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STAGE-DISCHARGE RELATIONSHIPS OF COMBINED FLOW OVER BROAD CRESTED WEIR AND BOTTOM OPENING UTILIZING INCOMPLETE SELF-SIMILARLY TECHNIQUE

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Abstract

The combination of the weir and the bottom opening is advanced applied hydrology which has real advantages such as reducing the problems related to sedimentation and deposition. Also, the presence of bottom openings is useful in times of flood and water scarcity. in this research, the stage-discharge relationship was determined by a combined flow based on the method of dimensional analysis and incomplete self-similarity (ISS) as well as using the previous method of dimensional analysis. Thirty experiments were conducted in a flume of dimensions (50×50) cm and a length of 15 m using six models of broad crested weir with rounded edges and a different-shaped bottom opening (orifice) (rectangular, square, and circular orifice) and five discharges pumped for each model. when comparing the two methods and calculating the discharge for each of them, it was showed that R^2 is high for both methods, but MARE in the ISS method is equal (4%) less than the other method is equal (18%). Moreover, the outcomes revealed the highest coefficient of discharge (C_{Dc}) for a circular orifice weir, followed by a square orifice, and finally with the rectangular orifice.

Keywords: Broad-crested weir, Discharge coefficient, Incomplete self-similarly, Rounded edge, Stage-discharge formula.

1. Introduction

Upstream weirs raise water levels and send it to irrigation canals. More farmland necessitates because the weirs cannot hold enough water. Building new structures or alter old ones, some are expensive, the other is not. Orifices in weir bottoms increase efficiency, especially during floods, and help clean the weir [1-6].

Many researchers conducted several investigations about the combined flow. However, few studies have dealt with a broad-crested weir with a rounded edge. Fu et al. [7] proposed an equation to calculate the discharge coefficient of the combined flow based on theoretical analysis and physical experiments. The results were investigated by statistical indicators. Fahmy and Mowafy [8] investigated the effect of composite structures on the flow characteristics of a broad-crested weir with a sharp edge and a circular bottom opening of 2 cm diameter. Various discharges were pumped (12, 15, 18 and 21 litres/s) and the head values over the weir were 42, 60, 68 and 90 cm, respectively. A radial gate was also used.

Ferro [9] obtained correlations using dimensional analysis and incomplete self-similarity for geometric forms (Bazin, broad-crested, labyrinth, and parabolic weirs). Salehi and Azimi [10] studied the hydraulic properties of the combined flow that investigated using dimensional analysis method. Seven different models containing gates were selected, including a broad-crested weir. Empirical formulas for. The discharge coefficient was determined when the flow was shared above the weir and under the gate. Al-Saadi [11] studied the hydraulic characteristics of the combined flow of different types of weir and the best shape in terms of hydraulic components was determined. The results showed that CD value for the semicircular weir and gate was the best. Bijankhan et al. [12] experimentally verified the free-flowing process of finite crest length (long, short, broad, sharp) with a sharp edge. Based on the aforementioned studies, the application of most stage-discharge relationships depends on type of the weir.

2. Materials and Method

2.1. Laboratory experiments

All experiments were performed at the Middle Technical University's hydraulic laboratory in Kut, Iraq, utilizing a 12 m long, 50 cm wide, and 50 cm high laboratory flume Fig. 1.

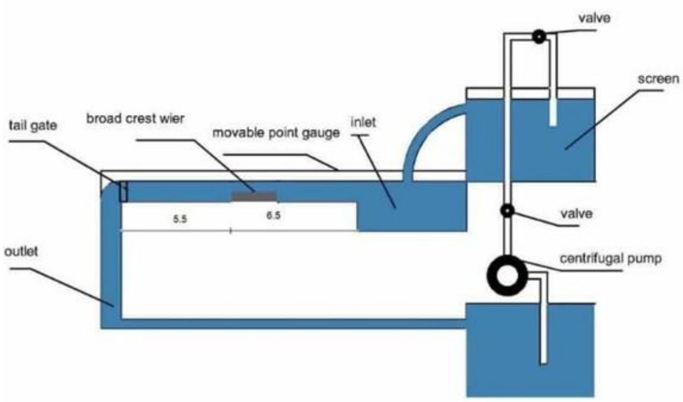

Fig. 1. A schematic diagram to illustrate the experimental setup.

The glass channel wall features a 90° V-notch at the entry to measure the downstream water output. It features a gate to change the tailwater depth. Two moveable carts measure water depths to within 1 mm [13].

