Journal of Engineering Science and Technology
21st EURECA 2024 Special Issue August (2025) 172 - 186

ARTIFICIAL INTELLIGENCE BASED NEW APPROACHES FOR IMPULSIVE NOISE REDUCTION TECHNIQUES

V. SARAVANAN*, N. SANTHIYAKUMARI

Department of Electronics and Communication Engineering,
Knowledge Institute of Technology, Salem , India
*Corresponding Author: vsece@kiot.ac.in

Abstract

This research presents a novel artificial intelligence-based strategy for alleviating impulsive noise within active noise control (ANC) systems. The approach integrates adaptive thresholding and soft thresholding methodologies with Median Absolute Deviation (MAD) analysis. Initially, an adaptive threshold $T(n)$ is defined using user-specified parameters α and β. This dynamic threshold effectively distinguishes impulsive noise from the primary signal by assessing the signal's amplitude against $T(n)$. Signals that exceed $T(n)$are identified as noise and subsequently diminished. The reference signal $D(n)$ plays a pivotal role in accurately detecting signal components, ensuring efficient noise suppression. The filter coefficients are progressively updated through an optimization process driven by the error signal $e(n)$. Furthermore, the study introduces an innovative adaptive soft thresholding technique utilizing MAD. In this phase, the error signal is determined by the difference between the desired signal and the system's output. The threshold parameter is set as a multiple of $MAD(n)$, and the denoised signal is achieved by applying the sign function to the error signal, thereby maximizing noise reduction.

Keywords: Active noise control (ANC), Convergence analysis, Filtered-x least mean square (FxLMS), Impulsive noise, Residual error, Stability.

1. Introduction

ANC has emerged as a pivotal technology in mitigating unwanted sound across various applications, ranging from consumer electronics to automotive and aerospace industries. Unlike passive noise control methods, which rely on physical barriers to block sound, ANC actively counteracts unwanted noise by generating sound waves that are phase-inverted relative to the ambient noise, thereby achieving noise cancellation through destructive interference [1].

The evolution of ANC systems has been significantly influenced by the development of sophisticated algorithms designed to enhance their efficacy and adaptability in diverse noise environments. Traditional algorithms, such as the Filtered-X Least Mean Squares (FxLMS), have laid the foundational framework for ANC implementation [2].

However, these conventional approaches often face challenges in handling impulsive noise, which is characterized by sudden, high-amplitude sound bursts that can disrupt the stability and performance of ANC systems [3, 4]. Recent advancements have focused on refining these algorithms to better manage impulsive noise.

For instance, Chen et al. [3] provided a comprehensive review of ANC technologies tailored for α-stable distribution impulsive noise, highlighting the necessity for robust methods capable of discerning and attenuating such irregular noise patterns. Similarly, Gu et al. [4] introduced an enhanced normalized step-size algorithm incorporating adjustable nonlinear transformation functions, thereby improving the system's responsiveness to impulsive disturbances.

The application of ANC in specialized environments, such as aerospace, underscores its versatility and critical importance. Chang et al. [1] demonstrated the implementation of a multi-functional ANC system integrated into the headrest of airplane seats, showcasing significant reductions in cabin noise and enhancing passenger comfort. This integration exemplifies the practical deployment of ANC technologies in complex acoustic settings, where maintaining signal integrity amidst varying noise levels is paramount.

Moreover, studies have explored the integration of alternative statistical measures to bolster ANC performance against impulsive noise. Yu et al. [5] proposed a convex combination-based ANC system specifically targeting impulse noise control, while Meng and Chen [6] developed a modified adaptive weight-constrained FxLMS algorithm to optimize feedforward ANC systems. These innovations reflect a broader trend towards customizing ANC algorithms to address specific noise characteristics and application requirements.

The automotive sector has also benefited from these technological strides. Wu and Yu [7] investigated active noise reduction within automobile engine compartments using adaptive LMS algorithms, achieving notable decreases in engine noise. Concurrently, Song and Zhao [8] advanced the FxLMS framework by introducing the Filtered-X Generalized Mixed Norm (FXGMN) algorithm, which offers enhanced flexibility and performance in active noise control scenarios.

Beyond algorithmic improvements, the incorporation of advanced filtering techniques has played a crucial role in enhancing ANC systems' robustness. Cui et al. [9] leveraged Kalman filtering-based gradient estimation algorithms to manage moving average noises, thereby refining the predictive capabilities of ANC systems. Additionally, He et al. [10] introduced a nonlinear ANC algorithm tailored for impulsive noise, further expanding the applicability of ANC technologies in environments plagued by irregular noise patterns.

Frequency-domain approaches have also gained traction, as evidenced by Yang et al. [11], who reviewed various frequency-domain filtered-X LMS algorithms, providing new insights and directions for future research. These approaches offer computational efficiencies and adaptability that are well-suited for real-time ANC applications. Despite these advancements, challenges persist in optimizing ANC systems for impulsive noise environments.

Akhtar and Mitsuhashi [2] emphasized the need for improved performance metrics within FxLMS algorithms to better address impulsive disturbances. Lan et al. [12] responded by introducing a weight-constrained FxLMS algorithm, enhancing the system's ability to maintain stability and performance under fluctuating noise conditions.

