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AMERICAN CALL OPTION PRICING WITH VARIABLE MATURITY DATES AND THE BINOMIAL METHOD IMPLEMENTATION

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Abstract

The American option has no analytical solution for pricing options. It only uses statistical approaches, such as the Binomial Method to determine the derivative values. The influential factors of American call option price to use the binomial method include the current stock price (S_0), the strike price (K), the central bank rate (r), the expiration date (T), the factor of stock price increase (u), the factor of stock price decrease (d), the probability of an increase in stock price (p), and the probability of a decrease in stock price (q). The empirical case study on various stock prices found the mean square error, MSE, of the American call option price, with the implementation of a binomial method, was nearly close to the market call option with the implementation of fixed maturity date and different maturity date.

Keywords: American call option, Binomial method, Mean square error.

1. Introduction

Current financial investments grow significantly. The evidence is observable from many investors investing their capital and financial assets. Investors may apply various methods to invest, such as investing in stocks, bonds, or derivative instruments. A derivative instrument is derived from other related securities. Derivative instrument value depends on the predetermined principal securities as measures. Several derivative instruments include options, warrants, and future contracts. One of the most traded derivative instruments is an option, a contract between two parties. With this contract, a party shares the rights with the other party to buy or sell an asset at a certain price at a certain moment. Options trading has been growing significantly, seen from many traded options in exchange centres, such as the European Options Exchange, Amsterdam; and a small-sized exchange in Sydney.

From the perspective of the right transfer to another holder, an option has two types: a call option and a put option. The call option shares the holder the right to buy certain stocks at a certain price at a certain moment. On the other hand, the put option shares the holder the right to sell certain stocks at a certain price at a certain moment.

Based on the period, stock option consists of American stock option and European stock option. The American stock option refers to a stock option that is exercisable before the expiration or within the expiration. Besides, European stock refers to an exercisable stock option only within the expiration. In the market, the most traded stock option is the American stock option because the American stock option is flexible [1]. This stock also generates higher profits than the European stock option. In this perspective, investors may have an opportunity to earn profits in every market situation as long as they can make a proper investment strategy on the stock option contracts. The key to getting profits from American stock options is the accuracy of price determination and the execution limit of the stock option.

Many applicable methods for option pricing. In 1973, Black and Scholes [2] published option price calculations that became the basis for the development of option price calculations. Many researchers have developed pricing options. Shi et al. [3] formulated a simple option price based on the Fourier cosine formula. Qian et al. [4] introduce a genetic algorithm, namely the GA-BP neural network, an excellent pricing option to determine option prices. Glau and Wunderlich [5] combined the parametric PDE method with a deep neural network for calculating option prices. Hu et al. [6] presented the calculation of option prices with a generalized Jarrow-Rudd method. Phaochoo et al. [7] provided solutions to the fractional Black-Scholes equation using the Meshless Local Petrov-Galerkin (MLPG) numerical technique constructed by the moving kriging approximation to European call or put option pricing. In the same year, they also provided numerical experiments on the European option pricing using the MLPG method with the moving kriging interpolation [8]. Moretto et al. [9] introduced an option pricing model with stochastic volatility by underlying the fat-tailed assets. Orosi [10] established a basic interpolation model for calculating the price of the American option. Alghalith [11] developed a simple, exact, explicit, and analytical solution for partial differential equations in the American option using the Black Scholes pricing formula.

Another alternative calculation model is the binomial model of European option calculations, proposed by Cox, Ross, and Rubinstein (CRR) [12]. Muroi and Saeki [13] introduced a binomial tree model for calculating option prices using the Discrete Carr and Madan formula approach. In 2022, Muroi and Suda [14] introduced the calculation of option prices using a binomial tree with a new approach to discrete cosine transformation. Ghafarian et al. [15] presented the European option pricing using binomial trees by entering Greek letters to correct the accuracy of the interval price. Researchers have developed various binomial models of option pricing calculations. Shvimer and Herbon [16] compared some methods of calculating the binomial model options to determine the model provided maximum profit. Kim et al. [17] developed a binomial model of CRR in 2 ways. Those were a binomial model with time parameters and a trinomial model with the geometric Brownian motion adaptation.

