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TEACHER'S KNOWLEDGE OF FRACTIONS IN THE PERSPECTIVE OF DIDACTIC-MATHEMATICAL KNOWLEDGE

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Abstract

The purpose of this research was to investigate in depth the knowledge of elementary school teachers which was regarded to the concept of fractions, both content knowledge and didactic. This research was qualitative research with a case study design involved to elementary school teachers as research participants. The data were collected through a fraction test which was developed based on the Framework of Didactic-Mathematical Knowledge and also in-depth interviews to understand in more detail the knowledge and explanations of the participants. Then, the data were analysed by using quantitative and qualitative analysis. The results of the research found in the mathematical aspect, teachers had limited knowledge and even had wrong understanding beliefs, especially on the meaning of fractions as part of the whole. In the didactic aspect, there was a tendency for teachers to teach the topics and parts of the thinking process exacted in the textbooks. In other words, the learning was carried out as an imitation of what was in the textbook.

Keywords: Didactic mathematical knowledge, Fractions, Teacher's knowledge.

1. Introduction

Fraction is one of the difficult mathematical concepts to be taught to elementary school students [1]. Some research results have shown fractions are difficult for students to understand and many have misconceptions [2]. There have been indications the misconceptions experienced by the students tend to be related to the teacher's knowledge, the way the teacher presents fractions, and how the teacher teaches them including the interventions given [3]. Many reports also state teaching fractions in elementary schools is difficult for both teachers and prospective teachers due to a weak understanding of the content of the fractions themselves [4].

During the last two decades, the main focus in education has been to prepare and develop quality teachers [5]. In the field of mathematics education, research and studies in recent years have focused on the knowledge mathematics teachers need to make their learning effective [6]. Various models were proposed to identify and describe the elements of knowledge mathematics teachers must possess, one of which is Didactic-Mathematical Knowledge (DMK), which is used to categorize and analyse the knowledge needed by teachers to teach a mathematical topic [7]. DMK interprets teacher knowledge based on three dimensions, namely the mathematical dimension, the didactic dimension, and the meta-didactic-mathematical dimension.

The mathematical dimension pertains to the knowledge in carrying out mathematical activities on certain topics and connecting them with other mathematical topics that will be studied at the next level according to the school mathematics curriculum. In addition to mastering mathematical content, teachers must know various influencing factors when planning and carrying out learning related to the mathematical content being taught. These factors are contained in the didactic dimension. The last DMK dimension is a meta-didactic-mathematical dimension that pertains to the knowledge teachers need to reflect on their teaching practices and find potential improvements to improve the learning process [8].

Based on the research findings showing students' low understanding of the concept of fractions and its relation to knowledge and the way teachers teach fractions to their students, this study was aimed at an in-depth investigation of how teachers' knowledge of fractions by using DMK. The results of this investigation would be the bases for creating strategies and appropriate references and sources to help teachers improve their understanding, especially on the concept of fractions.

2. Literature Review

The definition of fractions as “parts of a whole” dominates almost all textbooks and learning [9]. Generally, this definition is introduced by dividing a whole into several equal parts [10]. The whole is generally visualized with circles, and rectangles and rarely uses other forms [11]. Learning and explanations in elementary school textbooks are not accompanied by an in-depth explanation of what is meant by "equal parts" which means the size is the same (the area is the same) and does not have to be congruent. The emphasis on the ‘part-whole’ concept is not wrong, but it is not entirely correct because the ‘part-whole’ concept has several limitations and causes mixed numbers and fractions greater than one (improper fraction) to be unreasonable.

Fractions were not a single concept and have other interconnected constructs namely: part-whole, ratio, operator, quotient, and measure [12]. However, the part-whole concept underlies the other four interpretations of the concept. Part of a whole refers to a continuous quantity or an area divided into parts of equal area. This refers to how much area a fraction symbol represents. A fraction as a division indicates a fraction is the result of the division of two whole numbers. A fraction as a ratio is based on the comparison of two separate quantities and is usually represented as a : b or a/b. Fractions as operators describe functions that transform line segments, images, or numbers. Fractions as a measure refer to the fact fractions are numbers and can be ordered on the number line [10].

3. Research Method

This research is qualitative research with a case study method. Case study research is investigative research to describe, explore, and understand a certain phenomenon in depth. The phenomenon referred to in this study is the state of teachers' knowledge of the concept of fractions in elementary schools. The participants in the study were the teachers of grades IV and V of the elementary schools in Singaraja-Bali, totalling 15 people. The criteria for selecting participants are teachers must have a bachelor's background in primary school teacher education, they are civil servants, and already has a professional educator certificate.