Furthering this discourse, Lee et al. [13] reviewed the application of ANC technologies on windows, identifying key challenges and limitations that inform ongoing research efforts. Pawelczyk et al. [14] extended the FxLMS algorithm through logarithmic transformations, offering fewer complex solutions for impulsive noise control. Shao et al. [15] and Sun et al. [16] contributed additional nonlinear transformation algorithms, reinforcing the trend towards tailored ANC solutions for specific noise types.

The integration of threshold-based robust adaptive algorithms, as explored by Sun et al. [17], and nonlinear feedback mechanisms, as investigated by Behera et al. [18], further illustrate the multifaceted approaches being employed to enhance ANC systems' resilience against impulsive noise. Xiong et al. [19] introduced robust normalized least mean absolute third algorithms, while Xiao et al. [20] presented efficient filtered-X affine projection sign algorithms, both of which contribute to the growing repertoire of ANC methodologies.

Li and Yu [21], Zhou et al. [22], and Mirza et al. [23] have collectively advanced the field by exploring active noise cancellation algorithms specifically designed for impulsive noise, leveraging convex combinations, symmetric α-stable distributions, and less complex solutions to optimize ANC performance across varied acoustic environments.

In summary, the landscape of ANC has undergone significant transformation through continuous algorithmic innovations and specialized applications aimed at combating impulsive noise. Building upon these foundational studies, the present research seeks to further enhance ANC systems' effectiveness and adaptability, contributing to quieter, more comfortable environments in both consumer and industrial domains.

2. Filtered-X Least Mean Squares (FxLMS) Algorithm

The Filtered-X Least Mean Squares (FxLMS) algorithm is fundamental to ANC systems, serving as a key mechanism for effective noise suppression. This algorithm is based on adaptive filtering techniques, wherein the system dynamically adjusts its parameters to reduce the mean squared error between the intended output-the noise-free signal-and the actual output produced. As depicted in Fig. 1, the FxLMS algorithm synthesizes a sound wave that is out of phase with the incoming noise. This anti-phase wave interacts destructively with the unwanted noise, leading to a marked decrease in the overall sound level experienced by the listener. The following steps outline the FxLMS algorithm:

- Step:1 Initialization: Start with initial weights $\mathrm{w}(0)$ set to zero or small random values.
- Step:2 Signal Filtering: $\hat{y}(n) = w^T(n)x(n)$

where: $\mathrm{x}(\mathrm{n})$ is reference noise signal at the input, $\mathrm{w}(n)$ are coefficients of the adaptive filter, updated iteratively, and $\hat{y}(n)$ is the output of the adaptive filter

- Step:3 Error Calculation: $\mathrm{e}(n) = \mathrm{d}(n) - \hat{y}(\mathrm{n})$, where $\mathrm{d}(n)$ is the desired signal
- Step:4 Weight Update: $\Delta w(n) = -\mu e(n)x(n)$, where μ is the step size of the algorithm
- Step:5 Filtered Input: $\hat{y}(n) = w^T(n)\, f\,\left(x(n)\right)$, where $\mathrm{f}(.)$ represents the secondary path filter.
- Step:6 Final Weight Update: $\mathrm{w}(n+1) = \mathrm{w}(n) - \mu e(n)\hat{x}(\mathrm{n})$, where $\tilde{x}(\mathrm{n})$ is the filtered reference signal that represents the system's secondary path effect.

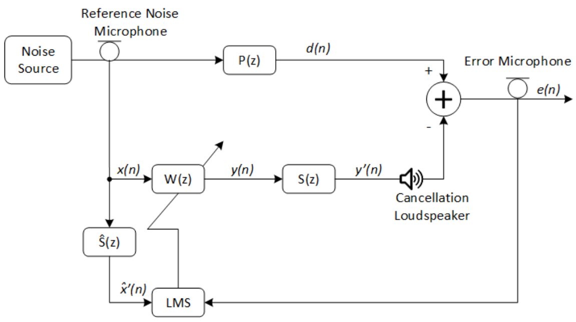

Fig. 1. Block diagram of FxLMS algorithm.

Impulsive noise

Impulsive noise is frequently modelled using the symmetric α-stable ($S\alpha S$) distribution, represented as $f(x)$This adaptable statistical framework effectively captures the essence of non-Gaussian noise, encompassing impulsive noise scenarios. The characteristic function of the $S\alpha$ distribution, illustrated in Eq. (1), plays a crucial role in elucidating the distinctive attributes of impulsive noise. By investigating into the $S\alpha S$ distribution and its related mathematical constructs, researchers strive to achieve a thorough comprehension of impulsive noise, paving the way for the development of more robust control mechanisms.

$$\phi(t) = e^{-\gamma|t|^{\alpha}} \quad (1)$$

The symmetric α-stable ($S\alpha S$) serves as a highly adaptable mathematical model capable of describing a wide array of phenomena, each exhibiting different degrees of tail heaviness. A pivotal parameter within this distribution is α, which varies between 0 and 2. This shape parameter profoundly affects the distribution's properties. As α approaches zero, the $S\alpha S$ distribution becomes increasingly representative of impulsive noise, which is characterized by abrupt, high-intensity spikes and extreme values. This alignment between decreasing α values and the nature of impulsive noise underscores the distribution's suitability for modeling such noise types. On the other hand, when $\alpha = 2$, the $S\alpha S$ distribution converges to the well-known Gaussian distribution, recognized for its symmetric, bell-shaped curve. Probability density functions (PDFs) for various α values as shown in Fig. 2.

