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ZONE OF CONCEPT IMAGE DIFFERENCES IN MATHEMATICS: A CASE STUDY OF DERIVATIVE CONCEPT

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Abstract

This study aims to investigate the zone of differences between the concept image and the formal concept definition of derivative concepts based on procedural and conceptual understanding. The study was conducted with a qualitative method with an interpretive phenomenological approach to explore the experience and knowledge of the participants. Data were obtained through tests and clinical interviews were given to 10 prospective mathematics teachers from a university in West Java, Indonesia. Ideally, the concept image formed in the participants' cognitive structure of a concept must be the same as the existing formal concept definition. However, the results of the study indicate that there are still differences occur between the concept image of prospective mathematics teachers and the formal concept definition causes cognitive conflict in the understanding of the derivative concept.

Keywords: Concept image, Derivative concept, Formal concept definition.

1. Introduction

The term concept image was first introduced to describe the total cognitive structure associated with the concepts, characteristics, and mental action processes that exist in the individual's mind [1, 2]. On the other hand, a formal concept definition is a concept accepted or agreed upon by the mathematician community on a large scale [3, 4]. In the context of the learning process, individuals must make interpretations when studying a mathematical concept. The concept image formed in the cognitive structure can be used to identify the extent of understanding of the formal concept definition [5]. The results show the concept image formed by individuals cannot be the same or even different from the formal concept definition should be [6-11]. This is because the concept image formed is not only influenced by the knowledge provided by the teacher but can also be influenced by the experience and previous knowledge of students [1, 3, 5, 12].

In the process of receiving knowledge, students can provide various responses, including (i) replacing the concept image that has been formed with a new concept image; (ii) maintaining the old concept image; (iii) or separately using both concept images to understand the formal concept definition. The term zone of concept image differences (ZCID) is used to examine the gap between concept image and formal concept definition based on the responses given by students [3]. Based on the mathematics education curriculum in Indonesia, one of the most important formal concept definitions to learn is the derivative concept which is a basic concept in studies in the field of Calculus [6]. The formal concept definition of the derivative concept can refer to the geometric representation of the meaning of the secant line and tangent lines on a curve as shown in Fig. 1.

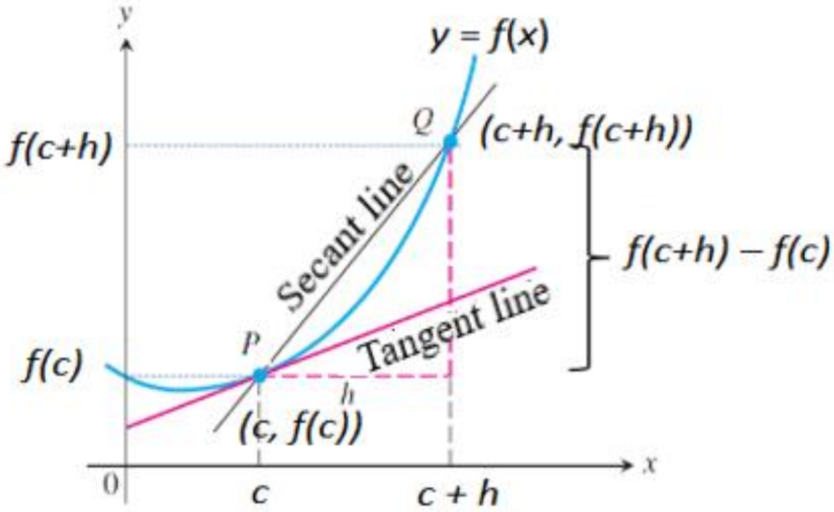

Fig. 1. Geometric representation of the derivative concept.

Suppose point P is a fixed point on a curve and Q is a contiguous point whose position on the curve can be varied. The line passes through the points P and Q is called a chord or secant line. The tangent at P is the limit of the chord if the point

Q moves in the direction P along the curve. If the curve is the curve of the equation $y = f(x)$, P has coordinates $(c, f(c))$, nearby point Q has coordinates $(c + h, f(c + h))$, and the chord passes through points P and Q has a slope of $= \frac{f(c+h) - f(c)}{h}$. If the tangent line is not perpendicular, then it is a line that passes through P with a slope of m_{tan}satisfies $= \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ which is the definition of a derivative concept [11, 13].

Therefore, a review of ZCID and formal concept definition of derived concepts is an important step for educational practitioners to know students' thinking processes. They can be used as references in designing learning designs to be more meaningful [1, 3, 5]. In this study, we aim to investigate the concept image has been formed in the cognitive structure as a result of the given instructional design, as well as to analyze the extent to which the gap between ZCID and formal concept definition has the potential to trigger cognitive conflicts in learning derivative concepts.