The data collected in this study were quantitative and qualitative. The data were collected using 9 question items developed based on the dimensions of DMK, namely the mathematical and didactic dimensions. The mathematical dimension consisted of two aspects, namely common knowledge and extended knowledge. Meanwhile, the didactic dimension consisted of six aspects, namely epistemic, interactional, cognitive, affective, mediational, and ecological aspects. In-depth interviews were conducted to confirm participants' answers and arguments and were used as the bases for interpretation.

The data in this study were analysed using quantitative and qualitative methods. Quantitative methods were used to analyse participants' answers to each question displayed in the form of a percentage of the number of participants who answered correctly, especially the questions related to the use of knowledge of the concept of fractions in problem-solving activities. Meanwhile, qualitative methods were used to analyse the data collected from the interviews. The stages of data analysis in this study are as follows. First, examine participants' answers to each question in detail. The next stage is grouping the same or similar participant responses. The third stage is conducting in-depth interviews with representatives of participants in each answer group. The final stage is categorizing and presenting in the form of percentages.

4. Results and Discussion

4.1. Results

The mathematical dimension was accessed through three question items; two items for common knowledge and the other one for extended knowledge. In the aspect of common knowledge, the first question posed was related to the general understanding of fractions and the second question was a question in which the participants were asked to identify the fraction 1/4 from several visualizations given. In the results of participants' responses to the first question, as many as 80% of participants answered fractions were the same part of the whole and as many as

20% answered the definition of fractions in formal form, i.e. numbers can be expressed in the form a/b where a is called the numerator and b is called the denominator. Meanwhile, the participants' success in answering the second question is shown in Table 1.

Table 1. The number of participants answering the second question on the aspect of common knowledge.

Item No.	Aspect	Correct (%)	Incorrect (%)
2a	Common knowledge	6.67	93.33
2b	Common knowledge	100.00	0.00
2c	Common knowledge	20.00	80.00
2d	Common knowledge	100.00	0.00

Table 1 shows the errors that mostly occurred in items 2a and 2c. In general, there were two types of answers to these items, namely the first type which was answering the whole items correctly with the understanding the parts do not have to be congruent, the most important thing is the area is the same. The second type of answer was to only be able to correctly answer question items whose parts were partitioned into congruent parts. This type of answer was only able to be answered correctly on items 2b and 2d. Look at an example of an answer in Fig. 1. The participants' responses on all aspects of common knowledge show that there was a misunderstanding in interpreting fractions as part of a whole where the same parts were interpreted the same in shape and size.

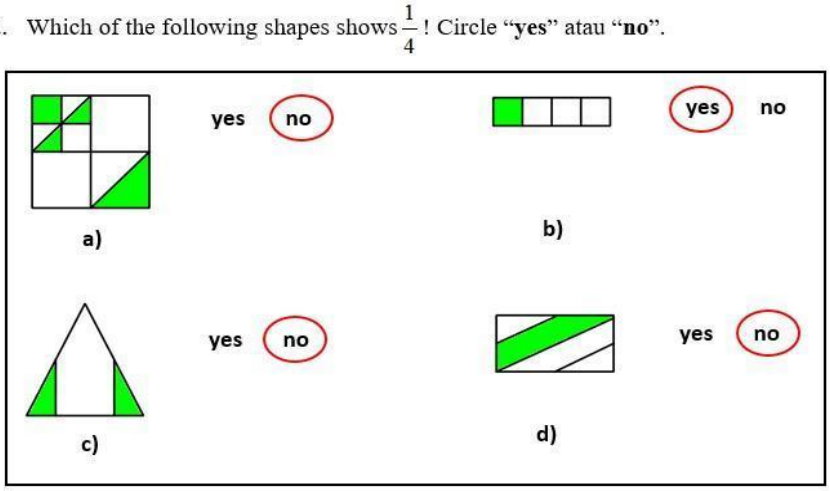

Fig. 1. Examples of participants' answers on the aspect of common knowledge.

In the aspect of extended knowledge, the questions asked were related to the formal definition of fractions, namely numbers can be expressed in the form where a and b are integers and $b \neq 0$. This definition is found in textbooks according to the curriculum and is used as a guide for teachers in teaching. As many as 80% of the participants agreed with the given formal definition and the rest expressed disagreement. The results of the interviews showed participants who agreed did not provide logical reasoning and tended to provide arguments that led to the belief

everything presented in the textbook was true. Meanwhile, those who disagreed provided arguments when using integers, then fractions can be negative and would look like a rational number.

The didactic dimension was accessed through questions specifically related to the knowledge in planning and executing the teaching of fractional concepts in elementary school. In the epistemic aspect, the questions related to the problem-solving ability of the participants in the form of mathematical problems were asked involving the concept of part of the whole in its completion.

