Journal of Engineering Science and Technology
EURECA 2013 Special Issue August (2014) 16 - 27
© School of Engineering, Taylor's University

THE EFFECT OF WING GEOMETRY ON LIFT AT SUPERSONIC SPEEDS

ABDULKAREEM SH. MAHDI AL-OBAIDI*, ERIC TAN NAN KUI

School of Engineering, Taylor's University, Taylor's Lakeside Campus,
No. 1 Jalan Taylor's, 47500 Subang Jaya, Selangor DE, Malaysia
*Corresponding Author: Abdulkareem.mahdi@taylors.edu.my

Abstract

The effect of various wing geometry parameters on lift generation at supersonic speeds has been studied and analysed. These parameters include the aspect ratio, taper ratio, leading edge sweep angle, relative thickness and the airfoil shape. The parametric study considered only four wing plan forms which were the straight rectangular, sweptback, delta and trapezoidal wings. Both analytical and semi-empirical approaches are used for determining the lift curve slope. The obtained results were validated using published experimental results. Based on the study, it was concluded that the amount of lift generated by the wing is a strong function of the wing planform rather than any single wing parameter. At supersonic speeds, it was also noted that the relative thickness and airfoil shape have negligible effect on the lift. The overall effect of the investigated parameters has been summarised in a design table. This table can be used as an efficient tool during the preliminary design stage for a quick estimation of lift based on certain wing design criteria outlined by a pre-defined set of tactical-technical requirements.

Keywords: Wings, Supersonic speed, Aerodynamics, Lift.

1. Introduction

Aerodynamic surfaces such as wings have traditionally been used to produce lift or as ailerons, rudders or stabilizers as means to control and stabilize a flying body. Although the main function of the aerodynamic surfaces is to produce lift; they vary in terms of shapes and therefore vary in performance characteristics. Performance of aerodynamic surface is measured in terms of the amount of lift it is able to generate in specific operating conditions as outlined by a set of tactical-technical requirements (TTR).

Nomenclatures

A	Aspect ratio
b	Wing span, m
C_L	Lift coefficient
$C_{L\alpha}$	Lift-force-curve slope, 1/rad.
C_{MAC}	Mean aerodynamic chord, m
C_r	Root chord, m
C_t	Tip chord, m
c	Speed of sound, m/s
M	Mach number
M_∞	Free stream Mach number
S_W	Wing surface area, m^2
$\bar{t}_{\max}$	Wing airfoil relative thickness
V	Air velocity, m/s

Greek Symbols

α	Angle of attack, rad.
β	Mach number parameter, $\sqrt{M^2-1}$
$\Lambda_{c/2}$	Mid chord sweep angle, rad.
Λ_{LE}	Leading edge sweep angle, rad.
λ	Taper ratio

Tactical-technical requirements are a set of instructions that governs how a flying body is expected to behave in flight, which is usually associated with target data such as projectile height, trajectory, flight speed and/or payload. An example of such instruction can be for example a cruise missile flying at 8 km above sea level at a speed of Mach 4 over a distance of 2000 km. At a glance it is a rather simple requirement, however there are a multitude of solutions which are usually accompanied by time consuming deductive process to deduce the most effective solution. In addition conducting experiments for each suggested configuration using wind tunnel or true flight is very expensive and complex logistical operations. Hence the study is designed to create a design table which enables a quick and efficient method of predicting the aerodynamic performance of a particular wing design during preliminary design stage.

Lift is generally derived from the pressure difference between the upper and lower surface of the airfoil. This is true for subsonic flows. In subsonic flows, air flows smoothly across the wing section and hence the air can be considered to be incompressible in which the ideal gas assumption and the potential flow with small disturbance theory can then be applied to deduce the aerodynamic characteristic of the wing. However when the flows enter the supersonic regime, the air becomes compressible and shock waves begins to forms on the leading edge and trailing edge of the wing section. The potential flow theory with regards to small disturbances can no longer be applied due to the drastic change in pressure distribution as result of the oblique shock waves.

