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JUNIOR HIGH SCHOOL STUDENTS IN SOLVING MATHEMATICAL ILL-STRUCTURED PROBLEMS: ANALYZING USING HAREL THEORY

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Abstract

This study explored how students think when solving ill-structured mathematics problems. This research method was a qualitative case study involving class VIII junior high school students. Data collection techniques were done by giving ill-structured mathematics problems in the form of geometry material, observation, and in-depth interviews. Furthermore, after students solved the problem; to further describe the student's answers, in-depth interviews were conducted to explore and dig up further information. The results showed that some issues were found: the complexity of the problems given to students became a challenge in solving unstructured geometry problems, students had difficulties interpreting the problem due to limited prior knowledge, especially regarding geometry material, and the problems given required creativity in finding solutions. Students' constraints were in the lack of prior knowledge, especially geometry materials, and creativity to find and establish strategies to build solutions according to their knowledge. This research contributed to uncovering students' thought processes when constructing solutions based on their prior knowledge to identify learning difficulties in more depth. This research also showed that the diversity of students' ways of thinking provides different problem-solving opportunities.

Keywords: Geometry, Junior high school, Mathematics ill-structured, Student, Ways of thinking.

1. Introduction

Ill-structured mathematics problems are problems that have incomplete identities, have ambiguous understanding, and require students to be able to organize information before solving the problem [1, 2]. Geometry is a branch of mathematics that studies the spatial aspects of shapes and planes in both 2D and 3D planes [3, 4]. Geometry has basic skills that must be owned, especially by students [5]. These basic skills relate to how students think, describing how cognitive students are. This cognitive is the primary provision for students to be able to transform the knowledge they have with the problem at hand. This way of thinking is adapted from Harel's theory, which uses characteristics as a framework for analysis [6, 7]. Some previous studies are reported in Table 1.

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Table 1. Studies on ways of thinking, ill-structured mathematics problem.

No.	Title	Ref.
1	Exploring Mathematical Representations in Solving Ill-Structured Problems: The Case of Quadratic Function	[8]
2	Critical Thinking Disposition: Students Skeptic in Dealing with Ill-Logical Mathematics Problem	[9]
3	Worked Example Using Ill-Structured Problem: Trained High Order Thinking Skill.	[10]
4	Karakteristik ways of thinking (wot) dan ways of understanding (wou) siswa berdasarkan teori harel	[11]
5	The different way of understanding and way of thinking between gender on the problem the linear equations of two variables.	[12]

2. Literature Review

Geometry studies various aspects of space and shape as they relate to the properties of space, ranging from points, lines, and curves to images formed in space. Some geometry studies such as (i) plane geometry, which focuses on studying objects on a flat plane such as lines, circles, and various types of polygons; (ii) solid geometry, which concentrates on three-dimensional shapes, including polyhedra, lines, planes, and various types of surfaces, (iii) spherical geometry which specifically studies the characteristics of lines and shapes depicted on the surface of a sphere. Table 2 shows an illustration of space exploration in geometry.

Geometry has skills that must be possessed at every level of mental development in geometry. Geometry skills and levels are interrelated, such as:

Visual skills, focus on recognition, analysis and relationships between visual figures with the ability to recognize, analyse and relate geometric shapes as well as being able to differentiate and identify visual properties; Verbal skills, emphasis on linguistic abilities in describing geometry by using language to explain geometric properties and relationships as well as accuracy in defining and interpreting concepts; Drawing skills, skills in drawing and constructing geometric figures by transforming verbal information into visual representations as well as the ability to make sketches and geometric constructions; logical skills, mathematical reasoning and classification by using logic to build evidence, understand relationships between groups of figures, evaluate assumptions and postulates; Applied skills, linking geometry with real phenomena by recognizing geometric shapes in physical objects and creating mathematical models to explain phenomena [5].

Problem-solving in terms of its elements are puzzle problems, structured problems, and ill-structured problems. Ill-structured problems have different characteristics from other problems. The problems contained in the problem and the dimensions of the problem are also measured and adjusted to the characteristics. Although this unstructured problem is a lot of research in fields other than mathematics education, in mathematics science, linking between concepts, determining strategies, and finding solutions, as well as mathematical applications in everyday life, began to be applied in the current era of education [1, 13, 14].

The theoretical framework of basic instructional principles D (Duality), N (Necessity), and R (Repeated-reasoning). The theory has three main systems: premises, concepts, and claims. In the concept art (concepts), there are mental actions, which are the essential elements of human cognition by observing people's statements and actions. Ways of understanding (WoU) are certain cognitive products of mental actions performed by an individual, and ways of thinking (WoT) are cognitive characteristics of mental actions. The way of thinking (WoT) has a relationship with the following characteristics: (i) proof schemes, (ii) problem-solving approaches, and (iii) beliefs about mathematics [6, 7, 15].