The American stock option, on contrary, has no analytic solution for determining its option price. Thus, the option price determination applies a numerical approach [18-23]. The American option price receives various numerical solutions, including the Brennan-Schwartz algorithm [18], the efficient numerical method [23], the front-fixing exponential time differencing (ETD) [24], and discontinuous Galerkin finite elements [25]. One simple, efficient, and applicable numerical approach to determine the value of the derivative is the Binomial Method [19, 26]. Some researchers, such as Primandari and Abdurakhman [27], applied the binomial method to determine the option price. The researchers discussed the Repeat Richardson Extrapolation Technique on Flexible Binomial Model to determine the American Put option price. The binomial model provided a convergent value in the calculation of the American option price [28, 29]. The second researcher, Orosi [30], developed the binomial model by introducing the alternative derivation with binary call assumption on determining option price. Merdekawati et al. [28] applied the binomial model to compare the value of American options with discrete and continuous dividends.

This article discusses the American call option pricing with the Binomial Method implementation. Stock prices in the market change and fluctuate from time to time. In this research, the researchers used the two-way change possibility as the base of the binomial method. The researcher determined the American call option price by assuming the traded stock option did not pay the dividend. The discussed problems in this research are the determination of the American call option with the binomial method implementation and the difference between the American call option price with binomial method implementation and the option price in the market.

2. Material and Method

This research applied a literature study approach by analysing the problem, simulating the final result of the model, and drawing a conclusion. In this literature study, the researcher identified the problem; studied the related supporting theories of the option - American option; and implemented the binomial method. The researchers determined the American option price because the calculation had no analytical solution. Therefore, the calculation required numerical completion. Primandari and Abdurakhman [27] applied a repetitive-numerical solution, the Richardson Extrapolation technique, on a flexible binomial model to determine the American put option price. This technique, the Richardson Extrapolation technique, applied two steps to determine the American put option price. The steps were: determining the row of option value approaches with the flexible binomial implementation and extrapolating the row of option values repetitively. The implementation of the Richardson Extrapolation on the flexible binomial model showed the convergent row. As a consequence, the process required a faster

American put option price determination. Zhao [19] compared eight numerical methods in determining the value of American options. The results showed that the binomial method was a numerical method with better efficiency and accuracy than other numerical methods.

Emmanuel et al. [26] used the binomial model in calculating European and American option prices. The underlying reason for applying the binomial model was the simplicity and flexibility of the model to be applied in various types of options. Emmanuel et al explained that the binomial model was an excellent model for American option implementation with early exercise property because the model considered the cash flow for each period. Merdekawati et al. [28] used the binomial method to calculate American options with discrete and continuous dividends. The results indicated that the larger partition implementation (*n*) led to more accurate and convergent American option prices at a certain value. This result showed the advantage of applying the binomial model. Primandari and Abdurakhman [27], Zhao [19], Emmanuel et al. [26], and Merdekawati et al. [28] used the numerical solution on determining American option prices. They found the advantages of the binomial method in calculating options prices, such as the implementation ease and the compatibility with the American call option.

The researcher analysed and simulated the problem by analysing the literature. This process allowed the researcher to formulate the determination of American option prices with the binomial method implementation. After establishing the formula, the researcher simulated the formula with the required data to determine the American option price. Then, the researcher concluded the results of the analysis and the discussion promoted in the previous phases.

3. Binomial Method

Stock prices in the market fluctuate from time to time. In this research, the base of the binomial model was the two-way change possibility. Figure 1 illustrates the given stock price, $t = 0$, while the option making is S_0. Based on the binomial method, the stock price at $t = T$ fluctuates. The stock price increases along with the probability p to be $S_u = u\,S_0$ or the stock price decreases along with probability $q = 1 - p$ to be $S_d = d\,S_0$ in which $u > 1$ and $d < 1$.