At subsonic speeds, having an angle of attack is beneficial since the flow separation creates a suction force at the nose of the wing section to bring the resultant air force relative to the chord axis to a position that is at right angles relative to the wind direction which in effect creates lift. However at supersonic speeds, the nature of air flow changes and hence these benefits are lost along with it as indicated in Ref. [1].

Based on Jones [1] findings, the lift of supersonic wings is only generated when the wing is regarded to have a subsonic leading edge. The subsonic leading edge phenomenon can only be achieved when the wing is sufficiently swept behind the Mach cone; this not allows the wing to generate lift as it would at subsonic speeds however it can also be used to overcome the pressure drag that is caused by formation of shock waves. Based on analytical investigations, Jones [1] deduced that the wing's lack of ability to generate lift at supersonic speed can be rectified by having the wing swept to keep its wetted surface area within the confines of the Mach cone and hence giving the wing its subsonic leading edge. It is also postulated that by introducing some degree of leading edge sweep angle, the effect of pressure drag caused by the shock waves can be minimized.

Bonnie [2] conducted an analytical investigation to deduce the effectiveness of the Busemann's second order approximation, which accounts for the shape and airfoil thickness of the wing based on a single generalized expression. The investigation was conducted based solely on the rectangular unswept planform. Based on the Busemann's second order, it was deduced that the lift is directly proportionate to the aspect ratio of the wing; the results also showed that increasing the airfoil thickness also increases the amount of lift generated by the wing although this increment proves to be negligible.

Hall [3] presented results of a parametric study on the various wing parameters and wing planform based on a combination of analytical and experimental approach. The study was aimed at studying the feasibility of the linearized formulation in predicting the aerodynamic characteristics of various wing planforms. The results obtained shows that the linearized theory does perform satisfactory at only subsonic speeds; performs poorly when benchmarked with experimental data at transonic Mach number; and was proven true for the changes in aspect ratio and wing planforms. However the results ruled out the effect of thickness and airfoil shape as having no effect on the lift coefficient but they do affect the other coefficients.

Al-Obaidi [4] performed a parametric study on the wing's aerodynamic characteristic based on a combination of semi-empirical and numerical methods. The parameters that were included in his study cover only the aspect ratio, taper ratio and maximum thickness. In his report, a program called "New_KAR" that uses linear interpolation method to determine the lift coefficient based on a set of existing data was created. Aside from the semi-empirical approach, the numerical panel method was also used to validate the accuracy of the program "New_KAR" and it was determined to be within an acceptable 10% error tolerance which is acceptable for preliminary design stages.

The objectives of this paper is to predict the aerodynamic characteristics of wings using analytical and semi-empirical methods and develop a design table for that correlates the effects of each individual geometrical parameter on the aerodynamic characteristic of the wing and if possible a mathematical expression that is able to describe the lift characteristic as a function of each of the wing's geometrical parameter as well as Mach number. The motivation for this study is to address the inadequacy of previous studies which up to present which is usually an isolated study that only involves a single approach on a specific wing planform, this in effect places the assumption that each wing geometrical parameter are independent of each other; hence for this purpose five widely used projectile shapes are investigated. The wing geometry and airfoil shapes are shown in Fig. 1, whereas the parametric range of the study are shown in Table 1.