3. Method

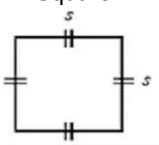
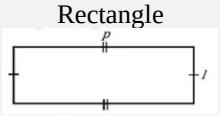
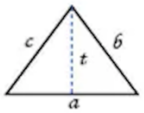
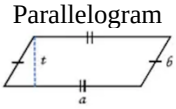
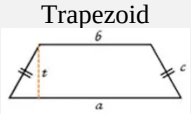
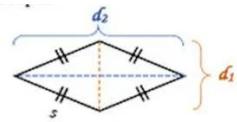
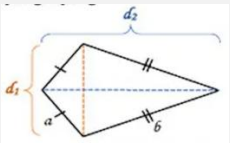
This research method is a qualitative case study involving class VIII junior high school students. Data collection techniques by giving ill-structured mathematics problems in the form of geometry material, observation, and in-depth interviews. Furthermore, after students solve the problem; to further describe the student's answers, in-depth interviews are conducted to explore further information

4. Results and Discussion

Students are given ill-structured mathematics problems on geometry, as in Fig. 1. The problems that have been solved by the student then be analysed using the framework in Fig. 2. This framework briefly reveals the students' answers and the results of the interviews. The strategy students choose to solve the answer affects their way of thinking and geometry skills. In the problem identification process, students must design the shape of the land according to their creativity to organize the land to get maximum area. After students create the land, students then look for solutions to get crop yields by building solutions that are by applied skills. Students who do not design the land are because students need to read carefully the problem

instructions and students are not used to problems that require analysis before determining the strategy in answering.

Table 2. Geometry shape and formula.

No.	Shape Name	Area | Formula	Perimeter/ Circumference Formula
1	Square 	$A = s^2 P = 4s$	$A = s^2 P = 4s$
2	Rectangle 	$A = l \times w$	$P = 2(l + w)$
3	Triangle 	$A = ½ \times base \times height$	$P = a + b + c$
4	Circle 	$A = \pi r^2 C = 2\pi r$	$A = \pi r^2 C = 2\pi r$
5	Parallelogram 	$A = base \times height$	$P = 2(a + b)$
6	Trapezoid 	$A = ½(a + b) \times height$	$P = a + b + c + d$
7	Rhombus 	$A = ½ \times d1 \times d2$	$P = 4s$
8	Kite 	$A = ½ \times d1 \times d2$	$P = 2(a + b)$

Note: *A* = Area, *P* = Perimeter, *C* = Circumference, *s* = side, *l* = length, *w* = width, *r* = radius, *d1*, *d2* = diagonals, *π* (*pi*) ≈ 3.14

The analysis continued on the student's way of thinking where the results found problem-solving steps and mathematical beliefs were not in line with the proof. As found students managed to design the maximum yield but did not design the shape of the land. The geometry skills process, and the ability to create the land, are not

completed. Land that has not been determined in shape results in the maximum are of the harvest. For more clarity, the following is the linkage of answers with the results of the analysis in Table 3.

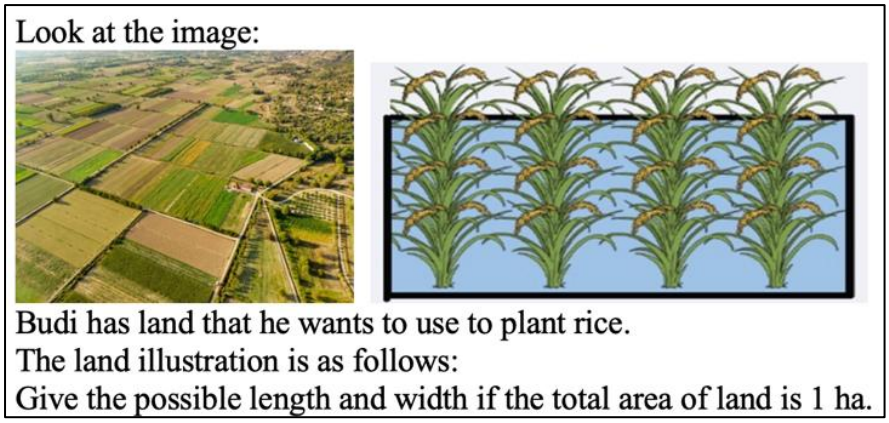


Fig. 1. Ill-structured mathematics problem on geometry.