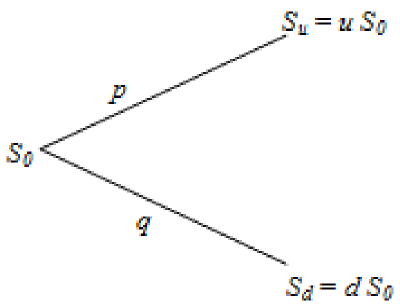

Fig. 1. Stock price changes in one binomial period.

The change in stock price at $t = T$ influences the option values. The call option value at $t = 0$ has the option of V. The stock price change at $t = T$ increases to S_u and makes the call option value to be $V_u = \max(0, S_u - K)$. The decreased stock price to S_d makes the call option value to be $V_d = \max(0, S_d - K)$. Figure 2 illustrates the fluctuation of the option value.

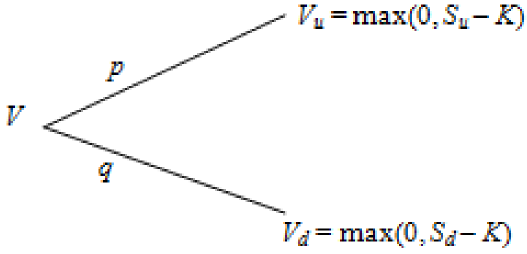

Fig. 2. Option price changes in one binomial period.

The binomial method changes T continuously to be (t), time, which has discrete elements of i. The nearly continuous result of I requires a partitioned T by n, a large number with ($n \rightarrow \infty$). The n variable refers to the number of time intervals, in which $\Delta t = T/n$, $t_i = i \, . \, \Delta t$, with i as the number of a partition $i = 0, 1, 2, \ldots, n$, and $S_i = S(t_i)$. S_i is an asset base price at t_i.

Some applicable assumptions in this model are:

X1) Price S, as the initial price, at a period of Δt can only change in two probabilities: increasing S_u or decreasing S_d with $0 < d < u$. In this assumption, u and d respectively are the constant fluctuation factors for each Δt.
X2) The probability of increase is p, $P(\text{up}) = p$. Therefore, $P(\text{down}) = q = 1 - p$.
X3) Stock price expectation is random and continuous, with risk-free interest rate r, from S_i at t_i to be S_{i+1} at t_{i+1} is:

$$E(S_{i+1}) = S_i . e^{r\Delta t} \tag{1}$$

X4) The subsequent assumption is - no dividend payments during a certain period.

The binomial model fosters a scheme (tree) to fluctuate stock prices discretely, as seen in Fig. 3.

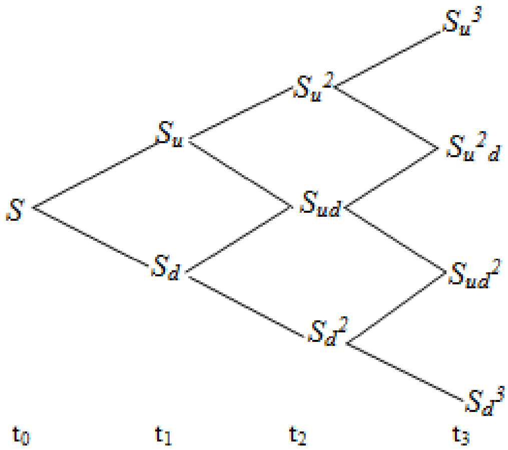

Fig. 3. Scheme of stock price fluctuation in binomial.