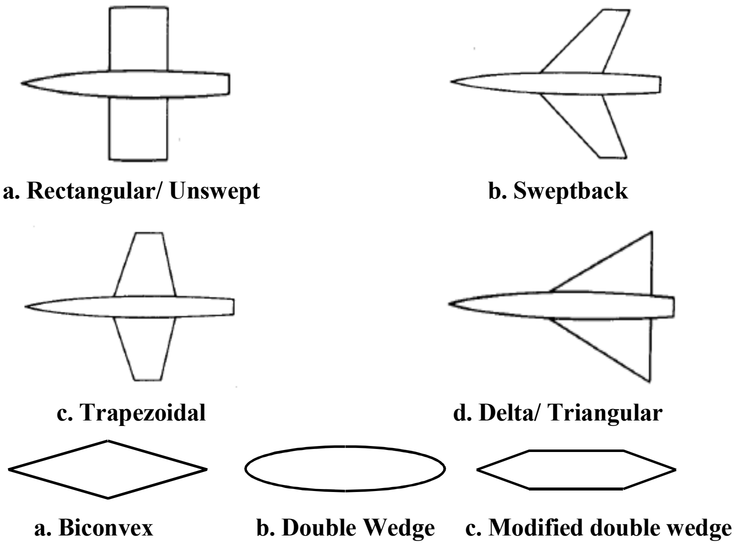

Fig. 1. Investigated Wing Planforms and Airfoil Shapes.

Table 1. Range of Parameters Studied.

Design Parameters	Range
Mach Number, M	$1.2 \leq M \leq 4$
Aspect Ratio, A	$2 \leq A \leq 4$
Taper Ratio, λ	$0 \leq \lambda \leq 1$
Leading Edge Sweep Angle, Λ_{LE} (deg.)	$10° \leq \Lambda_{LE} \leq 50°$
Relative Thickness, $\bar{t}_{max}$	$3\%, 5\%, 8\%$

2. Study Methodology

As described in the previous section, the study was conducted based on a combination of both analytical and semi-empirical method. The analytical method is based solely on both the linearized supersonic theory and the Busemann's second order approximation. The concept behind usage of analytical formulation is to formulate a generalized mathematical expression that accounts for all wing parameters as well as to study the effect of individual parameters. However it was later revealed that some parameters are highly coupled and that any attempt at a generalized mathematical expression will only prove to complicate the nature of the study rather than simplifying it. The analytical formulation by itself has its limitations; hence the semi-empirical method which uses the linear interpolation and extrapolation was used to validate the results obtained from the analytical formulation. It is also to be noted that certain parameters cannot be accounted for by the analytical formulations and hence these parameters can only be studied using the semi-empirical method. The figures and charts used for the purpose of semi-empirical method were obtained from Ref. [4] and converted to numerical data which can be referred to in *Appendix A*.

3. Prediction of Lift Coefficient

The total generated lift of a wing is usually expressed in terms of dimensionless coefficients in which are usually in terms of coefficient of lift, C_L, or in terms of lift-force curve slope, C_{L_α}, however both these dimensionless coefficients are related and are used interchangeably depending on the nature of the study. The relation between these two coefficients is as follows:

$$C_L = C_{L_\alpha}.\alpha \tag{1}$$

In this study, there are two other formulations that were used. These formulations are the *Linearized Supersonic Theory* and the *Busemann's Second Order Approximation*. Based on the linearized theory, the coefficient of lift can be expressed as:

$$C_L = \frac{4\alpha}{\sqrt{{M_\infty}^2 - 1}} \tag{2}$$

As shown in Eq. (2), the linearized theory only accounts for the angle of attack as well as the Mach number of the flying body, it does not account for the overall geometry of the wing. Hence this is the limitation of the linear theory in which the wing is assumed to be of infinite span and of infinitesimally thin, however this assumption can only be justified at hypersonic velocities where the trajectory of the flying body is governed purely by momentum and the amount of lift is independent of the shape of the wing and hence the lift of the wing can be safely predicted using Eq. (2).

The Busemann's second order approximation can be considered as an expanded variant of the linearized theory and is used to overcome the limitation of the linearized expression. The second order approximation is expressed as:

$$C_L = \frac{4\alpha}{\beta}[1 - \frac{1}{2\beta A}(1 - C_3 A')] \tag{3}$$

$$\beta = \sqrt{{M_\infty}^2 - 1} \tag{4}$$

$$C_3 = \frac{\gamma M^4 + (M^2 - 2)^2}{2(M^2 - 1)^{\frac{3}{2}}} \tag{5}$$

Equation (5) itself is a correctional coefficient that accounts for the influence of the airfoil thickness on the formation of expansion and compressive shock waves along the surface of the wing. The thickness effect of the airfoil is highly dependent on the cross sectional geometry of the airfoil and are expressed as:

Biconvex:

$$A' = \frac{2t}{3c} \tag{6}$$

Double Wedge:

$$A' = \frac{t}{2c} \tag{7}$$

Modified Double Wedge:

$$A' = \frac{2t}{3c} \tag{8}$$

The limitation of the Busemann's second order is that it is only accurate for rectangular unswept wings, it does not account for the effect of swept and trapezoidal wing. Hence it is due to this limitation that the semi-empirical method is relied upon heavily to fill the short comings of the analytical method.

4. MATLAB Computer Programme

For the purposes of facilitating the analysis of aerodynamic characteristics as well to reduce the effects of human error, a computer programme is developed in MATLAB code. The programme was designed to use the numerical data converted from the figures from Ref. [4]. The working concept of the programme is to use linear interpolation and extrapolation method to predict the aerodynamic characteristic of a particular wing planform based on a set of user defined inputs. An outline of the flow chart and capabilities of the program can be referred to in *Appendix. B*

To ensure the accuracy and validity of the results obtained by from the programme, the results are benchmarked against actual wind tunnel results obtained from Ref. [3].

5. Results and Discussion

The aerodynamic performance of each wing planform as described in sections above are analysed in five independent sections which are intended to study the influence of each parameter on the aerodynamic performance of the wing; which was carried out using both analytical and semi empirical methods as well as the computer programme described above.

5.1. Effect of aspect ratio

As shown in Fig. 2, it shows that aspect ratio and lift-force curve slope share a linear relationship in which the amount of lift generated by the wing is proportionate to the increase in aspect ratio. This is to be expected as the aspect ratio increase, so does the amount of wing surface that is exposed to moving fluid.

There are however some discrepancies in the data shown in Fig. 2. The discrepancies begin to manifest itself at Mach numbers greater 3.5. This suggests that the Mach cone itself has some profound effect on the lift and that there is a limit to the aspect ratio at higher order of Mach number.

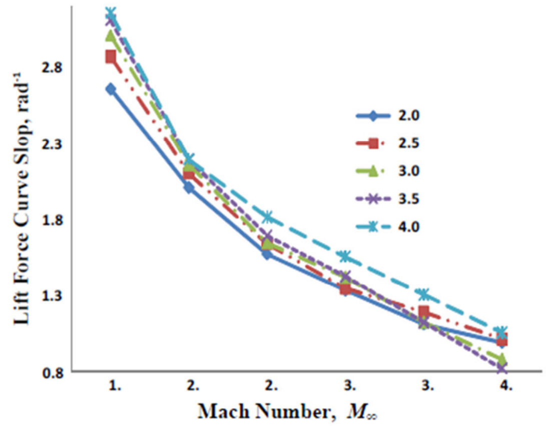

Fig. 2. Variation of Lift-Force Curve Slope vs. Mach Number for Different Aspect Ratios.

5.2. Effect of taper ratio

The influence of taper ratio, λ is rather quite ambiguous as shown in Table 2; at a glance, it is as though the taper ratio have no marginal effect on the lift force curve slope, C_{L_α}. The changes in C_{L_α} due to the change in taper ratio are miniscule.

As aforementioned in previous sections, some parameters are highly coupled. This is true in the case of taper ratio, λ, as it is nearly impossible to change taper ratio without changing the leading edge sweep angle, Λ_{LE}. This relationship between λ and Λ_{LE} needs to be further analyzed before any concrete conclusion can be drawn.

Table 2. Influence of Taper Ratio on the Lift-Force Curve Slope.

