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DIGITAL OPTIMAL CONTROL OF MULTI-STAND ROLLING MILLS WITH MEASUREMENT AND INPUT DELAYS

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Abstract

In this paper, the problem of designing an optimal control algorithm for rolling processes is addressed. Deriving a mathematical model for a general class of rolling mills, the optimal control problem is defined. In the design procedure, time delays in thickness measurment and control input are taken into account. The proposed strategy significantly improves the results of previous investigations from practical point of view. Simulation results are also studied to highlight the effictiveness of the method.

Keywords: Time delay, Optimal control, Rolling process, Thickness control.

1. Introduction

Rolling process plays an important role in steel manufacturing due to the increased demand for rolled products with high quality in aircrafts, automotive industry and so on. Although many control algorithms may theoretically meet such desired properties, but some practical limitations cause those methods not be implementable. In fact, rolling mills are some nonlinear systems with multiple inputs and multiple outputs [1-3]. Apart from the mechanical equipment, a big potential for improving the quality of the rolled products lies in the techniques adopted for process control. Meanwhile, high quality can be obtained by some control algorithms by using some sensors and actuators.

Tension and speed control of the continuous strip processing lines are the challenging problems to be devoted to get the desired performance. During the past decades, various control methodologies have been developed for rolling processes under different assumptions [4-7]. Inverse Linear Quadratic (ILQ) theory,

Nomenclatures

f	Forward slip
H	Entry thickness, mm
h	Exit thickness, mm
K	Elastic constant, N/mm
P	Roll force, N
R	Work roll radius, mm
r	Thickness reduction
S	Roll gap, mm
s	Yield stress
V_R	Roll velocity, m/s
v	Strip velocity, m/s
W	Strip width, mm
Greek Symbols	
μ	Coefficient of friction
σ	Tension, N/mm

as a multivariable and optimal control technique can be applied to mass flow control in rolling mills [5, 8]. As it is straightforward to design control systems based on ILQ theory and it is easy to tune the resulting system, this method is also used for actual plant control. Robust control may be also adopted to tackle bounded disturbances [9, 10].

From an optimization viewpoint, applying optimal control techniques can significantly decrease the production costs. System modelling, based on Petri nets, may be used to develop some optimal solutions in view of production schedules [11]. In fact, rolling mill is described as a discrete event system to be used in optimizing a prescribed objective function. Such methodology is highly dependent to system structure and parameters and cannot be applied to a wide class of rolling mills with any number of stands. Optimization in rolling program, e.g., in batch scheduling [12], and number of rolling profiles [13], has been also noticed in previous works. More recent investigations have focused on optimal thickness and tension control strategies e.g., based on solving riccati equations [14, 15], intelligent strategies [16], model predictive control [17], and stochastic optimization algorithms [18].

In spite of all the existing methods, considerable attention is still paid toward developing a controller, which effectively treats both the optimal performance and considering time delays [19]. In practice, time dalay which adds some right half plane zeros to open loop system and closed loop instability, inevitably arises from various sources, e.g., (i) the online data acquisition from sensors at different locations of the system, (ii) the time taken processing of the sensory data for the required control force calculation and transmission to the actuators, and (iii) the time taken by the actuator to produce the required control force. In this paper, a digital optimal control is formulated for a general class of rolling mills, taking the measurement and input delays into account.

The organization of the paper is as follows. In Section 2, the general behaviour of rolling mills is introduced. The dynamic equations of the system are presented in Section 3. The optimal control problem of time-delayed systems is formulated in

Section 4, and the proposed algorithm is then applied to a typical rolling process in Section 5. Finally, the concluding remarks are given in section 6.

2. Rolling Mill Behavior

Rolling process plays an important role in steel manufacturing due to the increased demand for rolled products with high quality in aircrafts, automobile, and other related industries. In the rolling process, strip is passed through a continuous production line, at high speed. Thickness reduction is managed by several stands, each consists of two work rolls and two backup rolls, as schematically shown in Fig. 1.

The basic equations which govern the relationship of system variables are formed by the characteristics of the material and the mechanical variables, in which the index *i* corresponds to the *i*th stand. The symbols used and appeared in the equations are listed in the Nomenclature.