Given the stock price at $t = t_0$ is $S_0 = S_{00} = S$, and the stock price at $t = t_1$ is $S_{01} = S_d$ or $S_{11} = S_u$. The moment of $t = t_2$, stock price becomes $S_{02} = S_d^2$, $S_{12} = S_{ud}$, or $S_{22} = S_u^2$. Then, at $t = t_3$, the stock price becomes $S_{03} = S_d^3$, $S_{13} = S_{ud}^2$, $S_{23} = S_u^2 d$, or $S_{33} = S_u^3$. Fig. 3 illustrates the stock price fluctuation. Generally, the stock price at $t = t_i$ has $i+1$ possibly with the general formula.

$$S_{ji} = S_0 u^j d^{i-j} \tag{2}$$

with $i = 0, 1, \ldots, n$ and $j = 0, 1, \ldots, i$.

The American option allows the option execution before the due date, $t \leq T$. Thus, the option execution needs the option value calculation for t_i in which $i = n - 1$, $n - 2, \ldots, 0$. The calculation shows a possibility of option values at the given times. In this case, the option values are better than at maturity.

Equation (2) is not recursive, or the equation takes a long calculation. The recursive shape of the option value, V, applies the Backward method starting from period t_{n-1}, t_{n-2}, until t_2, t_1, t_0 is

$$V_{ji} = e^{-r\Delta t} E\left(V_{j,i+1}\right) = e^{-r\Delta t}\left(V_{ji} e^{r\Delta t}\right) = e^{-r\Delta t}\left(pV_{j+1,i+1} + qV_{j,i+1}\right)$$
(3)

So, for the option values for American Call Option

$$V_{jn} = max\left(S_{jn} - K, 0\right) \text{ for } i = n \tag{4}$$

$$V_{ji} = max\left\{max\left(S_{ji} - K, 0\right), e^{-r\Delta t}\left(pV_{j+1,i+1} + qV_{j,i+1}\right)\right\} \text{ for } i \neq n \tag{5}$$

and for the American Put Option

$$V_{jn} = max\left(K - S_{jn}, 0\right) \text{ for } i = n \tag{6}$$

$$V_{ji} = max\left\{max\left(K - S_{ji}, 0\right), e^{-r\Delta t}\left(pV_{j+1,i+1} + qV_{j,i+1}\right)\right\} \text{ for } i \neq n \tag{7}$$

for $i = 0, 1, \ldots, n$ and $j=0, 1, \ldots, i$.

The previous explanation shows the steps of applying the Binomial method to calculate the American option price. The steps are:

1) Determining stock price S_{ji} using Eq. (2)

   $S_{ji} = S_0 u^j d^{i-j}$, $i = 0, 1, \ldots, n$ and $j = 0, 1, \ldots, i$.

2) Calculating the option profit on due date t_n with the equations of (4) and (6) for $i = n$.
3) Determining the option price with the backward method, starting from period t_{n-1}, t_{n-2}, until t_2, t_1, t_0, with Eqs. (5) and (7) for $i \neq n$.

In the binomial method, the unknown parameters are u, d, and p. Thus, the next step is - looking for the parameters of u, d, and p. In this study, the researcher applied the proposed step by Aziz [31] in determining the parameters, u, d, and p. The steps required three equations:

E1) Equating expectations of stock prices of discrete models with continuous models.
E2) Equating the discrete model variance with a continuous model.
E3) Equating $u. d = 1$.

The consequence based on X1) and X2) assumptions on the binomial method for the discrete model leads to Eq. (8).

$$E(S_{i+1}) = S_i(pu + (1-p)d) \tag{8}$$

S_i is any value for t_i, which randomly changes to S_{i+1}. Equation (1) is the continuous stock price expectation while Eq. (8) is the discrete stock price expectation. So, based on E1), the researcher obtained Eq. (9).

$$S_i e^{r\Delta t} = S_i(pu + (1-p)d) \tag{9}$$

This equation is the required first model to determine *u, d, p*. After completing Eq. (9) for *p*, the researcher obtains Eq. (10).

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{10}$$

where p is an opportunity that must meet $0 \leq p \leq 1$ with the conditions of $d \leq e^{r\Delta t} \leq u$.