2.2. Model description

The studied models of broad-crested weir were manufactured using a CNC machine to obtain the required dimensions of the model. It was made of foam, as shown in Fig. 2. Six models with a bottom opening (orifice) of a circular, rectangular and square shape were tested. Thirty experiments were performed in the free flow condition by pumping five discharges (5, 10, 15, 20 and 25 L/sec) for each model, as shown in detail in Table 1. Readings were taken after waiting for a minute to ensure the stability of the water. All model dimensions conformed with ASTM specifications and boundaries [14]: $h \geq 0.06m$, $R \geq 0.2\ h_{max,}$ $L \geq 1.75\ h_{max},$, $L+R \geq 2.25\ h_{max}$., $0.05 \leq\ H/L \leq 0.57$., $\frac{H}{p}<1.5$., $P \geq 0.15$ m, $B \geq 0.3m\ or \geq H\ or \geq L/5$.

Fig. 2. The broad-crested weir model with an orifice (rectangular shape).

Table 1. The dimensions of the used models (broad-crested weir).

No.	Shape of the opening	Dimension of the weir (cm)		Dimension of the orifice (cm)		
		P	***L***	***B***	***Z***	***D***
1	Rectangular			10	5	-
2	Circular	25	40	-	-	5
3	Square			5	5	-
4	Rectangular			10	5	-
5	Circular	35	40	-	-	5
6	Square			5	5	-

3. Results and Discussion

The analysis and discussion of the results include various criteria affecting the characteristics of the combined flow and deducing stage-discharge relationships.

3.1. First Method

3.1.1. Dimensional analysis of the combined flow condition

Dimensional factors, which affect the state of the combined flow as shown in Fig. 3, can be expressed using the functional connection:

$f\,(Q, B, h, a, g, \rho, \sigma, \mu) = 0$ (1)

The functional link, Eq. (1), is a heterogeneous physical phenomenon [15]. The dimensional independent variables ρ,g and z, as well as the -theorem, may be written as follows:

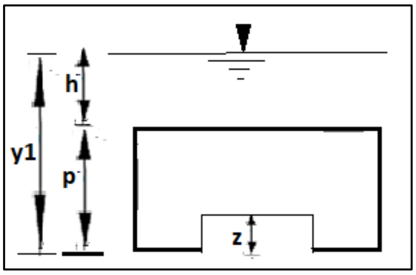

Fig. 3. Cross-section for combined flow (weir + rectangular orifice).

$\pi_1 = \frac{h}{z}$; $\pi_2 = \frac{Q}{g^{1/2}\, z^{5/2}}$; π_3= B/ z; $\pi_4 = \frac{\mu}{z^{3/2} g^{1/2}\, \rho}$; $\pi_5 = \frac{a}{z^2}$ and $\pi_6 = \frac{\sigma}{z^2 g \rho}$

To simplify the solution, the following relationships can be found:

$\pi_{(2,3)} = \frac{\pi_2^{2/3}}{\pi_4^{2/3}} = \frac{(Q)^{2/3}}{g^{1/2}\, z^{5/2}} \quad \frac{z^{2/3}}{B^{2/3}} = \frac{Q^{2/3}}{g^{1/3}\, B^{2/3}\, z}$. Introducing the essential depth for a rectangular channel cross-section, $k_b^* = \frac{Q^{2/3}}{g^{1/3}\, B^{2/3}}$. Thus $\pi_{(2,3)} = \frac{k_b^*}{z}$, $\pi_{2,3,4} = \frac{\pi_2}{\pi_5 \pi_3} = \frac{(Q)}{g^{\frac{1}{2}} z^{\frac{5}{2}}} \; \frac{z^{\frac{3}{2}} \rho g^{\frac{1}{2}}}{\mu} \; \frac{z}{B} = \frac{\rho\,(Q)}{\mu\, B} = Re,$ where Re = Reynold number.

