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LEARNING OBSTACLES OF PROSPECTIVE MATHEMATICS TEACHER STUDENTS ON THE CONCEPT OF CHAIN RULES AND ALTERNATIVE DIDACTIC DESIGNS

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Abstract

This study aims to: (1) identify student learning obstacles in understanding the chain rule concept and (2) design alternative didactic designs based on learning obstacle findings. Didactic Design Research (DDR) with two interpretive and critical paradigms was chosen in this study. The study involved 46 students of second-semester mathematics teacher candidates at one of the universities in Indonesia. Data analysis of test results, interviews, and document studies was carried out through identification, clarification, reduction, and verification techniques and then presented narratively. The results showed that students still experienced the ontogenic type of learning obstacle; mentally, students were not ready to accept knowledge because the didactic design did not accommodate the linkage of prerequisite material with chain rules. Epistemological learning obstacles are also found due to limited understanding and mastery of the chain rule concept, which is only associated with specific contexts. Based on the findings of these learning constraints, an alternative didactic design was then compiled as a hypothetical didactic design for the chain rule concept.

Keywords: Didactical design research, Didactic design, Epistemological learning obstacles, Ontogenic obstacles.

1. Introduction

The derivation is a fundamental concept in differential calculus courses and is a prerequisite for several other ideas. In college, derivation is one of the mathematical concepts needed to learn other concepts, other subjects, or apply to the real world [1-3]. In the study program producing prospective mathematics teachers, differential calculus is a prerequisite for several courses and significantly affects learning outcomes [4]. Nevertheless, most undergraduate students still consider derivation a difficult concept [1, 2]. Some of the causes of these difficulties include the lack of conceptual understanding and the given learning system [5-7]. Students still have difficulty determining the derivatives of rational functions and chain rules, maximum and minimum values, and their application to real-world contexts and problems related to limits [8-10].

Each material in mathematics certainly has different characteristics, but the learning flow of all the materials is relatively the same. They generally start from a situation of action, formulation, validation, and institutionalization. These stages are recommended by the Theory of Didactical Situations (TDS) [11]. The Socratic Questioning technique can be used at any stage of TDS, where learning is guided by questions posed to promote students' independent thinking. Higher-order thinking skills are present when students think, discuss, debate, evaluate, and analyse concepts through their thoughts and those around them [12, 13].

As revealed from various studies, the difficulties experienced by students are expected to be experienced by prospective mathematics teachers. There are differences in the image of the teacher's concept with the scientific conception of the derived concept [14]. It is necessary to conduct a study related to learning obstacles for prospective mathematics teachers because they will spearhead the success of the learning process. Knowing prospective mathematics teachers' learning obstacles will help lecturers develop a hypothetical didactic design. This study will focus on prospective mathematics teachers, especially learning obstacles on the topic of chain rules, considering that this concept requires a good understanding of composite functions. The aims of this study are (1) to identify various types of learning obstacles experienced by prospective mathematics teachers on the topic of chain rules and (2) to design alternative didactic designs based on the findings of learning obstacles. This qualitative research used the Didactical Design Research (DDR) design developed in 2010 [15]. DDR is a form of educational innovation that explores learning design characteristics and its impact on developing students' thinking processes [16, 17]. The paradigm used in this qualitative research is the interpretive paradigm [18, 19].

2. Literature Review

Some students may have difficulty understanding the material, which is caused by various factors. Problems caused by external factors are called learning obstacles. Students' prerequisite knowledge, how to teach or present material in textbooks, and students' limited understanding cause obstacles to student learning [16]. These obstacles are classified as ontogenic obstacles, didactic obstacles, and epistemological obstacles [16]. Meanwhile, other researchers distinguish four types of obstacles: cognitive obstacles, psychological obstacles, didactical obstacles, and epistemological obstacles [20]. This research will focus on ontogenic and epistemological learning obstacles.

The derivative of the function $y = f(x)$ is defined as the rate of change in x, denoted as $\frac{dy}{dx} = f'(x)$. By understanding the derivative as the rate of change, the derivative of the composition function of fog is $(fog)'$. If $u = g(x)$ is differentiable at x and f is differentiable at $u = g(x)$, then the composition function $fog)(x) = f(g(x))$ is differentiable at x, with the function notation and based on the chain rule, then the derivative of fog is $(fog)'(x) = f'(g(x)).g'(x)$[21]. Meanwhile, using Leibniz notation, the derivative of the composition function $y = (fog)(x) = f(u)$ with $u = g(x)$ is expressed in the form $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