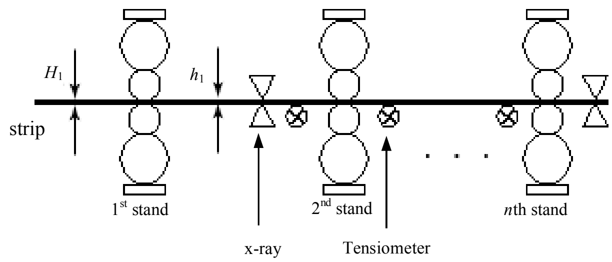

Fig. 1. A Typical Rolling Mill With *n* Stands.

The basic equations which govern the relationship of system variables are formed by the characteristics of the material and the mechanical variables, in which the index *i* corresponds to the *i*th stand.

The rolling force equation for the *i*th stand is given by an implicit function as [20]

$$P_i(H_1, H_i, h_i, \sigma_{fi}) = 0 \tag{1}$$

where H_1 is the entry thickness of the first stand, and h_i, H_i, and σ_{fi} denotes respectively the exit thickness, the entry thickness and the tension at front of the stand.

More precisely, the exit strip thickness is governed by

$$h_i = S_i + \frac{F_i}{K} \tag{2}$$

in which S represents the roll gap and K is the elastic coefficient. The rolling force equation is

$$P_i = W\kappa_i \bar{k}_i \sqrt{R_i'(H_i - h_i)} D_{pi} \tag{3}$$

where

$$\kappa_i = g_1(H_1, H_i, h_i, \sigma_{bi}, \sigma_{fi}) \tag{4}$$

$$\bar{k}_i = \alpha(\bar{r}_i + \beta) \tag{5}$$

$$D_{pi} = g_2(\mu, R, H_i, h_i) \tag{6}$$

in which, $\bar{r}_i$ is the mean value of r_i in the back and front of the ith stand. Thickness reduction at each stand, r_i, is given by

$$r_i = \frac{H_i - h_i}{H_i} \tag{7}$$

1) Flattened roll radius R_i' is determined by

$$R_i' = g_3(P_i, R_\mathrm{i}, W, H_i, h_\mathrm{i}) \tag{8}$$

2) The strip velocity is related to roll velocity and forward slip by

$$v_{fi} = (1 + f_i)V_{Ri} \tag{9}$$

where

$$f_i = \tan^2\left(\frac{H_{ni}}{2}\sqrt{\frac{h_i}{R_i'}}\right) \tag{10}$$

and

$$H_{ni} = g_4(\mu, R_i, H_i, h_i, \sigma_{fi}, \sigma_{bi}) = \sqrt{\frac{R_i'}{h_i}} sin^{-1}\sqrt{r_i} - \frac{1}{2\mu} ln\left(\frac{H_i}{h_i}\frac{1-\frac{\sigma_{fi}}{s_{fi}}}{1-\frac{\sigma_{bi}}{s_{bi}}}\right) \tag{11}$$

$$s_{fi} = \alpha(r_{fi} + \beta)^\gamma \tag{12}$$

$$s_{bi} = \alpha(r_{bi} + \beta)^\gamma \tag{13}$$

In the aforementioned equations, α, β, γ are material dependent constants and the indices b and f denote the parameter values in the back and front of stands respectively.

3. Dynamic Equations

The dynamic behaviour of rolling mills is mostly determined by the equations, related to interstand tensions, roll gap and roll velocity dynamics at each stand. The two simplified dynamics related to roll gap and roll velocity are [14, 20]

$$\dot{S}_i = -\frac{1}{T_s}(S_i - U_{si}) \tag{14}$$

$$\dot{V}_{Ri} = -\frac{1}{T_v}(V_{Ri} - U_{vi}) \tag{15}$$

where U_{si} and U_{vi} are the reference values and, T_s and T_v are the related time constants. Tension is governed by the following nonlinear dynamics:

$$\dot{\sigma}_{fi} = \frac{E}{L}\left(v_{b(i+1)} - v_{fi}\right) \tag{16}$$

where E and L are Young's modulus of the strip and the distance between each two stands respectively.