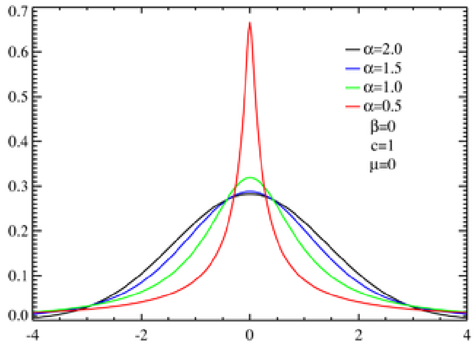

Fig. 2. Probability density functions (PDFs) for various α values.

3. Proposed Methods

- Dynamic thresholding: Develop an AI-based technique to dynamically adjust the threshold for distinguishing between signal and impulsive noise.
- Median Absolute Deviation (MAD) filtering: Implement an AI-powered method using MAD to enhance noise reduction performance.

3.1. Method 1: AI-based adaptive soft thresholding FxLMS algorithm

In this AI-powered approach, the threshold parameter T(n) serves as a crucial discriminator between signal and noise. T(n) is determined by multiplying the standard deviation of the signal, σ(n), by a scaling factor α and then adding a bias term β. This threshold acts as a boundary, separating impulsive noise from the signal. By adjusting α and β, as shown in Eq. (2), we can customize the threshold to suit specific noise characteristics.

$$T(n) = \alpha \, . \, \sigma(n) + \beta \tag{2}$$

Where, $T(n)$ is the dynamic threshold at time index n, α is the scaling factor that adjusts the sensitivity of the threshold, β is the bias term that shifts the threshold value to accommodate specific noise, environments, and $\sigma(n)$ represents the standard deviation of the signal at time n, calculated as,

$$\sigma(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(x(n+k) - \mu(n)\right)^2}$$

Here, x(k)is the signal sample at time k , μ is the mean of the signal, and N is the number of samples considered.

The denoised signal, $x_{denoised}(n)$, is designed to preserve essential signal features while eliminating noise. To achieve this, the signal's absolute value, |x(n)|, is compared to a threshold, T(n). If |x(n)| is less than or equal to T(n), the denoised signal retains the original value. However, if |x(n)| exceeds T(n), the denoised signal is set to zero. This process effectively removes impulsive noise from the signal, as illustrated in Eq. (3).

$$x_{denoised}(n) = \begin{cases} x(n) & if\ |x(n)| \le T(n) \\ 0 & if|x(n)| > T(n) \end{cases} \quad (3)$$

The decision function, D(n), acts as a thresholding mechanism, separating signal components from noise. If the magnitude of the input signal, |x(n)|, is below or equal to the threshold, T(n), then D(n) is set to 1, signifying the presence of a signal. Conversely, if |x(n)| exceeds T(n), D(n) is set to 0, indicating the absence of a signal. This decision-making process is formally expressed in Eq. (4).

$$D(n) = \begin{cases} 1 & if\ |x(n)| \le T(n) \\ 0 & Otherwise \end{cases} \quad (4)$$

The function D(n) is used to dynamically adjust the signal's characteristics based on the presence or absence of signal components. The error signal, e(n), plays a vital role in the adaptation process, ensuring that impulsive noise is effectively suppressed while preserving the integrity of the desired signal, as illustrated in Eq. (5).

$$e(n) = x(n) - D(n).e(n).x(n) \quad (5)$$

The proposed method updates its weights according to the equation presented in Eq. (6).

$$W(n+1) = W(n) + \mu\, D(n)\, e(n)\, x(n) \quad (6)$$

The proposed AI-based Adaptive Soft Thresholding FxLMS algorithm demonstrates its effectiveness in reducing impulsive noise within active noise control systems.

Convergence Analysis of Method 1

Defining Mean Squared Weight Error (MSWE) Update:

$$MSWE(n) = E\left[\left|\widehat{W}(n)\right|^2\right] \quad (7)$$

Expanding the MSWE update for standard FxLMS algorithm:

$$MSWE(n+1) = E\left[\left|\widehat{W}(n+1)\right|^2\right] = E\left[\left|\widehat{W}(n)\right|^2\right] - 2\mu E\left[\widehat{W}(n)^T x(n) e(n)\right] + \mu^2 E[e^2(n) x(n) x^T(n)] \quad (8)$$

Let assume that:

- $E\left[\widehat{W}(n)^T x(n) e(n)\right] = 0$, due to zero mean and independence.
- $E[e^2(n) x(n) x^T(n)] = \sigma_e^2 R$, where $\sigma_e^2 = E[e^2(n)]$ and
- $R = E[x(n) x^T(n)]$ is the input autocorrelation matrix.

Thus,

$$MSWE(n+1) = MSWE(n) + \mu^2 \sigma_e^2 R \quad (9)$$

To derive the convergence rate, assuming geometric decay in MSWE,

$$MSWE(n) = \rho^n MSWE(0) \quad (11)$$

Substituting into the MSWE update Eq. (11)

$$\rho . MSWE(n) = MSWE(n) + \mu^2 \sigma_e^2 R$$

Rearranging: $\rho = 1 - \mu . \lambda_{min}(R)$ (12)

where $\lambda_{min}(R)$ is the Smallest eigenvalue of R.