The inequalities are correlated to the fluctuation, rising, and falling of the asset prices to the risk-free interest rate *r*. The inequalities of $d \leq e^{r\Delta t} \leq u$ are not new assumptions with the no-arbitrage principle of $0 < d < u$. From the continuous stock price expectation model, the researcher obtained $E(S_{i+1}^2)$.

$$E(S_{i+1}^2) = S_i^2 e^{(2r+\sigma^2)\Delta t} \tag{11}$$

The researcher obtained the variance of the stock price for a continuous model from Eqs. (1) and (11).

$$Var(S_{i+1}) = S_i^2 e^{2r\Delta t}(e^{\sigma^2 \Delta t} - 1) \tag{12}$$

By contrast, the implementation of Eq. (8) makes the variance for the discrete model fulfil Eq. (13).

$$Var(S_{i+1}) = S_i^2(pu^2 + (1-p)d^2 - e^{2r\Delta t})Var(S_{i+1}) = S_i^2(pu^2 + (1-p)d^2 - e^{2r\Delta t})Var(S_{i+1}) = S_i^2(pu^2 + (1-p)d^2 - e^{2r\Delta t}) \tag{13}$$

So, by making two variances from Eqs. (12) and (13) equal, the result becomes Eq. (14).

$$p = \frac{e^{(2r+\sigma^2)\Delta t} - d^2}{u^2 - d^2} \tag{14}$$

Next, by making Eqs. (10) and (14) equal also with the given $u \,.\, d = 1$, the result becomes Eq. (15).

$$u^2 - 2\beta u + 1 = 0 \tag{15}$$

where $\beta = \frac{1}{2}\left(e^{-r\Delta t} + e^{(r+\sigma^2)\Delta t}\right)$

Roots of Eq. (15) are $u = \beta \pm \sqrt{\beta^2 - 1}$ with $\beta^2 - 1 > 0$.

Since $d < u$, the researcher selected $u = \beta + \sqrt{\beta^2 - 1}$.

Next, with an approximation of exponential numbers $e^x \approx 1 + x$, the researcher obtained the values of *u* and *d*.

$$u = e^{\sigma\sqrt{\Delta t}} \tag{16}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \quad (17)$$

4. Results and Discussion

The researcher took two study case examples with the implementation of a binomial method to determine the American call option price. In this research, the applied stock price data was from Yahoo Finance [32]. On the other hand, the researcher obtained the Federal Reserve Interest Rate data from Global-rates.com [33]. The first study case applied to 5 stocks with the same maturity period but different contract prices. The study took the data on July 25, 2022. The applied maturity period was T = 53 days. On the day, the Fed-Interest Rate was r = 1.75% while the partition was n = 100. Table 1 presents the stock price data and the calculation result. From Table 1, the MSE binomial method option price is close to the market option price, 10.69%. Empirically, the results showed the American call option price with the same maturity period with binomial method approach calculation of the market option price.

The second study case applied to one stock, the MSFT (Alphabet Inc.) stock with the same contract price but a different maturity period. The researcher took the stock price on July 25, 2022. On the day, the stock price S_0 was 258.83 with the chosen contract price of K = 265, r = 1.75%, and taken partition of n = 100. Table 2 shows the calculation results of the option call price with different maturity periods. Empirically, the table shows the price of the American call option with different maturity periods based on the binomial method approach calculation of the market option price. The obtained Mean Square Error is 12.76%. From the tables, Tables 1 and 2, the binomial method approach could calculate the American call option on the market option price. Thus, the binomial method was applicable in determining the price of the American call option with the same maturity period and also with different maturity periods.

Table 1. Calculation table of American stock option price using binomial method.

No.	Stock	S_0	K	C Market (X_1)	C Binomial (X_2)	$(X_1 - X_2)^2$
1	MSFT (Alphabet Inc.)	258.83	270	7.07	6.6437	0.1817
2	AAPL (Apple Inc.)	152.95	160	0.92	0.7813	0.0192
3	ADBE (Adobe Systems Incorporated)	391.96	405	1.87	1.7835	0.0075
4	AON (Aon plc)	281.16	290	8.2	8.5183	0.1013
5	META (Meta Platform, Inc)	166.65	175	10.20	10.6742	0.2249
					SSE (sum square error)	0.5346
					MSE (mean square error)	0.1069

Table 2. Calculation table of American call option prices with different maturity periods.