Mach Number	*Taper ratio*					
	1.0	**0.8**	**0.6**	**0.4**	**0.2**	**0.0**
1.5	2.6524	2.6487	2.6519	2.6515	2.6515	2.6524
2.0	2.0046	2.0036	2.0046	2.0045	2.0045	2.0046
2.5	1.5724	1.5705	1.5727	1.5729	1.5729	1.5724
3.0	1.3329	1.3341	1.3328	1.3328	1.3328	1.3329
3.5	1.1122	1.1153	1.1122	1.1122	1.1122	1.1122
4.0	0.9912	0.9928	0.9911	0.9910	0.9910	0.9912

5.3. Effect of leading edge sweep angle

As shown in Fig. 3, it is to be noted that changes in leading edge sweep angle, Λ_{LE} decrease the magnitude of lift force curve slope. This is especially true at low Mach numbers ($M \leq 3.5$). At higher Mach number, the increase in Λ_{LE} increases the aerodynamic performance of the wing.

This change in relationship suggests that the Mach cone generated by the flying body has some degree of effect on the aerodynamic performance of the wing. This suggests that lift is only generated at supersonic speeds when the wing has a subsonic leading edge (portions of the wing that is still enclosed within the Mach cone).

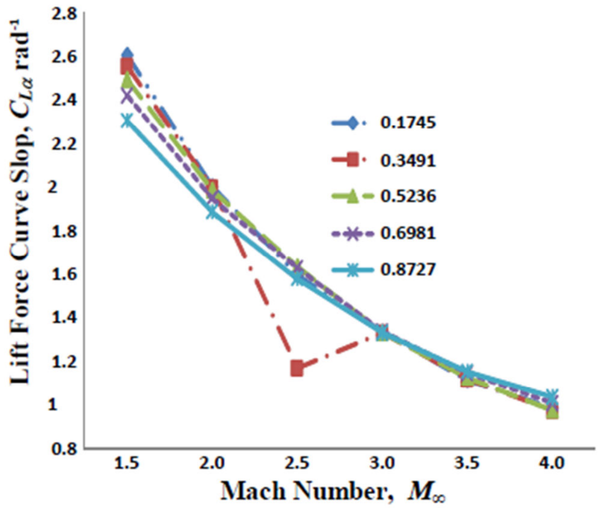

Fig. 3. Variation of Lift-Force Curve Slope vs. Mach Number for Different Leading Edge Sweep Angles (rad.).

It is known that the Mach angle, μ is inversely proportionate to the Mach number, M. This relationship can express as:

$$\mu = \sin^{-1}\frac{1}{M} \tag{9}$$

Hence based on Eq. (9), this relationship shows that the Mach cone will become increasingly slender as the Mach number increases; hence at higher order of Mach number, having a swept wing will increase the amount of lift the wing is able to generate. However there is a secondary effect of the Mach cone, which is that at lower Mach numbers, having a swept wing reduces the aerodynamic performance and that the wing is most efficient when its Λ_{LE} is identical to the Mach angle. This suggests that the air velocity that passes the compressive shock wave (Mach cone) has two components an axial and a normal component. The normal component is believed to be in subsonic velocities whilst the axial component is in supersonic velocity. It is the subsonic component that allows the wing to generate lift as it is.

5.4. Effect of wing thickness

The wing thickness, as postulated by Bonnie [2], has negligible effects on the amount of lift the wing is able to generate. This notion is proven to be true by both analytical and semi-empirical methods. As shown in Fig. 4, the increase in airfoil relative thickness,$\bar{t}_{MAX,}$ has no effect on the aerodynamic performance of the wing.

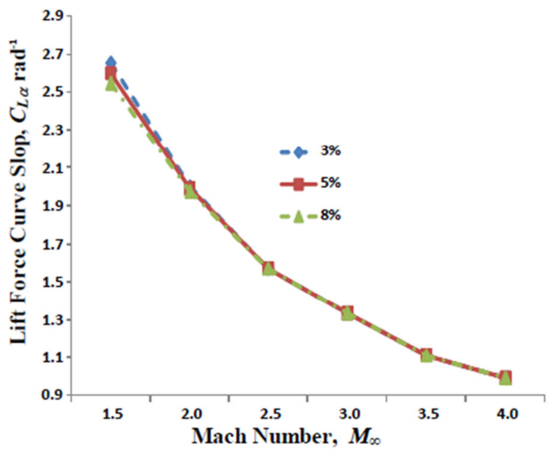

Fig. 4. Variation of Lift-Force Curve Slope vs. Mach Number for Different Airfoil Thicknesses.