2. Methods

This research uses a case study approach with Interpretive Phenomenological Analysis (IPA) to interpret various phenomena that occur based on experience [13]. The participants involved in the research are 10 prospective mathematics teachers from a university in West Java, Indonesia. Determination of participants is done based on a purposive technique by considering the criteria of individuals who have taken differential calculus courses. Research data obtained from test results related to the concept of derivatives. Furthermore, clinical interviews were conducted in the form of sequential questions from easy to difficult levels to confirm participants' answers comprehensive information was obtained about the concept image that had been formed, as well as to find out the participants' thinking flow based on the knowledge and experience, they had.

3. Results and Discussion

The results of filling out the tests about derivative concepts by 10 prospective mathematics teachers with questions showed different interpretations. The concept image of the derivative still does not fully refer to the existing formal concept definition. The other indications show the concept image from the participants is still in the experience to solve problems in the context of procedural understanding than conceptual understanding. The following are examples of answers to each question shown in Fig. 2 and Fig. 3.

1. Show the process to determine the derivative of $y = 2x(x^3 + 1)$!

In question number 1, we want to see the concept image that has been formed on each participant by asking questions about the derivative of a function. The questions given fall into the routine category, which is a problem that is often found when learning derivative concepts at the beginning of the discussion. Answers from all respondents are relevant to the formal concept definition of concepts of derivatives. The steps taken are to use a general derivative formula $f'(x) = nax^{n-1}$ for $f(x) = ax^n$ based on multiplication property$f'(x) = u.v' + u'.v$ to derive $y = 2x(x^3 + 1)$. Suppose $u = 2x$ and $v = x^3 + 1$, then the derivation of u is $u' = 1.2x^{1-1} = 2x^0 = 2.1 = 2$, and the derivation of v is $v' = 3x^{3-1} + 0 = 3x^2$.

Based on the results of derivation u and v, the derivative of $y = 2x(x^3 + 1)$ is $y' = 2x.3x^2 + 2.(x^3 + 1) = 6x^3 + 2x^3 + 2 = 8x^3 + 2$.

The decision of respondents used the general formula derived and the multiplication property in answering the questions given in line with the experience in the cognitive structure they have when solving similar questions. The conceptions made by students in learning or responding to something are involved with the interpretation and mental picture. The links concepts are based on the impressions and experiences they get in the previous learning process [14-18].

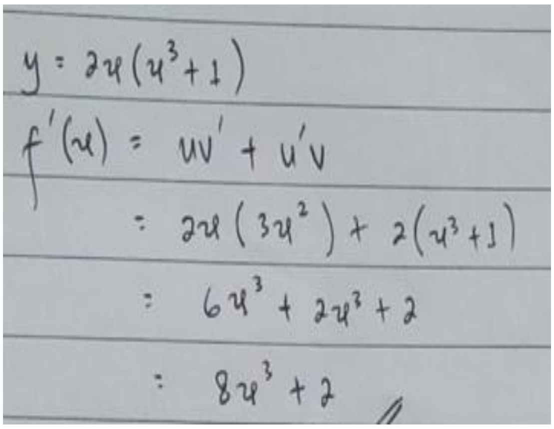

Fig. 2. The participant's answer to derive the function.

2. Investigate the value of $\frac{g(x+h)-g(x)}{h}$ for $g(x) = x^2 + 2$ at $x = 3$.

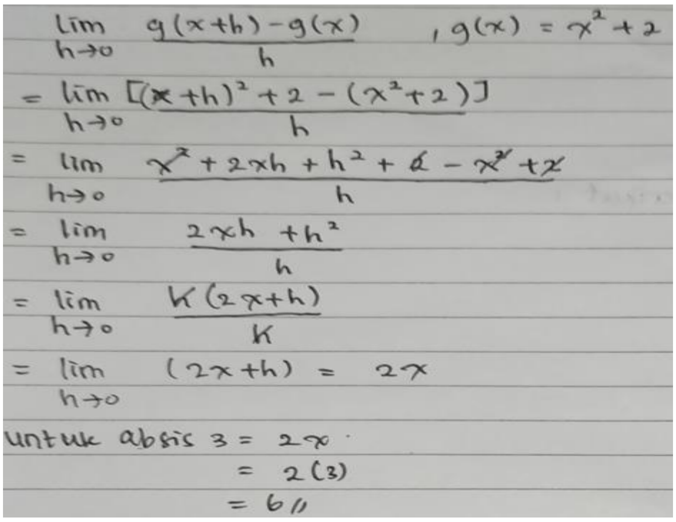

Fig. 3. The participant's answer to derive the function by limit.

