration points and w_j is the associated weighting factor.

Similarly, the difference in the deflections between two points along the column can be related to the slopes using

$$v_{i+1} - v_i = \frac{z_{i+1} - z_i}{2}\sum_{j=1}^{n_g} w_j \cdot \theta_{i+1}(z_j) \qquad i = 0 \text{ to } L \quad (16)$$

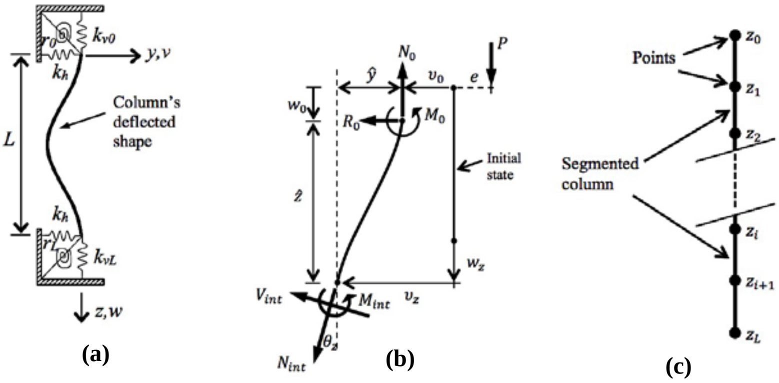


Fig. 3. (a) Deflected shape of column. (b) Free body diagram taken at distance z from top of column. (c) Column's discretization.

The relationship between the axial deformations at two points within the column can be obtained equating Eqs. (3) and (4), so that

$$w_{i+1} - w_i = \frac{z_{i+1}-z_i}{2}\sum_{j=1}^{n_g} w_j \left[\left(y_{n(i+1)}(z_j) - \bar{y}_{i+1} \cdot \kappa_{i+1}(z_j)\right) - \frac{1}{2}\left(\theta_{i+1}(z_j)\right)^2\right] \tag{17}$$

where $i = 0$ to L

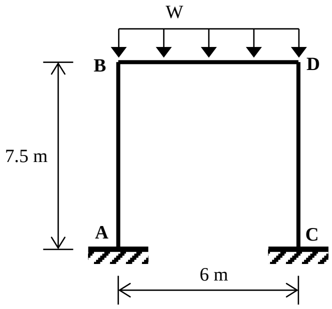
in which $y_{n(i+1)}(z_j)$ is the location of the neutral axis at $z = z_j$ given by Eq. (7).

The magnitude of the reactions at the bottom of the column can be related to the reactions at the top by substituting $z = L$ and $\theta = \theta_L$ into Eq. (12). Then, Eqs. (15)-(17) can be solved simultaneously to find the unknowns M_0, R_0, N_0 and its associate deflections and rotations at each z_i. The solution procedure is iterative and can be programme using mathematical package such as MAPLE.

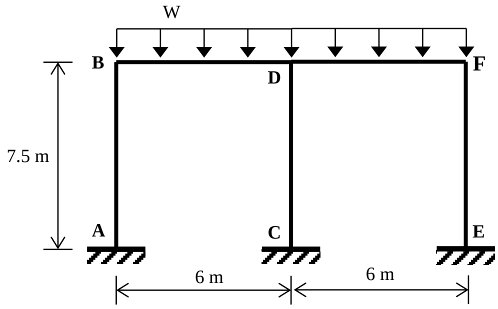
4. Results and Discussion

The behaviour of a semi-rigidly connected column obtained from the proposed model were compared and verified against finite element model. One bay one story (1B1S) and two bay one story frames (2B1S) as shown in Figs. 4(a) and (b) were chosen for the comparison. The beams and columns were made of 457x191x98UB and 305x305x97UC for both frames. The mechanical properties of steel are taken as 200GPa, 280MPa and 380MPa for the Young's modulus, yield stress and ultimate stress respectively.