No.	Maturity Dates	T	C Market (X_1)	C Binomial (X_2)	$(X_1 - X_2)^2$
1	29 July 2022	4	3.8	3.4836	0.1001
2	5 August 2022	11	4.8	4.5720	0.0520
3	12 August 2022	18	5.5	5.1923	0.0947
4	19 August 2022	25	6.43	6.0341	0.1567
5	26 August 2022	32	6.95	6.4942	0.2078
6	2 September 2022	39	8.05	7.5832	0.2179
7	16 September 2022	53	9.59	9.2635	0.1066

8	21 October 2022	88	12.95	12.6584	0.0850
				SSE (sum square error)	1.0208
				MSE (mean square error)	0.1276

5. Conclusions

The result and discussion showed that the binomial method was applicable to determine the American call option price. The influential factors of the American call option price included the stock price (S_0), the contract price (K), the central bank interest rate (r), the maturity period (T), the stock price increasing factor (u), the stock price decreasing factor (d), the stock price increase probability (p), and the stock price decrease probability (q). This study determined the performance of the binomial method in the calculation of American call option price. In the first case, the researchers took 5 stocks with the same maturity period. In the second case, the researcher took one stock with different maturity periods. The empirical findings of the first and second study cases showed the obtained Mean Square Error, MSE, of the American call option price with binomial method approach implementation the market option price with the same maturity period and different maturity period.

References

1. Hull, J.C. (2021). *Options*, futures, and other derivatives. Global edition, (11^{th} ed.), Prentice-Hall.
2. Black, F.; and Scholes, M. (1973). The pricing of options corporate liabilities. *The Journal of Political Economy*, 81(3), 637-654.
3. Shi, Z.; Lyu, P.; and Ma, J. (2022). High-order methods for option pricing under multivariate rough volatility models. *Computers & Mathematics with Applications*, 139, 173-183.
4. Qian, L.; Zhao, J.; and Ma, Y. (2022). Option pricing based on GA-BP neural network. *Procedia Computer Science*, 199, 1340-1354.
5. Glau, K.; and Wunderlich, L. (2022). The deep parametric PDE method and applications to option pricing. *Applied Mathematics and Computation*, 432, 127355.
6. Hu, Y.; Lindquist, W.B.; Rachev, S.T.; Shirvani, A.; and Fabozzi, F.J. (2022). Market complete option valuation using a Jarrow-Rudd pricing tree with skewness and kurtosis. *Journal of Economic Dynamics and Control*, 137, 104345.
7. Phaochoo, P.; Luadsong, A.; and Aschariyaphotha, N. (2016). The meshless local Petrov-Galerkin based on moving kriging interpolation for solving fractional Black-Scholes model. *Journal of King Saud University - Science*, 28(1), 111-117.
8. Phaochoo, P.; Luadsong, A.; and Aschariyaphotha, N. (2016). A numerical study of the European option by the MLPG method with moving kriging interpolation. *SpringerPlus*, 5(1), 305.
9. Moretto, E.; Pasquali, S.; and Trivellato, B. (2016). Option pricing under deformed gaussian distributions. *Physica A: Statistical Mechanics and Its Applications*, 446, 246-263.