In general, the dynamic behaviour of the process represents nonlinearities. Although the relationships between some of the process variables are nonlinear, but at steady state the equations governing the system would be linear [1, 3, 8]. The linearized model around the operating point presents a state-space description of the form

$$\begin{cases} \dot{X} = AX + Bu \\ y = CX \end{cases} \tag{17}$$

where X, y and u represent the vectors of states, outputs and inputs respectively. Moreover A, B, C are some matrices whose dimensions depend on the number of stands n. More precisely, X contains the interstand tensions, roll velocities and gap between work rolls. The dimension of state vector X depends on the number of stands in rolling mill, i.e., for n stands one can select

$$X = [\sigma_{f1},...,\sigma_{fn}, V_{R1},...,V_{Rn}, S_1,...,S_n]^T \tag{18}$$

In most of rolling processes, strip thickness is measured directly by x-ray measurement after stands. Although this method gives exact gauge value, the time delay involved in measurement cannot be ignored in the design procedure. The delay τ is determined by the strip velocity v_f at the exit and the distance l between each stand and x-ray measuring device, i.e.,

$$\tau = \frac{l}{v_f} \tag{19}$$

4. Opimal Controller Design

In order to develop the optimal control algorithm for rolling mills, LQR method is first introduced for linear systems without delay. Then, incorporating the input and measurment delays into dynamical equations, the proposed controller is designed.

4.1. Classical LQR algorithm revisite

Consider a linear system, described by

$$\dot{x} = Ax + Bu \tag{20}$$

where x and u denote the state and control vector respectively. Choosing the control input as

$$u(t) = -K\,x(t) \tag{21}$$

the optimal control problem is to find the vector matrix K such that the quadratic criterion

$$J = \int_0^{\infty} (x^T Q\, x + u^T R\, u)dt \tag{22}$$

is minimized. In performance index (22), Q and R are weighting matrices which respectively show the significance of obtaining small states and control signals with respect together. In other words, the constraints on admissible control signals, imposed by the actuators, can be considered by choosing suitable weighting matrices. Equivalently, the digital implementation of the optimal LQR method may be stated as follows.

For discrete time linear system

$$x(k+1) = Ax(k) + Bu(k) \tag{23}$$

the performance index takes a form as

$$J(u) = \sum_{n=1}^{\infty}[x^T(n)Qx(n) + u^T(n)Ru(n)] \tag{24}$$

It can shown that the optimal gain vector in

$$u(k) = -K(k)\, x(k) \tag{25}$$

is obtained as

$$K_{\text{opt}}(k) = R^{-1}B^T S \tag{26}$$

where S is the solution of Riccati equation

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{27}$$

Now, the classical LQR method is extended to a class of time delay systems by which rolling processes dynamics can be described.

4.2. Optimal control of time delay systems

As a general case, time delay may exist in both states and inputs. More precisely, the linear system dynamics (23) takes the form

$$x\,(k+1)=Ax(k)+\sum_{i=1}^{n_1} A_i x(k-\gamma_i) + Bu(k)+\sum_{j=1}^{n_2} B_j u(k-\tau_j) \tag{28}$$

in which $\gamma_i, i = 1,2,\dots,n_1$ and $\tau_j, j = 1,2,\dots,n_2$ denote the dealys in the states and inputs respectively.

The key idea is transforming the dynamical equation (28) into a nominal form by defining

$$e(k) = r(k\text{-}1) - x(k\text{-}1) \tag{29}$$

$$\Delta x\,(k) = x(k) - x(k\text{-}1) \tag{30}$$

and

$$\Delta u(k) = u(k) - u(k-1) \tag{31}$$

where r is the reference input. Replacing the error variables (29)-(31) in (28), yields

$$\Delta x\,(k+1)=A\Delta x(k)+\sum_{i=1}^{n_1} A_i \Delta x(k-\gamma_i) + B\Delta u(\text{k})+\sum_{j=1}^{n_2} B_j \Delta u(k-\tau_j) \tag{32}$$

Now, defining the new state vector as

$$Z(k) = \begin{bmatrix} e(k) \\ \Delta x(k) \\ \Delta x(k-1) \\ \vdots \\ \Delta x(k-\gamma_1) \\ \Delta u(k-1) \\ \vdots \\ \Delta u(k-\tau_1) \end{bmatrix} \quad (33)$$

results in a discrete time linear system as

$$Z(k+1) = \bar{A}Z(k) + \bar{B}\,\Delta u(k) + \Delta r(k) \quad (34)$$

where $\bar{A}$ and $\bar{B}$ are some matrices with appropriate dimensions, depending on the elements on A and B in (23). In fact, the discrete system (23) has a form, similar to the linear system without delay (20). Hence, the classical LQR algorithm for (34) can be applied as the optimal controller of time delay system (28).