There is alternative answer to the question, students must convert 1 hectare into 10,000 m^2. Then you can calculate the possible length and width to obtain a maximum area of 10,000 m^2. Possible answer I: long (l) = 100 m, wide (w) = 100 m; then $A = l$ x w, A = 100 m x 100 m, A = 10,000 m^2. Possible answer II: l = 50 m, w = 200 m; then $A = l$ x w, A = 50 m x 200 m, A = 10,000 m^2. Possible answer III : l = 25 m, w = 400 m; then $A = l$ x w, A = 50 m x 200 m, A = 10,000 m^2. Possible answer IV: l = 10 m, w = 1.000 m; then A = l x w A = 10 m x 10.000 m A = 10,000 m^2. Possible answer V: l = 20 m, w = 500 m; then A = l x w, A = 50 m x 200 m, A = 10,000 m^2. The result of student answered are: (i) Students do not convert 1 hectare into 10,000 m^2, (ii) students only write down one possible answer without calculating other possibilities, (iii) students who answer like possibility IV: l = 10 m, w = 1,000 m; then $A = l$ x w, A = 10 m x 1.000 m, A = 10,000 m^2. Based on the interview results, students could not describe the land and could not determine the position of l and w.

The mistakes made by students were not about designing the land that should be the basis for determining maximum profit. This is because students were not familiar with problems that require analysis, reasoning, and critical thinking to solve. The ability to solve math problems is closely related to several higher-order thinking skills. These skills include the ability to reason logically, the ability to make the right decisions, the ability to analyse critically, and the ability to think creatively and innovatively. All aspects are interconnected and support each other in the process of solving mathematical problems [16].

Another impact is that it allows educators to better understand in detail the learning challenges that students face. Students' success in learning is the result of a complex interaction between the teacher's teaching methods, previous academic experiences, individual capacity differences, and the specific context of learning that affects their understanding [17]. Students' basic geometry ability affects their confidence and math problem-solving skills. Female students, especially those using direct learning methods, have greater difficulties in visualizing geometry

concepts than male students [18]. Finally, this adds new information regarding teaching and learning in mathematics, as reported elsewhere [19-25].

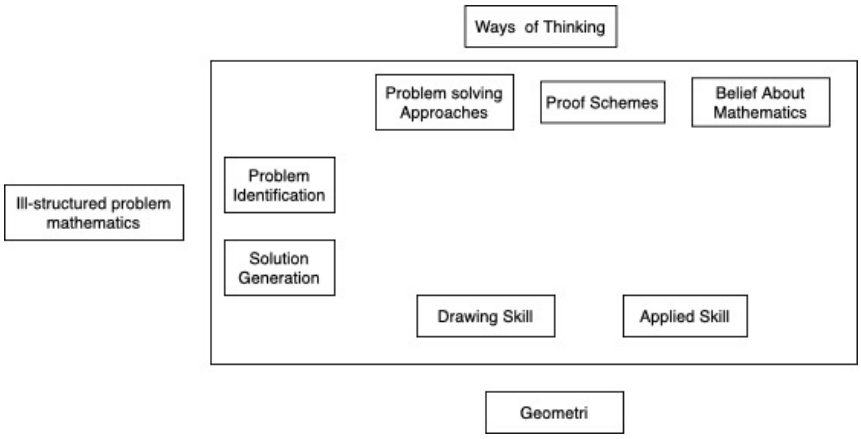


Fig. 2. Framework for analysing student answers.

Table 3. Student answer results.

	Problem-solving approaches	**Proof schemes**	**Belief about mathematics**
Problem identification and drawing skill	Students can identify the identity required but fail to design the land.	Students can prove the answer, but the proof of land design affects the maximum yield.	Students are confident in solving the problems given because they have the appropriate prior knowledge.
Solution generation and applied skill	Students solve according to the right steps.	The proofs provided by the students were in accordance with the steps. Although the shortcomings are in the land design.	Confidence in solving the problem is reinforced through interviews with students regarding responses before the problem is given.

5. Conclusion

The important points in the conclusion of this study include: (i) The complexity of the problems given to students to face significant challenges in solving ill-structured geometry problems. This kind of problem requires complex thinking skills rather than just the use of simple mathematical formulas or procedures, (ii) students have difficulties interpreting the problem due to limited prior knowledge, especially regarding geometry material, (iii) the given problem requires creativity in finding solutions. These findings provide recommendations that (i) the need to strengthen mathematical concepts so that mathematical material adapted in the form of any problem can be solved appropriately, and (ii) the need for students to

train creativity in thinking to solve unstructured problems that require flexible strategies and solutions.

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