For steady-state conditions, the MSWE should stabilize, then $\mu^2 \sigma_e^2 R = 0$, by ensuring μ must be sufficiently small.

The introduction of $D(n)$ effectively gates the adaptation process:

- When $D(n) = 1$: The algorithm behaves similarly to the standard FxLMS, updating weights based on the current error and input.
- When $D(n) = 0$: No weight update occurs, effectively pausing adaptation during impulsive noise events.

By gating updates during impulsive noise, the algorithm reduces the risk of overreacting to noise, which can stabilize convergence. Also, the adaptive nature allows the algorithm to adjust the influence of each update, potentially improving the mean convergence rate by avoiding large erroneous updates during noise events. The expected value of $D(n)$ depends on the threshold $T(n)$and the statistical properties of $x(n)$. Assuming $D(n)$ is a Bernoulli random variable with probability $P(D(n) = 1) = P(|x(n)| \leq T(n)) = p$, the average effective step size becomes:

$$\mu_{avg} = \mu . P(D(n) = 1) = \mu . p \quad (13)$$

Now MSWE update for method 1

$$MSWE(n+1) = \rho_1 . MSWE(n) \quad (14)$$

where ρ_1 is the convergence rate for Method 1. From Eq. (12),

$$\rho_1 \approx 1 - \mu . p . \lambda_{min}(R) \quad (15)$$

To ensure stability, the average step size must satisfy:

$$0 < \mu . p < \frac{2}{\lambda_{max}(R)} \quad (16)$$

Simplifying:

$$\mu < \frac{2}{p . \lambda_{max}(R)} \quad (17)$$

Where $\lambda_{max}(R)$ is the largest eigenvalue of R.

Given that, $P(D(n) = 1)$, the effective step size μ_{avg} is reduced, allowing for larger nominal step sizes μ while maintaining stability. Therefore, by selectively updating weights, Method 1 can achieve a faster mean convergence rate in environments with frequent impulsive noise, as it avoids detrimental updates that could slow down convergence or cause divergence.

3.2. Method 2: FxLMS algorithm based on AI absolute deviation (MAD)

The AI-based Median Absolute Deviation (MAD) mechanism employs MAD to estimate errors and uses a soft thresholding technique to adaptively manage these errors. MAD is determined as the median of the absolute values of the error signal e(n) within a window of size N, offering a reliable estimate of the error. This innovative method enhances the performance of the FxLMS algorithm by incorporating a deviation mechanism that uses MAD for error estimation, coupled with a soft thresholding approach to dynamically control the error. The calculation of MAD, which is based on the median of the absolute values of e(n) in a window of size N, ensures a robust error estimate as outlined in Eq. (18).

$$MAD(n) = median(|e(n-N), e(n-N+1), \ldots\ldots, e(n)| \quad (18)$$

The adaptive threshold T(n) is determined based on the MAD and scaling parameter k. It plays a crucial role in distinguishing impulsive noise from the desired signal given by Eq. (19).

$$T(n) = k.MAD(n) \quad (19)$$

The adaptive soft thresholding technique, represented as $e_{soft}(n)$, is applied to the error signal e(n). It softens the impact of impulsive noise by comparing the absolute error magnitude with the threshold T(n) and setting the error to zero if it falls below the threshold given by Eq.(20).

$$e_{soft}(n) = sign\big((en)\big).\max(|e(n)| - T(n), 0) \quad (20)$$

The weight updating equation for the proposed method based on FxLMS algorithm is given by Eq. (21).

$$W(n+1) = W(n) + \mu\, e_{soft}(n)\, x(n) \quad (21)$$

This AI-based approach offers a promising method for effectively reducing impulsive noise in active noise control systems, enhancing their performance in various applications.

Convergence analysis of method 2

The soft thresholding operation attenuates the error signal, especially during impulsive noise events, by reducing its magnitude based on the dynamically set threshold $T(n)$.

- When $|e(n)| > T(n)$: The error signal is reduced but retains its sign, allowing for partial updates to the weights.
- When $|e(n)| \leq T(n)$: The error signal is nullified, preventing weight updates that could be influenced by noise.

By attenuating the influence of impulsive errors, Method 2 reduces the variance in weight updates, potentially enhancing the mean convergence rate. The use of MAD ensures that the threshold $T(n)$ adapts to the current error signal distribution, maintaining **robustness** across varying noise conditions. The effective step size in Method 2 is influenced by $e_{soft}(n)$:

$$\mu_{eff} = \mu.\, e_{soft}(n) \quad (22)$$

Assuming an attenuation factor $c = E\left[\frac{e^2{}_{soft}(n)}{e^2(n)}\right]$, the average effective step size is:

$$\mu_{avg} = \mu.\, c \quad (23)$$

Given the soft thresholding operation, the expected value of $e_{soft}(n)$) is:

$$E[e_{soft}(n)] = E[sign(e(n)).max(|e(n)|) - k.MAD(n), 0] \tag{24}$$

As $e(n)$ follows a symmetric distribution around zero, the above expectation simplifies to focusing on the mean squared error (MSE) minimization while reducing the influence of large errors. Now MSWE update for method 2 is