Equation (1) can be rewritten as follows:

$\pi_1 = f\,(\pi_{2,4}, \pi_5, \pi_{2,3,4}, \pi_7)$ (2)

Substituting in Eq. (2), it can be rearranged to be as the following:

$\frac{h}{z} = f\,(\frac{k_b^*}{z}, \frac{a^2}{z}, Re, We)$ (3)

Because the flow in the channels was turbulent and viscous forces were small compared to internal forces, Re was ignored in this work [16]. Also, according to Weber [17]. The water level over the weir (h) was roughly 6 mm, therefore we may ignore the Weber number (We) in Eq. (3):

$\frac{h}{z} = f\,(\frac{k_b^*}{z}, \frac{a^2}{z})$ (4)

3.1.2. Incomplete self-similarity (ISS) theory

The phenomena are expressed by the functional relationship $\pi_1 = f(\pi_2, \pi_3; \ldots, \pi_{n-1})$, where π_1 represents functional symbol, and the self-similarity is dubbed complete self-similarity (CSS) in a given π_n dimensionless group.When the function tends to a finite limit, and it differs from zero. The phenomenon is expressed by the following functional relationship: $\pi_n = \pi_n^{\varepsilon} f(\pi_2, \pi_3, \ldots.., \pi_{n-1})$. Incomplete self-similarity (ISS) is the word used in the parameter π_n to describe this situation [17, 18]. In Eq. (4) a/z^2 value, when $\frac{k_b^*}{z} \to 0$ then $h/z \to 0$ and when $\frac{K_b^*}{z} \to \infty$ then $h/z \to \infty$ the ISS occurs. Therefore, for a given a/z^2 value, Eq.(4) takes the following form:

$$\frac{h}{z} = c\left(\frac{k_b^*}{z}\right)^t \quad \text{according} \quad \frac{a^2}{z} \tag{5}$$

Note: c and t are numerical constants that must be determined by experimentation.

3.1.3. Development of stage-discharge relationship for the combined flow

To develop a special equation for the case of the combined flow . Equation (5) can be used depending on the experimental results, the relationship between (h/z) and critical depth ($\frac{k_b^*}{z}$) according to ($\frac{a^2}{z}$) is shown in Fig. 4. Also, the values of c and t are listed in Table 2, they can be obtained from Fig. 4. Note that c and t represent the constants in Eq. (5). Figures 5 and 6 are used to obtain Eqs. (6) and (7), respectively.

$$c = 1.377\left(\frac{a}{z^2}\right)^{-0.391} \tag{6}$$

$$t = 1.137\left(\frac{a}{z^2}\right)^{0.3848} \tag{7}$$

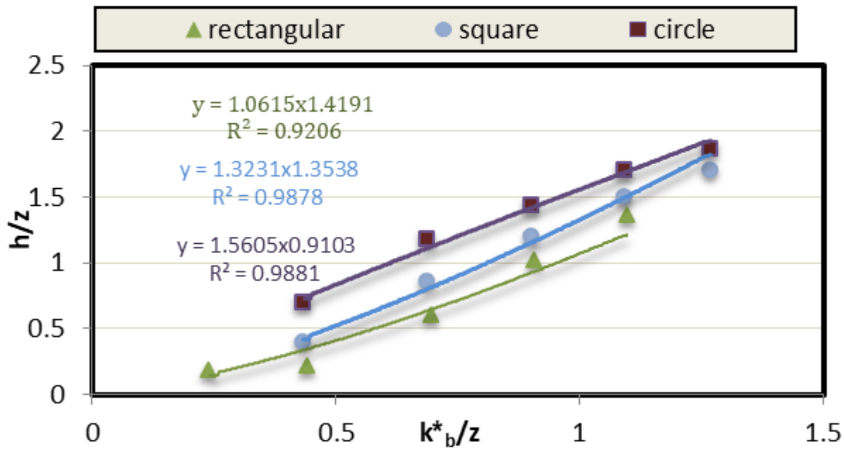

Fig. 4. Relationship between pairs of experimental ($\frac{h}{z}$, $\frac{k_b^*}{z}$).

Table 2. The values of c and t are from Fig. 4.

Shape of the orifice	C	T	a/z^2
Rectangular	1.0615	1.4191	2
Square	1.3231	1.3538	1
Circular	1.5605	0.9103	0.785

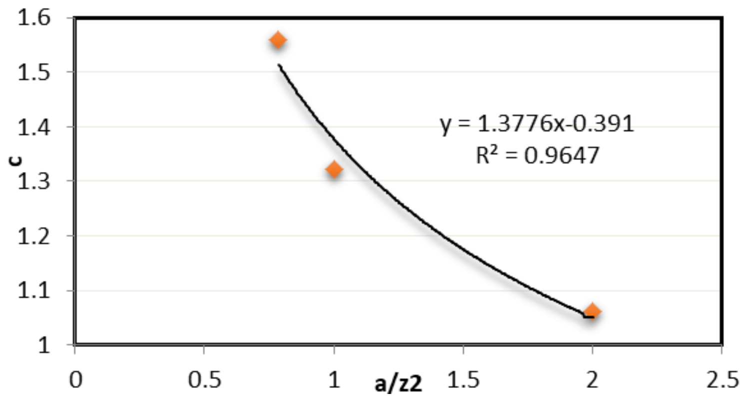

Fig. 5. The relationship between values of *c* and $\frac{a}{z^2}$.