5.5. Effect of airfoil shape

The airfoil shape was proven to have no significant or rather negligible effect on the aerodynamic performance of the wing as shown in Fig. 5. Although the biconvex airfoil does appear to perform slightly better compared to the double wedge, however this degree of improvement is too small to be of any significance.

There is virtually no difference in performance capabilities of both airfoils, and it is likely that airfoil selection for a supersonic wing is usually dependent on other factors such as drag, stability and to some extend economical perspective and some airfoils are harder to fabricate [5].

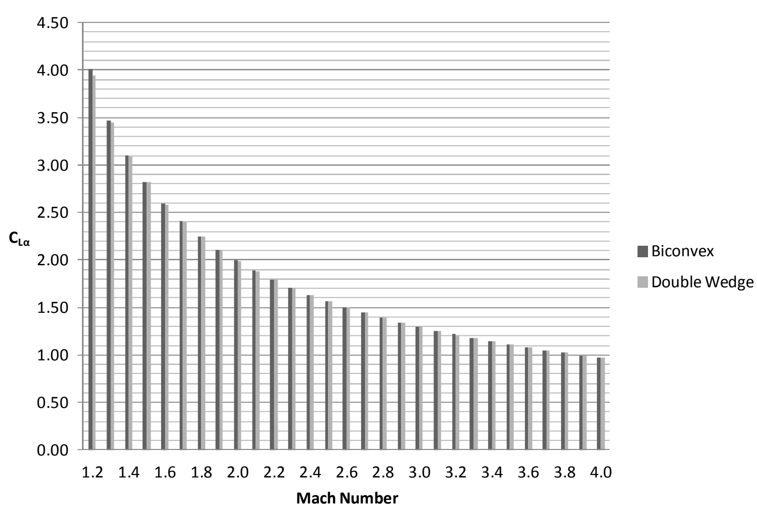

Fig. 5. Lift Generated for different Airfoil Shapes at Different Speeds.

6. Conclusions

Considering the results of the study for the four wing planform with regards to Mach number ranging from 1.5 to 4 and their respective effect on lift, the following conclusions are made.

- The airfoil shape and relative thickness of the wing has negligible effect on the generated lift.
- The taper ratio and leading edge sweep angle are highly coupled
- The leading edge sweep angle by itself is independent of the effects of taper ratio, λ.
- The leading edge sweep angle will only be beneficial at higher Mach numbers (Mach > 2.0) due to the effect of the Mach cone.
- The aspect ratio have a linear relationship with the generated lift; this relationship is only true for low to mid-range supersonic speeds (Mach < 3.5)
- The effect of the taper ratio is believed to be obscured by effect of the leading edge sweep angle.

References

1. Jones, R. T. (1947). *Wing plan forms for high speed flight*. Langley Memorial Aeronautical Laboratory, National Advisory Committee for Aeronautics, Langley Field, VA.
2. Snow, R.M.; and Bonney, E.A. (1947). *Aerodynamic characterisitcs of wings at supersonic speeds.* The John Hopkins University Applied Physics Laboratory, Maryland.
3. Hall, C.F. (1953). *Lift, drag, and pitching moment of low aspect ratio wings at subsonic and supersonic speeds*. Natonal Advisory Commitee For Aeronautics, Washington.
4. Al-Obaidi, A.S.M. (2005). *The influence of configuration design on the aerodynamics and stability of SSM at supersonic speeds*. Ph.D. Thesis, Al-Rasheed College of Engineering and Science, University of Technology, Baghdad, Iraq.
5. Kermode, A.C. (2006). *Mechanics of Flight*. Pearson Prentice Hall, New Jersey.
6. Lebedev, A.A.; and Chernobrovkin, L.S. (1973). *Dinamika poleta bezpilotnykh letatelykh apparatov*. "*Flight dynamics of unmanned vehicles*". Mashinostroenie, Moscow.
7. Jankovic, S. (1979). *Aerodynamika Projektila*. "*Aerodynamics of Projectiles*" Masinski Fakultet, Univerziteta u Beogradu, Belgrade.