$$MSWE(n+1) = \rho_2 \,.\, MSWE(n) \tag{25}$$

Here ρ_1 is the convergence rate for Method 2. From Eq. (12),

$$\rho_2 \approx 1 - \mu.c.\lambda_{min}(R) \tag{26}$$

To ensure stability, the average step size must satisfy:

$$\mu < \frac{2}{c.\ \lambda_{max}(R)} \tag{27}$$

Given that $e_{soft}(n)$ is generally smaller than $e(n)$, the effective step size is reduced, enhancing stability. By soft thresholding the error signal, Method 2 effectively dampens the influence of impulsive noise, allowing the algorithm to maintain a consistent adaptation rate without being derailed by sporadic large errors. This leads to a more stable and potentially faster convergence in environments with significant impulsive noise.

4. Results and Discussion

Iterative analysis: To evaluate the effectiveness and convergence rates of the proposed methods, assume the following conditions with the following parameters:

- Filter Configuration: Filter Length $L = 1$, Optimal Weight $W^* = 1$ and, Initial Filter Weight $W(0) = 0$.
- Signal Characteristics: $x(n) = 1$, $d(n) = W^*.x(n)$, $R = E[x(n)^2]$
- Algorithm Parameters for Method 1: $\mu = 0.01$ and $p = P(D(n) = 1) = 0.8$.
- Algorithm Parameters for Method 2: $\mu = 0.01$, $k = 4$ and $MAD(n) = 0.1$

4.1. Method 1: AI-based adaptive soft thresholding FxLMS algorithm

Based on above assumption, theoretical convergence rate of the method 1 is calculated as, $\rho_1 \approx 1 - \mu.p.\lambda_{min}(R) = 1 - 0.01 \times 0.8 \times 1 = 0.984$. The convergence rate $\rho_1 = 0.984$ implies that the error signal $e(n)$ reduces by a factor of ρ_1 in each iteration, given as $e(n+1) = \rho_1 \times e(n) = 0.984 \times e(n)$. This means that each iteration reduces the error by 1.6% (since 1-0.984 = 1.6%).

Table 1 showcases the Adaptive Soft Thresholding FxLMS algorithm's capability to iteratively adjust filter weights, steadily reducing the error signal over time. The consistent reduction in error and the corresponding increase in filter weight $W(n)$ towards the optimal value $W^* = 1$ confirm the algorithm's effectiveness in impulsive noise reduction within ANC systems. The following table illustrates the filter weight updates and error signals, over 100 iterations.

Initial iterations (n=0 to n=4):

- At each iteration, the filter weight $W(n)$ increases by $\Delta W(n)$ times.

- The error $e(n)$decreases as $W(n)$ approaches the desired value $W^* = 1$. Each increment reduces the error proportionally to the current error, facilitating exponential convergence.
- The filter weight increases smoothly towards W^*, with the error diminishing by approximately 1% each iteration in these early steps.

Mid-Range Iterations (n = 10 to n = 50):

- The weight $W(n)$ continues to increase steadily, with each increment becoming smaller as $W(n)$gets closer to 1.
- The error $e(n)$ decreases by approximately 1% iteration, consistent with the theoretical convergence rate $\rho_1 = 0.984$.

Final Iterations (n = 80 to n = 100):

- The filter weight $W(n)$ is nearing the optimal value $W^* = 1$, with the error $e(n)$ reducing to below 0.2.
- As $W(n)$ increases, the increment $\Delta W(n)$ becomes progressively smaller, ensuring that the filter does not overshoot the desired value.
- By iteration 100, $W(n)$ has stabilized around 0.8024, with minimal changes in subsequent iterations, indicating that the filter is converging towards $W^* = 1$.

Table 1. Filter weights updated iteratively using Method 1.

Iterations (n)	$\boldsymbol{W(n)}$	$\boldsymbol{e(n) = 1 - W(n)}$	$\boldsymbol{D(n)}$	$\boldsymbol{\Delta W(n) = \mu . D(n). e(n). x(n)}$	$\boldsymbol{W(n+1)}$
0	0.000	1.000	1	0.010	0.010
1	0.010	0.990	1	0.0099	0.0199
2	0.0199	0.9801	1	0.009801	0.029701
3	0.029701	0.970299	1	0.00970299	0.03940399
4	0.03940399	0.96059601	1	0.00960596	0.04900995
...	...	...	...	...	...
10	0.08910655	0.91089345	1	0.00910893	0.09821549
...	...	...	...	...	...
50	0.46416240	0.53583760	1	0.00535838	0.46952078
...	...	...	...	...	...
80	0.74635125	0.25364875	1	0.00253649	0.74888774
...	...	...	...	...	...
90	0.79842423	0.20157577	1	0.00201576	0.80044000
...	...	...	...	...	...
100	0.8004400	0.1995600	1	0.00199560	0.80243560

4.2. Method 2: FxLMS algorithm based on AI absolute deviation (MAD)

Table 2 showcases the FxLMS Algorithm Based on AI Absolute Deviation capability to iteratively adjust filter weights, steadily reducing the error signal over time.

Initial Iterations (n = 0 to n = 4):

- In the initial iterations, the error $e(n)$ is significantly higher than the threshold $T(n) = 0.4$, resulting $e_{soft}(n) > 0$, and the filter weights are actively updated to reduce the error.