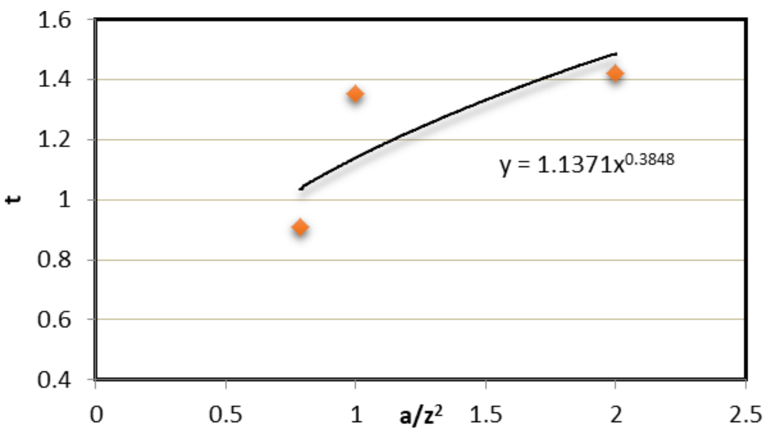

Fig. 6. The relationship between *t* and $\frac{a}{z^2}$.

Equations 6 and 7 must be substituted in Sq. (5) to obtain the general equation for combined flow:

$$\frac{h}{z} = 1.3776\ \left(\frac{a}{z^2}\right)^{-0.391}\ \left(\frac{K_b^*}{z}\right)^{1.1371\left(\frac{a}{z}\right)^{0.3848}} \quad (8)$$

The equations of that orifice can be determined.

- for rectangular orifice: $\frac{h}{z} = 1.05056\left(\frac{k_b^*}{z}\right)^{1.48469}$ (8a)
- for square orifice: $\frac{h}{z} = 1.3776\left(\frac{k_b^*}{z}\right)^{1.1371}$ (8b)
- for circular orifice: $\frac{h}{z}\ = 1.51436\left(\frac{k_b^*}{z}\right)^{1.03596}$ (8c)

3.2. Second method

Calculation of discharge coefficient for the combined flow C_{Dc}

To calculate the discharge, simultaneous flow over the broad-crested weir and through the opening can be determined by the following equation [18].

$$Q_{theo} = Q_b + Q_w \quad (9)$$

where, Q_b is the theoretical discharge through the bottom opening (orifice) can be written as:

$$Q_b = \sqrt{2gH}a \quad (10)$$

Maximum $H = y_1 - z/2$

Q_w is the theoretical discharge over the broad-crested weir, which can be expressed as the following [18].

$$Q_w = \frac{2}{3}\sqrt{\frac{2g}{3}}\, B\, h^{1.5} \quad (11)$$

The actual discharges are Q_b and Q_{weir} for the opening and the weir, respectively. $Q_{b\,act} = C_{db}.\, Q_b$, $Q_{w\,act} = C_{dw}\,.Q_w$

where C_{db} and C_{dw} are the weir and opening discharge coefficients, respectively.

The theoretical discharge Q_{theo} for both the opening and the weir is:

$$Q_{theo} = \sqrt{2gH}a + 2/3\sqrt{\frac{2}{3}g}\, Bh^{1.5} \quad (12)$$

and actual discharge Q_{act}

$$Q_{act} = C_{Db}\sqrt{2gH}a + 2/3\, C_{Dw}\sqrt{\frac{2}{3}g}B \quad (13)$$

By simplifying Eq. (13):

$$Q_{act} = C_{Dc}(\sqrt{2g}(H^{0.5}\, a + \frac{2}{3\sqrt{3}}\, B\, h^{1.5}) \quad (14)$$

where, C_{DC} is the combined discharge coefficient. Hence, this combined discharge coefficient will be estimated using Eq. (14)

$$C_{Dc} = \frac{Q_{act}}{(\sqrt{2g}(0.5H\, a + \frac{2}{3\sqrt{3}}\, B\, h^{1.5}))} \quad (15)$$

It can be taken as a relationship with $(\frac{h}{h+p})$ by dimension analysis. The following formula can be stated as a function of all other independent variables using the parameters:

$$C_{Dc} = (\rho, \mu, h, p, g) \quad (16)$$

Using ρ, g and h as repeating variables. Using Buckingham's theorem, the following functional relationship for C_{Dc} with ratio $(h/h+p)$ may be expressed as:

$$C_{Dc} = f(\frac{h}{h+p}) \quad (17)$$

Figure 7 illustrates the discharge coefficient (C_{Dc}) with the non-dimensional parameters $(\frac{h}{h+p})$.