5. Application to A Rolling Mill Process

In order to evaluate the performance of the optimal controller, the developed LQR algorithm is applied here to a 2-stand cold rolling process. Nevertheles, the procedure can be easily extended to a system with n stands, whose model has been introduced in Secions 2 and 3. The numerical values of the system are specified by Tables 1 and 2.

By using such parameter values, the matrices A and B of the state space description are calculated as

$$\mathbf{A} = \begin{pmatrix} 4.02 & -48 & -1.66\times 10^2 & 2.1\times 10^2 \\ 0 & -1.667 & 0 & 0 \\ 0 & 0 & -2.5 & 0 \\ 0 & 0 & 0 & -2.5 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 \\ 1.667 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.5 \end{pmatrix}$$

Assuming l is 2.5 m, Eq. (19) results in time delays $\tau_1 = 0.225$ s and $\tau_2 = 0.2$ s. Now, the discrete time model of system of the form (34) is determined by defining

$$Z(k) = \begin{bmatrix} e(k) \\ \Delta x_1(k) \\ \Delta x_2(k) \\ \Delta x_1(k-1) \\ \Delta x_2(k-1) \\ \Delta x_1(k-2) \\ \Delta x_2(k-2) \\ \Delta u(k-1) \end{bmatrix} \quad (35)$$

Table 1. Strip Mill Specifications.

Parameter	Value
K (N/mm)	4.6×10^9
T_v (s)	0.6
T_s (s)	0.3
E (N/mm^2)	2.1×10^5
L (mm)	4600
$\alpha(-)$	620
$\beta(-)$	0.008
$\gamma(-)$	0.3

Table 2. Tandem Mill Operating Points.

Parameter	First stand	Last stand
H (mm)	2.1	1.6
h (mm)	1.6	1.38
W (mm)	1000	1000
v_f (m/s)	10.08	11.44
σ_b (N/mm)	20	100
σ_f (N/mm)	100	44
R (mm)	285	285
f	0.0488	0.0166
μ	0.07	0.07

In order to evaluate the performance of the applied optimal controller for the underlying rolling system, some simulation studies are presented here. In order to make a comparison, a PID controller is applied to system and the time response is depicted in Fig. 2. Using the PID structure, a faster response and a more smooth output is achieved, as shown in Fig. 3, comparing to the original system (without PID) and a simple proportional controller, respectively. Nevertheless, from a practical viewpoint, the time response is not acceptable dut to the overshoots and fluctuations.

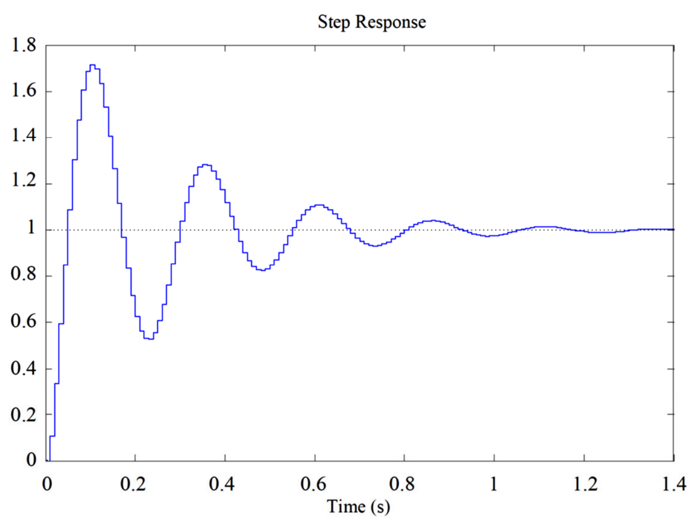

Fig. 2. Output Response by Applying PID Structure.

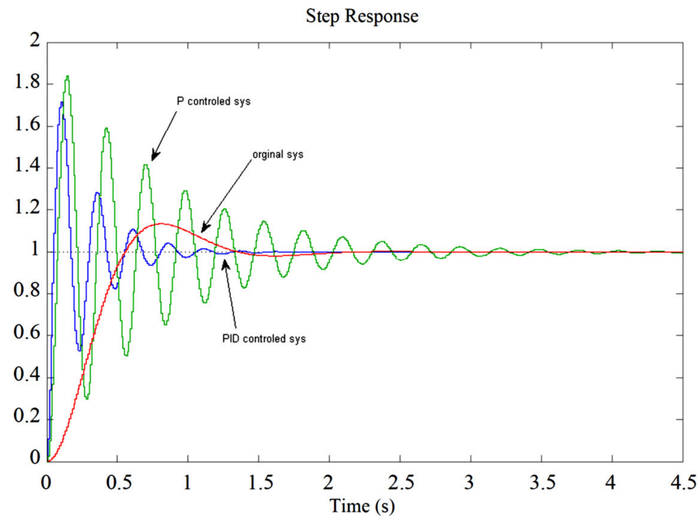

Fig. 3. Step Response Using Proportional and PID Controllers.