In question number 2, we want to do a review of whether the subject also has a concept image of the definition of a derivative concept by using the concept of a limit concept $f'(x) = \frac{f(x+h)-f(x)}{h}$ and use it to derive functions. Based on the answers given, it is known the participant can derive functions of $g(x) = x^2 + 2$ at $x = 3$with used $g'(x) = \frac{g(x+h)-g(x)}{h}$ and relevant to formal concept definition in the context of procedural understanding. The results of clinical interviews are carried out with each participant to reconfirm the answers to the tests that have been given previously. Questions in clinical interviews related to derivative concepts are given starting from easy questions to questions that are difficult to review the concept image has been formed on the cognitive structure of the participants and examine the extent of the difference with the formal concept definition of the derivative concept. The results of clinical interviews from participant 1 (P1) and participant 2 (P2) are shown in Table 1. According to test and clinical interview results which include questions ranging from procedural understanding to conceptual understanding, the majority of participants do not understand the basic ideas of the derivative concepts. These conditions are described from the answers P1 and P2 which demonstrate the conceptualization of the meaning of derivative definition and the connection between $f'(x) = nx^{n-1}$ with $f'(x) = \frac{f(x+h)-f(x)}{h}$ although P1 and P2 know the notations are tools to derive a function without understanding the relationship between each other. In addition, the other participants also can not explain the meaning of $f'(x) = \frac{f(x+h)-f(x)}{h}$ in the same with formal concept definition on derivative which can be explained through geometric and symbolic representations.

The tendency of individuals to respond to a condition will be influenced by the concept of image as a mental picture is partly generated by the experience the individual has [14, 16]. The concept of the image generated by external representations in the form of mathematical ideas can help a person to read, see, hear, or manipulate internal representations in the form of objects, images, graphics, images, or symbols, and associate them with certain meanings [17, 19].

Based on the findings from the tests and clinical interviews, a more in-depth study is required while learning the derived ideas during lecture activities. The research must not only cover the fundamental concepts of derivatives, but it must also cover high-level derivative concepts. Before presenting concepts in their informal form, educators must foster students' concept representations of the concepts to be taught [19]. Encouragement of students' idea pictures will give a chance for them to rediscover mathematics, allowing them to perceive concepts as independently acquired information that can justify knowing [20]. During concept creation, the link between the concept picture and the formal concept description must have reciprocal interactions and mutual advantages. Based on the findings from the questionnaire and clinical interview, a more in-depth study is required while learning the derived ideas during lecture activities. The research must not only cover the fundamental concepts of derivatives, but it must also cover high-level derivative concepts. Before presenting concepts in their informal form, instructors must foster students' concept representations of the concepts to be taught [14-18]. Encouragement of students' idea pictures will give a chance for them to rediscover mathematics, allowing them to perceive concepts as independently acquired information that can justify knowing [19]. During the concept creation, the link between the concept picture and the formal concept description must have reciprocal interactions and mutual advantages [20]. This study is in line with current literature regarding the strategies in teaching mathematics [21-29].

Table 1. The results of the clinical interview of two participant

Questions		Responses
"What is the derivative definition?	P1:	"The derivative is the opposite of the integral. In the integral concept, if a function is integrated, then the sum of its powers will increase. However, in the derivative concept, if a function is derived, then the number of exponents will decrease".
	P2:	"The concept of the derivative is an operation to derive a function and its role is to determine the maximum and minimum values".
"What is the meaning of $\boldsymbol{f'(x) = \frac{f(x+h)-f(x)}{h}}$?"	P1:	" $f'(x) = \frac{f(x+h)-f(x)}{h}$ is a notation for deriving linear and quadratic functions. Apart from these functions, we can use the notation $f'(x) = nx^{n-1}$".
	P2:	"Mmm, the notation of $f'(x) = \frac{f(x+h)-f(x)}{h}$ is the definition of a derivative concept, maybe. I do not understand the meaning of the notation"
"Is there a relationship between $\boldsymbol{f'(x) = \frac{f(x+h)-f(x)}{h}}$ with $\boldsymbol{f'(x) = nx^{n-1}}$?"	P1:	"Mmmm, wait, I remember, the notation $f'(x) = nx^{n-1}$ obtained from $f'(x) = \frac{f(x+h)-f(x)}{h}$, right? I do not know how it's related.
	P2:	"There is a relationship, both of those notations are equally used to derive functions. $f'(x) = nx^{n-1}$ more commonly used in the concept of derivatives than $f'(x) = \frac{f(x+h)-f(x)}{h}$."

4. Conclusion

Participants can only utilize the derivative notation as a tool to derive functions. Their grasp of the derivative concept is still limited to procedural understanding. The basic notion of the derivative, including its definition and the significance of its notation, remains the ideal image of the subject in conceptual comprehension. These results allow us to derive the following conclusions: (i) participants generally can perform derivative functions well using the concept image of the notation they normally use, namely $f'(x) = nx^{n-1}$ for $f(x) = x^n$; (ii) based on formal concept definitions, participants get a notion of the importance of mathematical ideas or concepts. construct fundamental derivatives ideas; (iii) participants are not accustomed to examining the usage of derivative ideas in non-routine issues; iv) The degree of research on the hereditary notion is not very high. The review of derivatives in this study is still restricted to fundamental ideas. The concept images of prospective mathematics teachers that are investigated have not yet attained a more in-depth study, particularly those that relate to differential calculus, which is taught at the collegiate level. We recommend that educators pay close attention to each participant's unique perception of the derivative notion and categorize any potential learning barriers to provide a learning design with many representations.

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