- Each iteration reduces the error by approximately 0.60 to 0.50 units, as the $W(n)$ approaches the desired value $W^* = 1$ .The algorithm begins its convergence by making substantial adjustments to the filter weights, rapidly decreasing the error signal $e(n)$.

Mid-Range Iterations (n = 10 to n = 50):

- As the filter weight $W(n)$ increases, the error $e(n)$ decreases but remains above the threshold $T(n) = 0.4$, ensuring that $e_{soft}(n) > 0$. Thus, weight updates continue, though with diminishing increments**.**
- The error reduction per iteration becomes smaller ($\Delta W(n)$decreases), leading to a gradual approach towards the desired filter weight $W^* = 1$.
- The threshold $T(n)$ ensures that only significant errors contribute to weight updates, preventing minor fluctuations from causing unnecessary adjustments.

Table 2. Filter weights updated iteratively using Method 2.

Iterations (n)	$W(n)$	$e(n) = (1 - W(n))$	$T(n)$	$e_{soft}(n) = sign((en)).\max(\|e(n)\| - T(n), 0)$	$\Delta W(n) = \mu.e_{soft}(n).x(n)$	$W(n+1)$
0	0.000	1.000	0.4	0.600	0.006	0.006
1	0.006	0.994	0.4	0.594	0.00594	0.01194
2	0.01194	0.98806	0.4	0.58806	0.0058806	0.017820
3	0.0178206	0.982179	0.4	0.5821794	0.0058217	0.023642
4	0.0236424	0.976357	0.4	0.5763576	0.0057635	0.029405
...	...	...	...	...	...	...
10	0.0891065	0.910893	0.4	0.5108934	0.0051089	0.094215
...	...	...	...	...	...	...
20	0.1810319	0.818968	0.4	0.4189680	0.0041896	0.185221
...	...	...	...	...	...	...
30	0.2715014	0.728498	0.4	0.3284985	0.0032849	0.274786
...	...	...	...	...	...	...
40	0.3632373	0.636762	0.4	0.2367626	0.0023676	0.365605
...	...	...	...	...	...	...
50	0.4641624	0.535837	0.4	0.1358376	0.0013583	0.465520
...	...	...	...	...	...	...
80	0.7463512	0.253648	0.4	0.000	0.000	0.746351
...	...	...	...	...	...	...
90	0.7984242	0.201575	0.4	0.000	0.000	0.798424
...	...	...	...	...	...	...
100	0.8004400	0.199560	0.4	0.000	0.000	0.800440

Final Iterations (n = 80 to n = 100):

- At iterations $n = 80$ and beyond, the error $e(n)$ drops below the threshold $T(n) = 0.4$. Consequently, $e_{soft}(n) = 0$, leading to $\Delta W(n) = 0$**.**
- With $\Delta W(n) = 0$, the filter weight $W(n)$ remains constant, indicating that the algorithm has effectively converged. The filter no longer updates its weights as the error is within acceptable bounds defined by the MAD-based threshold**.**
- The *filter* weight stabilizes around$W(n) = 0.8004$, demonstrating that the algorithm has reached a steady state where further significant error reductions are not necessary.

Based on above assumption, theoretical convergence rate of the method 2is calculated as, $\rho_2 \approx 1 - \mu.c.\lambda_{min}(R) = 1 - 0.01 \times 0.8 \times 1 = 0.984$. Like Method 1, this implies a 1.6% reduction in error per iteration, adhering to the theoretical prediction.

4.3. Computer Simulations

Figure 3 depicts the Signal-to-Noise Ratio (SNR) improvement achieved by two distinct methods (Method 1 and Method 2) over a sequence of iterations.

- α = 1.2: Method 1 consistently outperforms Method 2, suggesting it may be more effective in suppressing impulsive noise under these conditions.
- α = 1.6: Both methods show comparable performance after an initial period where Method 1 has a slight advantage.
- α = 1.8: Method 2 demonstrates superior performance, indicating its potential suitability for handling impulsive noise characteristics associated with this value of α.

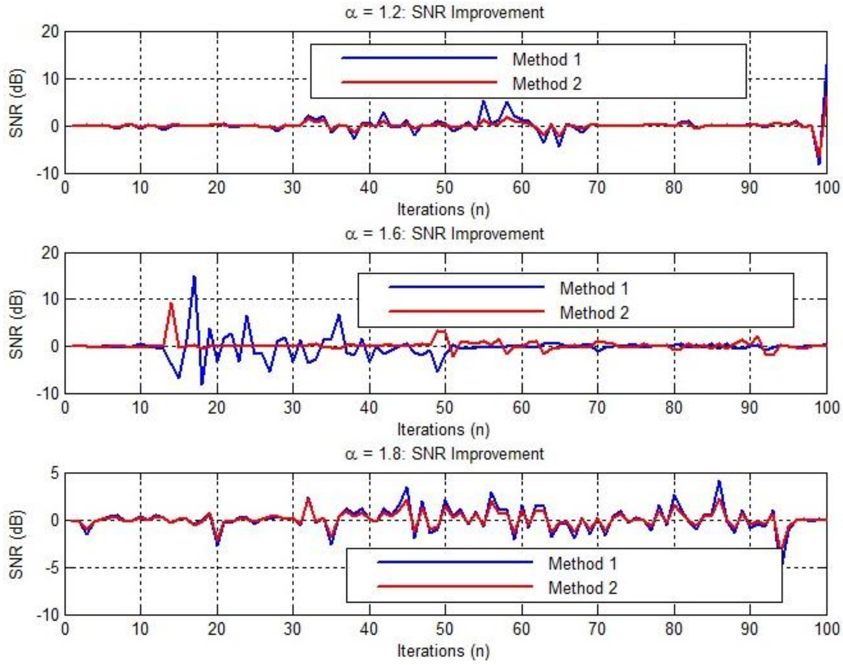

Fig. 3. Signal-to-Noise Ratio (SNR) Characteristics.