3. Method

This qualitative research used the Didactical Design Research (DDR) design developed in 2010. The interpretive paradigm underlies us to understand the didactic design problems of the topic of the chain rule in the sourcebook. Data sources are the results of the Respondent's Ability Test (RAT), interviews, and document studies. Data analysis of RAT results in learning obstacles was carried out qualitatively, analyzed simultaneously through data reduction techniques, and then presented narratively. Based on the learning obstacle found in the interpretive paradigm, an alternative didactic design was designed on the topic of chain rules based on the critical paradigm. The tasks developed were based on the TDS combined with the Socratic Questioning Technique, then tested on prospective mathematics teacher students. The research participants were 46 second-semester students of the Mathematics Education Study Program at one university in West Java, Indonesia, aged between 18 to 21 years, of which 39 were women, and 7 were men. The participants are taking a Differential Calculus course and have learned the basic concepts of derivation. Interviews with some students were conducted as an attempt to clarify and were completed after we gave tests related to the topic of chain rules.

4. Results and Discussion

The ability of students to determine the derivative of a function using the chain rule depends on their understanding of various derivative search rules and the concept of composite functions. In the reference book, the idea of compositional function is presented at the beginning of the chapter. At the same time, the derivative search rule is given before the concept of the chain rule. Based on this, the authors designed a test called the Responsive Ability Test (RAT), which was intended to identify various obstacles experienced by students. The test consists of 2 questions involving the concept of the chain rule, the product derivative, and the quotient of two functions. Based on the results of the RAT, we identified various learning obstacles experienced by prospective mathematics teacher students. In this study, two types of learning obstacles were found.

4.1. Learning obstacles encountered by students in the ontogenic obstacle type

Obstacles experienced by prospective mathematics teacher students occur because of mental readiness in receiving knowledge. This obstacle is identified when students work on the following questions: Determine the derivative of the following function to the given independent variable (i.e., $\frac{dy}{dx}$ for part a and $\frac{d\theta}{dr}$ for part b; corresponding to equations (1) and (2), respectively), presenting the result in its simplest form.

a. $y = (\frac{x^3-1}{2x^3+1})^3$ (1)

b. $\theta = r\sqrt{2-3r^2}$ (2)

Various problems were identified. Students could not solve the issues properly. These include (a) not mastering technical matters related to the chain rules; (b) weak understanding of the derivative of quotient and product two functions; (c) the didactic situation faced is different from the usual one. Some answers according to the identified problems are shown in Fig. 1.

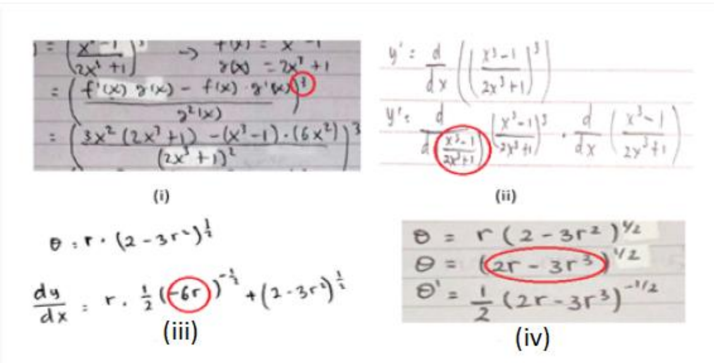

Fig. 1. Students do not understand the concept of the chain rule.

From Fig. 1, the students did not understand the chain rule concept due to their weak understanding of the composite function concept. Students do not understand which functions work first and which functions follow them. As shown in part (iv), students' answers occur in addition to not understanding the chain rule; and not mastering the exponent's nature.

Some students understand the chain rule concept but cannot solve the problem correctly. A weak understanding of the prerequisite material is related to the rules for finding the derivative of a function, especially the derivative of the product of two functions and the derivative of the quotient of two functions. Weak prerequisite knowledge is revealed from students' answers, as shown in Fig. 2.

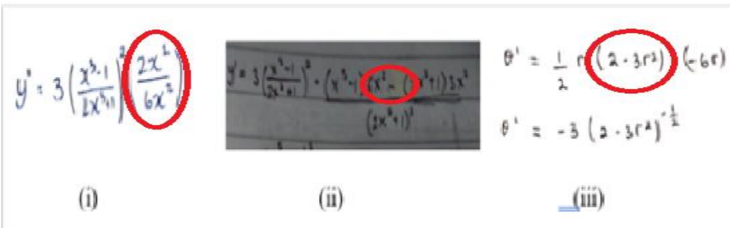

Fig. 2. Students do not understand the prerequisite material.