Appendix A

Figures of Design Charts Used for Numerical Data Conversion

Figures A-1 to A-4 used in the semi-empirical analysis of this study were obtained from Al-Obaidi [3]. It is to note that the figures and charts were originally provided by Lebedev and Chernobrovkin [6] and Jankovic [7].

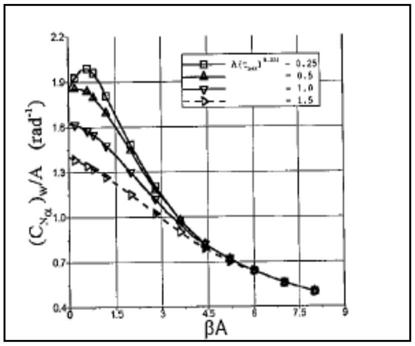

Fig. A-1. Normal Force Curve Slope for *A*.tan $\Lambda_{c/2}$= 0.0.

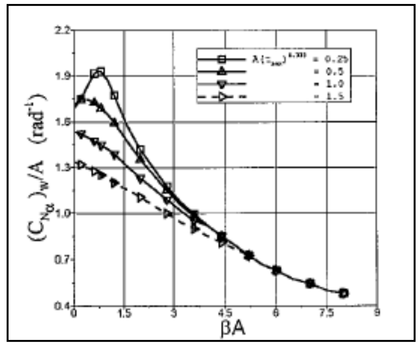

Fig. A-2. Normal Force Curve Slope for *A*.tan $\Lambda_{c/2}$= 1.0.

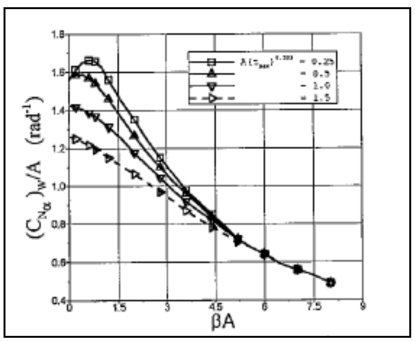

Fig. A-3. Normal Force Curve Slope for *A*.tan $\Lambda_{c/2}$= 2.0.

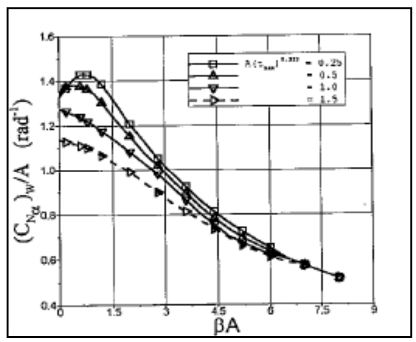

Fig. A-4. Normal Force Curve Slope for *A*.tan $\Lambda_{c/2}$= 3.0.

Appendix B

MATLAB Computer Programme

B.1. Introduction

A MATLAB computer code was written and used in the semi-empirical analysis of the study. The MATLAB programme was written to operate based on the concept of Multi-Array Matrix manipulation as well as linear interpolation and extrapolation methodology to enable a quick prediction of lift characteristic of a wing a wing as a function of Mach number, aspect ratio, leading edge sweep angle, taper ratio and relative thickness of the wing.

It is to be noted that the program is designed to operate with the accuracy standard suitable for preliminary design although an updated database will allow the program to function beyond preliminary design and be suitable for optimization stages.

B.2. Programme Limitations

The programme was created based on the available data provided by Lebedev and Chernobrovkin [6] and Jankovic [7] hence the accuracy of the prediction is limited to the data provided and that any input that is outside of the range will be predicted using extrapolation protocol embedded within the program. The accuracy of the extrapolated data has yet to be confirmed and hence user discretion is advised.