10. Orosi, G. (2017). An interpolation-based approach to American put option pricing. *Springer Proceedings in Mathematics and Statistics*, 190, 167-175.
11. Alghalith, M. (2018). Pricing the American options using the Black-Scholes pricing formula. *Physica A: Statistical Mechanics and Its Applications*, 507, 443-445.
12. Cox, J.C.; Ross, S.A.; and Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3), 229-263.
13. Muroi, Y.; Saeki, R.; and Suda, S. (2021). Binomial tree method for option pricing: discrete Carr and Madan formula approach. *International Journal of Financial Engineering*, 8(2), 2150024.
14. Muroi, Y.; and Suda, S. (2022). Binomial tree method for option pricing: Discrete cosine transform approach. *Mathematics and Computers in Simulation*, 198, 312-331.
15. Ghafarian, B.; Hanafizadeh, P.; and Qahi, A.H.M. (2018). Applying Greek letters to robust option price modelling by a binomial tree. *Physica A: Statistical Mechanics and Its Applications*, 503, 632-639.
16. Shvimer, Y.; and Herbon, A. (2020). A comparative empirical study of binomial call-option pricing methods using S&P 500 index data. *The North American Journal of Economics and Finance*, 51, 101071.
17. Kim, Y.S.; Stoyanov, S.; Rachev, S.; and Fabozzi, F.J. (2019). Enhancing binomial and trinomial equity option pricing models. *Finance Research Letters*, 28, 185-190.
18. Madi, S.; Cherif Bouras, M.; Haiour, M.; and Stahel, A. (2018). Pricing of American options, using the Brennan-Schwartz algorithm based on finite elements. *Applied Mathematics and Computation*, 339, 846-852.
19. Zhao, J. (2018). American option valuation methods. *International Journal of Economics and Finance*, 10(5), 1.
20. Liu, C.; Schellhorn, H.; and Peng, Q. (2019). American option pricing with regression: convergence analysis. *International Journal of Theoretical and Applied Finance*, 22(8), 1950044.
21. Lee, J.K. (2020). A simple numerical method for pricing American power put options. *Chaos, Solitons & Fractals*, 139, 110254.
22. Yan, D.; Zhu, S.P.; and Lu, X. (2020). A numerical study of the utility-indifference approach for pricing American options. *Computers & Mathematics with Applications*, 80(5), 894-905.
23. Lee, J.K. (2021). An efficient numerical method for pricing American put options under the CEV model. *Journal of Computational and Applied Mathematics*, 389, 113311.
24. Company, R.; Egorova, V.N.; and Jódar, L. (2021). A front-fixing ETD numerical method for solving jump-diffusion American option pricing problems. *Mathematics and Computers in Simulation*, 189, 69-84.
25. Kozpınar, S.; Uzunca, M.; and Karasözen, B. (2020). Pricing European and American options under the Heston model using discontinuous Galerkin finite elements. *Mathematics and Computers in Simulation*, 177, 568-587.

26. Emmanuel, F.S.; Adedoyin, A.O.; and Hammed, O.O. (2014). Performance measure of binomial model for pricing American and European options. *Applied and Computational Mathematics*, 3(6-1), 18-30.
27. Primandari, A.H.; and Abdurakhman. (2014). Teknik ekstrapolasi richardson berulang pada model binomial fleksibel untuk menentukan harga opsi jual Amerika. *IndoMS Journal on Statistics*, 2(1), 39-28.
28. Merdekawati, D.A.; Norasia, Y.; Kumalasari, C.J.; and Putri, E.R.M. (2020). Comparison of American binomial options with discrete and continuous dividend. *International Journal of Computing Science and Applied Mathematics*, 6(2), 54-58.
29. Riaman; Parmikanti, K.; Irianingsih, I.; Joebaedi, K.; and Supian, S. (2019). Convergence of binomial tree methods to Black Scholes model on determining stock option prices. *IOP Conference Series: Materials Science and Engineering*, 567(1), 012013.
30. Orosi, G. (2015). A simple derivation of risk-neutral probability in the binomial option pricing model. *International Journal of Mathematical Education in Science and Technology*, 46(1), 142-147.
31. Aziz, A. (2009). Empat model aproksimasi binomial harga saham model black-scholes. *CAUCHY*, 1(1), 15-24.
32. NasdaqGS. (2022). *NasdaqGS Real Time Price on Yahoo Finance*.
33. Global-rates.com. (2022). *FED federal funds rate, the American central bank's interest rate*.