In addition to the answers to the RAT, from the results of interviews with students and document studies, the following information was obtained: (a) students had difficulty in determining the derivative of a composite function, which involved the derivative of the product and the quotient of two functions; (b) some students stated the derivative as $y = \frac{u}{v}$ is $y' = \frac{u'}{v'}$; (c) students do not understand how the chain rule works because they do not understand which function should be derived first, and which function will follow it; (d) some students know how the chain rule works but are weak in prerequisite material; (e) The practice questions given are limited to simple composition functions.

The difficulties experienced by these students are ontogenic obstacles of various types. Students do not understand technical matters related to composite function derivation, which function is derived first, and which function follows it; as shown in Fig. 1(i), this is categorized as an instrumental type for ontogenic obstacle. The didactic situation presented is different from the one usually faced, causing students not to be able to solve the problem properly; as shown in Figs. 2(i) and 2(iii), this is categorized as an ontogenic obstacle of conceptual type. Weak prerequisite knowledge is the cause of student abstraction, which is classified as an ontogenic psychological type obstacle, as shown in Figs. 1(ii) and 2(ii). Learning obstacles can be caused by mental readiness and cognitive maturity in receiving knowledge or can also be related to the level of difficulty of didactic situations causing students to be hampered in participating in the learning process [17].

4.2. Learning obstacles experienced by students in the epistemological obstacle type

Obstacles occur due to students' limited understanding of concepts, problems, or others related to chain rules. Their understanding is only associated with a context that is too narrow according to their experience. From Fig. 1(iii), students do not understand the question, and the variables in the questions are not interpreted properly. The derivative notation is not written correctly, this is because students are accustomed to using derived notation with $\frac{dy}{dx}$. In Fig. 1(iv), students cannot use the exponential property because they are used to the powers of integers.

The interview results showed that students are familiar with the notation function $y = f(x)$ and its derivative as $\frac{dy}{dx}$. When the form in the equation in the form of $y = \sqrt{2 - 3x^2}$, students can use the chain rule and develop it into the form $y = x\sqrt{2 - 3x^2}$, and students can determine the correct derivative. Based on this, the learning obstacles experienced are categorized as epistemological obstacles.

There are still fundamental errors in the derivative concept of students majoring in mathematics [22]. The understanding of the derivative concept is still weak and is a concept that is considered difficult by students [1, 22]. According to TDS, the stages of didactic design are action, formulation, validation, and institutionalization [23]. We designed a didactic design based on the concept of the chain rule. Didactic design based teaching is one of the effective methods for improving students' understanding [24-26]. The didactic design developed includes (1) topics and sub-topics; (2) predicting student responses; (3) didactic and pedagogical anticipation; and (4) objects and mathematical abilities developed. The didactic and pedagogical anticipation components contain the stages of learning according to the TDS. TDS stages are (a) action situations, presenting problems related to composite functions to stimulate students to think and realize the importance of chain rules; (b) formulation situations, leading students to understand the concept of chain rules; (c) validation situations, improvement processes or strengthening of certain concepts if some students have different formulations or appear wrong constructions; and (d) institutionalization situations, students can apply the chain rule to other problems in different contexts. The Socratic Questioning technique guides students in understanding the concepts at each stage of alternative didactic design. The didactic design format includes didactic and pedagogical anticipation, objects, and mathematical abilities developed (see Table 1).

Table 1. Chain rule didactic design format snippet.

Didactic and Pedagogical Anticipation	Developed Maths Objects and Abilities
Action Situation Given the following didactic situation: a. $y = \sqrt[3]{x^2 + 3x}$ b. $y = (x^3 + x)^{100}$ c. $y = cos\ (2x^2 + 1)$ Can the various derivation rules of functions help you determine the derivatives of these functions? **Formulation Situation** The Socratic Questioning technique guides students in understanding why the Chain rule is needed and how to determine the derivative of a function using the chain rule Some key questions: ● Can you determine the derivatives of the above functions? Why? ● Etc **Validation Situation** Validation of the meaning of the Chain rule is done through class discussion with lecturer intervention if needed. Etc **The institutionalization** stage is facilitated by sample questions that require students to participate in determining solutions actively. Etc	Understanding the Chain rule with two different writing styles: 1. Functional Notation: $(fog)'(x) = f'\big(g(x)\big).\, g'(x).$ 2. Leibniz Notation $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

5. Conclusion

This study concludes that some prospective mathematics teacher's students still experience two types of learning obstacles in the chain rule. The two types of learning obstacles are:

(i) Ontogenic type, students cannot solve problems due to mental unpreparedness marked by weak mastery of prerequisite material.

(ii) Epistemological type, students cannot solve problems due to limited understanding of the chain rule concept, adapted to their experience.

The didactic design developed based on learning obstacles, following the TDS stage and Socrates Questioning Techniques, is considered capable of overcoming the learning obstacles experienced by prospective mathematics teacher students.

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