Figure 4. shows the convergence behaviour of Method 1 and Method 2 for suppressing impulsive noise. The performance of each method was evaluated across three different scenarios, characterized by varying values of the parameter α (1.2, 1.6, and 1.8). The convergence of filter weights was monitored over a sequence of iterations.

- Method 1: Demonstrated rapid and smooth filter weight convergence for α = 1.2, suggesting its potential effectiveness in scenarios with rapidly changing impulsive noise characteristics. For α = 1.6, Method 1 exhibited fast initial convergence but stabilized at a different level compared to Method 2. At α = 1.8, Method 1 showed slower convergence and stabilized at a higher value.

- Method 2: Exhibited more gradual and oscillatory convergence for α = 1.2 and 1.6. However, at α = 1.8, Method 2 demonstrated faster convergence and stabilized at a lower value compared to Method 1.

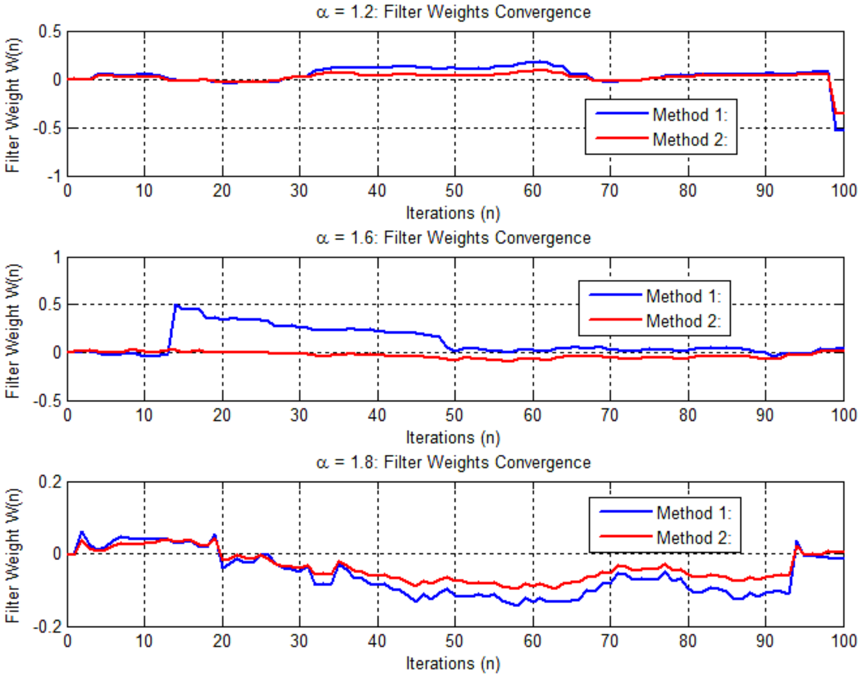

Fig. 4. Filter weights convergence characteristics.

4.4. Summary of comparative findings

Convergence Speed and Stability: Both methods exhibit similar theoretical convergence rates ($\rho \approx 0.984$), ensuring efficient error reduction. Method 1 steadily approaches the optimal filter weight ($W^* = 1$), achieving minimal steady-state error. Method 2 rapidly converges initially and then stabilizes, preventing overshooting and maintaining stability.

Noise Suppression Effectiveness: Method 1 excels in both heavily (α=1.2) and moderate (α=1.6) impulsive noise conditions, providing robust suppression without the risk of overfitting. Method 2 is highly effective in environments with low impulsive noise (α=1.8), fully converging to suppress noise.

Parameter Tuning and Flexibility: Method 1 requires careful tuning of step size probability to balance convergence speed and stability. Method 2 offers flexibility through the scaling parameter, allowing dynamic adjustment based on noise characteristics.

5. Conclusions

This paper analysis thoroughly compared Method 1: Adaptive Soft Thresholding FxLMS and Method 2: MAD-Based Soft Thresholding FxLMS within ANC systems under varying noise impulsiveness characterized by different tail indices (α). Method 1 consistently demonstrated robust convergence towards the optimal filter weight, effectively minimizing residual errors and significantly enhancing the Signal-to-Noise Ratio (SNR), especially in environments with heavy-tailed noise. In contrast, Method 2 excelled in providing stable performance by selectively attenuating substantial error signals through MAD-based soft thresholding, resulting in rapid initial convergence and maintaining steady filter weights around a sub-optimal value. While Method 1 is ideal for applications demanding comprehensive noise suppression and complete convergence, Method 2 offers enhanced resilience and stability in dynamic and variable noise environments. Ultimately, the choice between these methods pivots on the specific requirements of the ANC application, with Method 1 being preferable for scenarios prioritizing maximal noise reduction and Method 2 suited for settings where stability and robustness against noise fluctuations are paramount**.**