- for rectangular orifice: $C_{Dc} = 3.3747\,\frac{h}{h+p} + 0.4136$ (17a)
- for square orifice: $C_{Dc} = 4.8852\,\frac{h}{h+p} + 0.4518$ (17b)
- for circular orifice: $C_{Dc} = 5.3415\,\frac{h}{h+p} + 0.4509$ (17c)

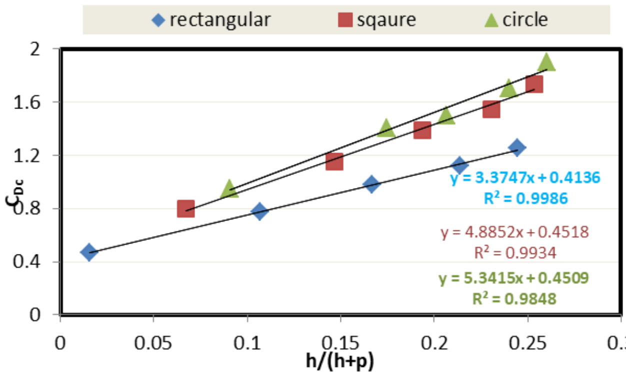

Fig. 7. The relationship between the values of ($\frac{h}{h+p}$)and values of (C_{Dc}).

3.3. Validation of new formula for combined flow

It is necessary to demonstrate the accuracy of the equations in the case of combined flow by comparing the stage-discharge formulas (Eqs. (8) and (17)). This can be obtained by summing the results of the study of Fahmy and Mowafy [8] and substituting them into Eqs. (8c) and (17c) for circular orifice because Fahmy and Mowafy [8] used the combined flow of a circular orifice. It was observed when compared with the experimental discharge of the study of Fahmy and Mowafy [8] as shown in Fig. 8 that R^2 was 0.98 and 0.96 But the relative mean error was 4.4% and 18% for Eqs.(8c) and (17c) respectively. Equation (8c) was more accurate than Eq. (17c) obtained by dimensional analysis.

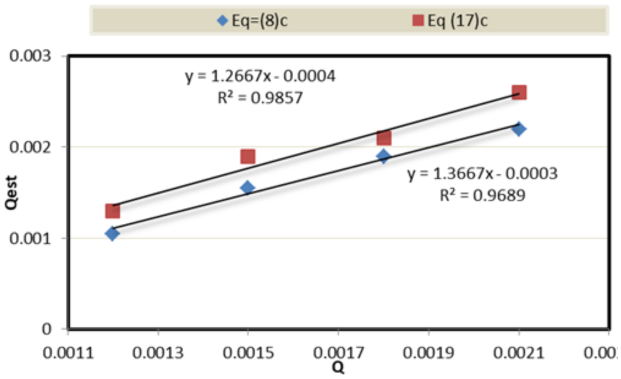

**Fig. 8. Comparison between (Eqs. (8c) and (17c)
using the result of Fahmy and Mowafy [8].**

4. Conclusions

The most important conclusions obtained through the experiments conducted and the data obtained within the scope of this study are:

- The general formula for the combined flow was developed by the method of (incomplete self-similarity (ISS) analysis, and for each shape of the openings (rectangular, square and circular)

- The discharge coefficient (C_{Dc}) was calculated, and the values ranged between (0.41 –1.9).The results also showed that the highest discharge coefficient (C_{Dc}) when the weir is with a circular orifice, where its value ranges (1.9-0.95), followed by the weir with a square orifice with values ranging (0.79-1.732), while the lowest value for discharge coefficient (C_{Dc}) when the orifice is rectangular and with values (0.4661-1.25).
- After calculating the discharge by both methods and comparing them with the experimental values of the study Fahmy and Mowafy [8]. The formula developed by incomplete self-similarity theory (ISS) is less valuable than the other method, thus is more accurate.

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