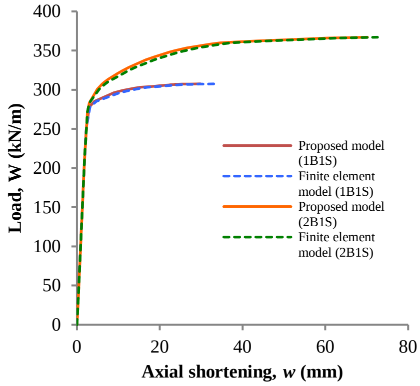
The finite element analyses in this study were conducted by using ABAQUS. The beam and column were connected by using connector element in which the translational and rotational type are Cartesian and Rotation with elastic behaviour, which was connected through their centroid. The stiffness of the springs can be found by using Eq. (2) and β_s is taken as 0.6. In which, the translational stiffness in y and z-axis were 63,200 N/mm and 196,800 N/mm respectively and the rotational stiffness was 3.56×10^9 Nm/rad. The axial shortening, lateral displacement and rotation at top of column AB for both 1B1S and 2B1S frames were shown in Figs. 4(b) and (c). The results obtained from the proposed model agree well with the results from finite element analysis. This shows that the proposed model is able to capture the behaviour of a semi-rigidly connected column.



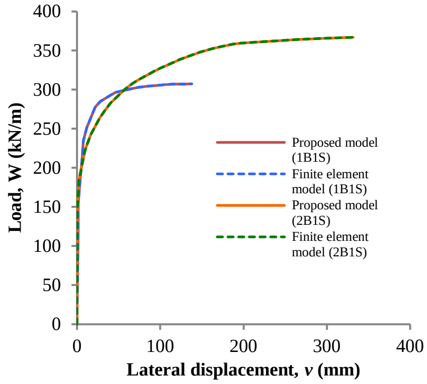
(a) One bay one story (1B1S).



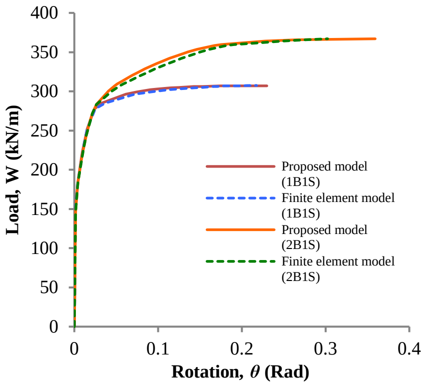
(b) Two bay one story (2B1S).



(c) Load vs Axial shortening.



(d) Load vs lateral displacement.



(e) Load vs rotation.

Fig. 4. Comparison between proposed model and finite element model.

5. Conclusions

An elaborate illustration on the development of the formulation used to model a steel column with semi-rigid frame connections has been laid up in this paper. The iterative nonlinear inelastic semi-analytical procedure was developed by utilizing mechanical based equations that can be solved using mathematical software packages. Translational and rotational springs were attached to each end of the columns to act as semi-rigid connections. The proposed model was compared against established finite element analysis. It was shown that the proposed model is able to accurately predict the nonlinear inelastic behaviour of a steel column with semi-rigid connections. A careful selection of the connections behaviour will lead to an efficient, optimum and economical steel frame design.

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References

1. Foley, C.M.; and Vinnakota, S. (1997). Inelastic analysis of partially restrained unbraced steel frames. *Engineering Structures*, 19(11), 891-902.
2. Li, G.; Qi, C.; and Luo, J. (2012). Nonlinear finite element analysis of steel frame with semi-rigid connections. *Applied Mechanics and Materials*, 226, 1199-1202.
3. Chiorean, C.G. (2017). Second-order flexibility-based model for nonlinear inelastic analysis of 3D semi-rigid steel frameworks. *Engineering Structures*, 136, 547-579.
4. Thai, H.T.; and Kim, S.E. (2015). Second-order distributed plasticity analysis of steel frames with semi-rigid connections. *Thin-Walled Structures*, 94, 120-128.
5. Hadianfard, M.A.; and Razani, R. (2003). Effects of semi-rigid behavior of connections in the reliability of steel frames. *Structural Safety*, 25, 123-138.
6. Kartal, M.E.; Basaga, H.B.; Bayraktar, A.; and Muvafik, M. (2010). Effects of semi-rigid connection on structural responses. *Electronic Journal of Structural Engineering*, 10(10), 22-35.
7. Batkov, E.; Tarasova, D.; Andreev, K.; and Limarenko, I. (2016). Technologies, equipment and construction materials for underground infrastructure development: shear forces in bolts of semi-rigid joints for steel constructions. *Procedia Engineering*, 165, 1595-1603.
8. Chen, Z.; Xu, H.; Zhao, Z.; Yan, X.; and Zhao, B. (2016). Investigation on the mechanical behavior of suspend-dome with semirigid joints. *Journal of Constructional Steel Research*, 122, 14-24.