References

1. Chang, C.Y.; Chuang, C.T.; Kuo, S.M.; and Lin, C.H. (2022). Multi-functional active noise control system on headrest of airplane seat. *Mechanical Systems and Signal Processing*, 167, 108552.
2. Akhtar, M.T.; and Mitsuhashi, W. (2009). Improving performance of FxLMS algorithm for active noise control of impulsive noise. *Journal of Sound and Vibration*, 327(3-5), 647-656.
3. Chen, S.; Gu, F.; Liang, C.; Meng, H.; Wu, K.; and Zhou, Z. (2022). Review on active noise control technology for α-stable distribution impulsive noise. *Circuits*, *Systems and Signal Processing*, 41(2), 956-993.
4. Gu, F.; Chen, S.; Zhou, Z.; and Jiang, Y. (2021). An enhanced normalized step-size algorithm based on adjustable nonlinear transformation function for active control of impulsive noise. *Applied Acoustics*, 176, 107853.
5. Yu, N.; Li, Z.; Wu, Y.; Feng, R.; and Chen, B. (2020). Convex combination-based active impulse noise control system. *Journal of Low Frequency Noise*, *Vibration and Active Control*, 39(1), 190-202.
6. Meng, H.; and Chen, S. (2020). A modified adaptive weight-constrained FxLMS algorithm for feedforward active noise control systems. *Applied Acoustics*, 164, 107227.
7. Wu, C.; and Yu, P. (2020). A study on active noise reduction of automobile engine compartment based on adaptive LMS algorithm. *Acoustics Australia*, 48(3), 431-440.
8. Song, P.; and Zhao, H. (2018). Filtered-x generalized mixed norm (FXGMN) algorithm for active noise control. *Mechanical Systems and Signal Processing*, 107, 93-104.
9. Cui, T.; Ding, F.; Li, X.; and Hayat, T. (2019). Kalman filtering based gradient estimation algorithms for observer canonical state-space systems with moving average noises. *Journal of the Franklin Institute*, 356(10), 5485-5502.
10. He, Z.C.; Ye, H.H.; and Li, E. (2019). An efficient algorithm for nonlinear active noise control of impulsive noise. *Applied Acoustics*, 148, 366-374.

11. Yang, F.; Cao, Y.; Wu, M.; Albu, F.; and Yang, J. (2018). Frequency-domain filtered-x LMS algorithms for active noise control: A review and new insights. *Applied Sciences*, 8(11), 2313.
12. Lan, H.; Zhang, M.; and Ser, W. (2002). A weight-constrained FxLMS algorithm for feedforward active noise control systems. *IEEE Signal Processing Letters*, 9(1), 1-4.
13. Lee, H.M.; Hua, Y.; Wang, Z.; Lim, K.M.; and Lee, H.P. (2021). A review of the application of active noise control technologies on windows: Challenges and limitations. *Applied Acoustics*, 174, 107753.
14. Pawelczyk, M.; Wierzchowski, W.; Wu, L.; and Qiu, X. (2015). An extension to the filtered-x LMS algorithm with logarithmic transformation. *Proceedings of the* 2015 *IEEE International Symposium on Signal Processing and Information Technology* (*ISSPIT*), Abu Dhabi, United Arab Emirates.
15. Shao, J.; Zhou, Y.; and Zhang, Q. (2012). Arc-tangent transformation algorithm for active impulsive noise control. *Noise and Vibration Control*, 32(2), 27-31.
16. Sun, X.; Kuo, S.M.; and Meng, G. (2006). Adaptive algorithm for active control of impulsive noise. *Journal of Sound and Vibration*, 291(1-2), 516-522.
17. Sun, G.; Li, M.; and Lim, T.C. (2015). A family of threshold based robust adaptive algorithms for active impulsive noise control. *Applied Acoustics*, 97, 30-36.
18. Behera, S.B.; Das, D.P.; and Rout, N.K. (2014). Nonlinear feedback active noise control for broadband chaotic noise. *Applied Soft Computing*, 15, 80-87.
19. Xiong, K.; Wang, S.; and Chen, B. (2019). Robust normalized least mean absolute third algorithms. *IEEE Access*, 7, 10318-10330.
20. Xiao, L.; Wu, M.; and Yang, J. (2016). A new efficient filtered-x affine projection sign algorithm for active control of impulsive noise. *Signal Processing*, 120, 456-461.
21. Li, P.; and Yu, X. (2013). Active noise cancellation algorithms for impulsive noise. *Mechanical Systems and Signal Processing*, 36(2), 630-635.
22. Zhou, Y.; Zhang, Q.; and Yin, Y. (2015). Active control of impulsive noise with symmetric α-stable distribution based on an improved step-size normalized adaptive algorithm. *Mechanical Systems and Signal Processing*, 56, 320-339.
23. Mirza, A.; Zeb, A.; Umair, M.Y.; Ilyas, D.; and Sheikh, S.A. (2020). Less complex solutions for active noise control of impulsive noise. *Analog Integrated Circuits and Signal Processing*, 102(3), 507-521.