To remove such drawbacks, the optimal controller based on LQR algorithm is applied to system with reference input 1.38 mm for thickness. Taking the time delays τ_1 and τ_2 into account, the capability of the poposed optimal controller is demontrated in Figs. 4 and 5. It is shown that the output which has been significantly imroved with respect to PID algorithm. A smooth response without steady state error is obtained by adopting the weighting matrices in objective function (15) as $Q = I$, and $R = 2I$. Moreover, a faster time response can be achived by $Q = 2I$, and $R = I$, at the expense of larger control effort. In fact, increasing the weighting matrix Q causes to increase the speed of convergence in the system states.

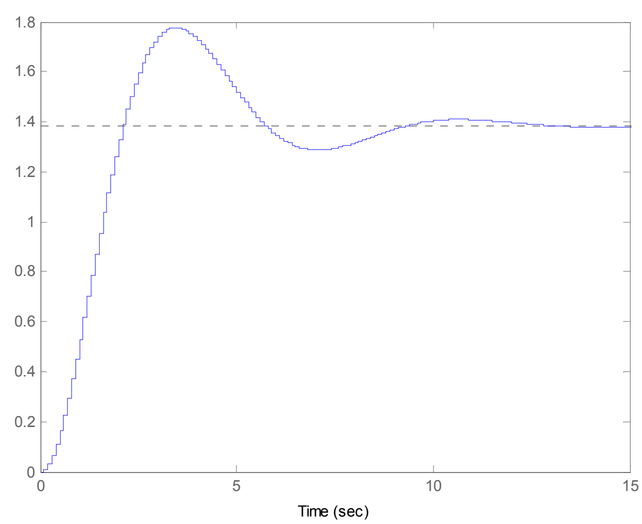

Fig. 4. Thickness Control by LQR Algorithm.

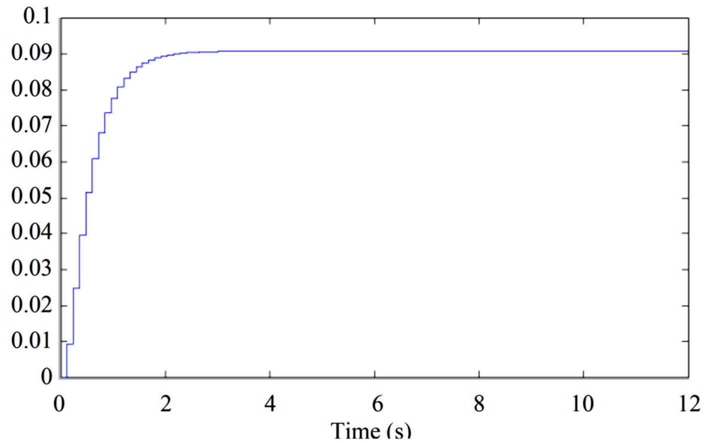

Fig. 5. Tension Regulation Using LQR Method.

Remark 1: Comparing with some previous investigations, developed for restricted rolling systems [11, 13, 14, 18], the developed optimal controller can be applied to a wide class of rolling mills.

Remark 2: Ensuring both optimal performance and simplicity of implementation for rolling mills, as a complex dynamical system, are of the main benefits of the developed algorithm. In fact, some theoretically powerful methods may result in a complicated design procedure [9, 14].

6. Conclusion

Assigning the state variables, inputs and outputs, a general class of rolling mills in the presence of measurement and output delays are modeled. The optimal control problem is formulated for discrete-time linear systems. Then, such method is modified and extended to the case, time-delays also exist is system dynamics. The developed digital optimal algorithm is applied to a typical rolling process. To show the benefits of the developed scheme, PI and PID strategies are also applied to the system. Making a comparison between simulation results show the effectiveness of the designed digital optimal scheme.

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