The research findings show the success rate of participants in answering questions was very low as much as 25%. There were three types of answers generated, namely the first type which was the correct answer with the part of whole concept approach, the second type which was the correct answer with a less relevant approach, namely the square and triangle area approach, and the third type which was incorrect answer with the argument the answer provided was not a fraction because the shaded parts were not congruent.

In the interactional aspect, the questions asked were related to how the participants introduced fractions including mixed fractions to students. All participants provided the same response, namely connecting the part of the whole concept with real or everyday experiences generally experienced by students. In the mediational aspect, the questions asked were regarding the media used by the participants in teaching the concept of fractions. The participants' responses to this aspect complemented the responses to the interactional aspect. All participants provided the same response, namely using concrete media in introducing fractions to their students. The use of concrete media that is generally used is pieces of fruit or cake and the tendency to use it when introducing fractions to students for the first time.

In terms of the cognitive aspect, questions were asked regarding the teachers' experiences in identifying students' difficulties in learning fractions. There were two types of difficulties according to the experiences of the participants. The first was the difficulty in converting common fractions to mixed fractions or vice versa. The second difficulty was when learning about the operations of fractions, especially multiplication and division of fractions. In the affective aspect, the questions asked were related to the teachers' solution or strategy in overcoming students' difficulties following the responses to questions in the cognitive aspect.

The responses of all participants tended to be homogeneous and less innovative, that is overcoming difficulties by providing re-explanations with examples and their solutions. In the Ecological aspect, the questions asked were related to the knowledge about the position of the topic of fraction in the school curriculum. There were two types of responses to the questions in this aspect. The first type of response is the topic is very appropriate to be taught because mathematics begins to be taught separately in grade IV. Thus, it would be easier to be taught considering the scope of the topic of fractions is quite wide (decimal, operations). The second type of response emphasized trust in the curriculum and textbooks without further explaining why it appears in grade IV.

4.2. Discussion

In this section, we compared the results and current literature [11-25]. In the mathematical dimension, the findings of this study indicate teachers had a lack of

knowledge and even experienced misconceptions regarding the most basic concept of fractions, namely fractions as part of the whole. Fractions were defined as parts of a whole where parts were defined as equal parts in both shape and size. This misunderstanding was termed “congruent parts” [2, 19]. The findings in this study have also strengthened the results of other research which found most prospective teachers interpret "equal parts" as congruent forms and have implications for examples when teaching fractions to their students [13].

The findings and analysis of the common and extended knowledge aspects in this study strengthen the study of other researchers who stated one of the factors that contributed to the low understanding and misconceptions of students in the concept of fractions was the teachers’ lack of understanding regarding the content of the fractions themselves [4, 11, 14]. Teachers’ lack of understanding has implications for the way the teachers’ present fractions and the interventions provided during learning [3, 15]. Knowledge of mathematical content is important for teachers to ensure that teachers know the material they are teaching and recognize when the students provide wrong answers or incorrect definitions in textbooks. In addition, mathematical mastery provides a basis for teachers in developing quality learning designs [16, 20].

In the didactic dimension, especially the epistemic aspect, the research findings indicate the teachers had weak knowledge in terms of using the part of the whole concept in problem-solving activities. Most teachers were able to find answers but were less able to explain the reasons behind the problem-solving procedures they used. The epistemic aspect is an aspect of specific knowledge that is deeper than the aspect of the mathematical dimension. This knowledge is the basis for analysing and interpreting students’ work and influencing students’ learning outcomes [17, 21-23].

Regarding the findings on the cognitive facet aspect, based on the experience of the teachers, students generally have difficulty in arithmetic processes and operations on fractions. The arithmetic process of fractions is largely determined by students' knowledge of the arithmetic of whole numbers and the concepts and processes of producing fractions [18, 24, 25]. Thus, it is important to ensure the students have the correct understanding of the basic concepts of fractions before entering into arithmetic processes involving rules and procedures. Meanwhile, in other aspects of didactic dimensions, there was a tendency for the teachers to provide learning such as transfer of knowledge in textbooks, both in terms of scope and depth of the topic as well as learning trajectories including their way of thinking.

5. Conclusion

The research findings show teachers had limited mathematical knowledge and even tended to lack the concept of fractions. The findings have shown many teachers experienced misconceptions. Didactic knowledge was also still limited, and the teachers taught the students as presented in the textbooks, both in the scope of the topic, illustration examples, and the flow of thinking. This study did not investigate the meta-didactic-mathematical dimension of DMK related to the knowledge the teachers needed to reflect on their learning practices and find potential improvements to improve the learning process. Thus, it is necessary to conduct other research that specifically investigates lesson plans and how teachers carry out learning to complement the findings of this study.

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