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RELIABILITY–REDUNDANCY ALLOCATION PROBLEM OF PHARMACEUTICAL PLANT

HARISH GARG[1,*], S. P. SHARMA[2]

[1]School of Mathematics and Computer Applications,
Thapar University Patiala 147004, Punjab, India
[2]Department of Mathematics, Indian Institute of Technology Roorkee,
Roorkee- 247667, Uttarakhand, India
*Corresponding Author: harishg58iitr@gmail.com

Abstract

The reliability design is related to the performance analysis of engineering systems. The reliability - redundancy optimization problems involve selection of components with multiple choices and redundancy levels that produce maximum benefits, can be subject to the cost, weight, volume and reliability constraints. Classical mathematical methods fail in handling non-convexities and non-smoothness in optimization problems. This drawback has been removed through meta-heuristics due to their ability of finding an almost global optimal solution in reliability-redundancy optimization problems. Particle swarm optimization (PSO) is one of such meta-heuristic algorithm. An efficient penalty based PSO algorithm has been proposed in this paper for reliability-redundancy optimization problems. This paper aims to present an application of the PSO algorithm for searching the optimal solution of reliability-redundancy allocation problems with nonlinear resource constraints of a Pharmaceutical Plant.

Keywords: Redundancy allocation, Particle swarm optimization, Industrial system, Reliability optimization.

1. Introduction

Due to advances in technology and growing complexity in technological systems, the job of system analyst has become more challenging as they characterize measure and analyse the system behaviour using quantitative and qualitative techniques for enhancing the production as well as productivity. The increasing need for highly reliable system further demands the study of reliability optimization. To design a highly reliable system, there are mainly two ways of

Nomenclatures

C	Upper limit of the cost of the system
c_i	Cost of each component in the subsystem i $(1 \le i \le m)$
g_i	i^{th} constraint function
M	Number of constraints
m	Number of subsystems in the system
n	Vector of redundancy allocation for the system, = $(n_1, n_2, \ldots, n_m)$
n_i	Number of components in subsystem
R_s	System reliability
r	Vector of component reliabilities for the system, = $(r_1, r_2, \ldots, r_m)$
r_i	Reliability of each component in subsystem, i $(1 \le i \le m)$
S	Set of feasible region
V	Upper limit of the volume of the system
v_i	volume of each component in the subsystem i $(1 \le i \le m)$
W	Upper limit of the weight of the system
w_i	weight of each component of the system i $(1 \le i \le m)$

Abbreviations

PSO	Particle swarm optimization
RAP	Reliability allocation problem
RRAP	Reliability-redundancy allocation problem

improving the system reliability one is -adding the redundant components and other is - increasing the component reliability. Both ways are usually increase the resources (cost, weight, volume, etc.). Therefore, at the stage of designing a highly reliable system, an important problem can be how to get the balance between reliability and other resources [1]. Besides the above two ways, the combination of the two approaches and reassignment of interchangeable elements are another feasible ways for increasing the system reliability. Such problem of maximizing system reliability through redundancy and component reliability choices is called “reliability-redundancy allocation problem (RRAP)."

During the last two decades, numerous reliability design techniques have been introduced to solve these problems. These techniques can be classified as implicit enumeration, dynamic programming, branch and bound technique, linear programming, Lagrangian multiplier method, heuristic method and so on [2-5]. Kuo et al. [6] and Tillman et al. [7] have extensively reviewed the several optimization techniques for system reliability design in their books. Since the reliability-redundancy allocation problem is a complex NP hard problem, so the computational complexity of the problem increases. Moreover, the all the above heuristic techniques require derivatives for all nonlinear constraint functions that are not derived easily because of highly computational complexity. Hence heuristic and meta-heuristic have become a popular alternative for it. These heuristics include genetic algorithms, simulated annealing, tabu search, particle swarm optimization, artificial bee colony, etc. Yokota et al. [8] and Hsieh et al. [9] applied genetic algorithms to solve these mixed-integer reliability optimization problems. Coit and Smith [10] combined GA and neural network (NN) to tackle the series-parallel redundancy problem. Chen [11] applied the immune algorithm (IA) for solving the reliability-redundancy allocation problem. Coelho [12] proposed an efficient PSO algorithm based on Gaussian distribution and chaotic sequence (PSO-GC) to solve the reliability-redundancy optimization problems.

In the present paper, the reliability - redundancy allocation problem of the pharmaceutical plant, situated in the northern part of India, is formulated as a non - convex integer nonlinear problem in which we maximize the reliability of the given system w.r.t. constraint functions associated with system weight, cost, volume and reliability. It is worth mentioning that Garg et al. [13] solved the reliability allocation problem (RAP) only for the same plant that too w.r.t. linear cost constraints only, while we have considered non - linear mixed integer RRAP, instead of RAP along with the non-linear constraints (weight, volume, cost and reliability of the system). PSO, is one of the meta-heuristic technique, is used to optimize the system reliability of the plant. Further the remaining content of the paper is organized as follow: Section 2 describes the used assumptions and notations. Section 3 deals with the description of the RRAP along with methodology. The case study of the pharmaceutical plant along with the mathematical formulation of the RRAP is described in Section 4 while conclusions drawn are discussed in Section 5.

2. Assumptions and Notations

Before introduce the reliability-redundancy allocation problem, we define the following assumptions and notation that have been used in the entire paper. The notations are as shown in the Nomenclature section, whereas the assumptions are stated as below:

- The supply of components is unlimited.
- Reliability, cost, weight and volume of each component in one subsystem are same.
- Failed components do not damage the system, and are not repaired.
- All redundancies are active: hazard function is the same when it is in use or not in use.
- Failures of individual components are independent.

3. Description of the Reliability-Redundancy Allocation Problem

The reliability-redundancy allocation problems (RRAP) determine the optimal component reliabilities and redundancy level of components in a system to maximize the system reliability subject to several resource constraints. The RRAP is formulated as follows:

$$\begin{aligned} &\text{Maximize} \quad R_s = f(r,n) \\ &\text{subject to}: \quad g_j(r,n) \le 0, \quad j = 1,2,....,M \qquad (1) \\ &\qquad 0 \le r_j \le 1, \quad r_j \in [0,1], \quad n_j \in Z^+ \end{aligned}$$

where R_s is the reliability of system, g is the set of constraint functions usually associated with system weight, volume and cost, $r = (r_1, r_2, \ldots, r_m)$ and

$n = (n_1, n_2, \ldots, n_m)$ are respectively the vectors whose i^{th} components are reliabilities and redundancy allocation for the i^{th} subsystem, i.e., r_i and n_i are the reliability and the number of components in the i^{th} subsystem, respectively; $f(\cdot)$ is the objective function for the overall system reliability. The goal is to determine the number of components and the reliability of each subsystem so as to maximize the overall system reliability. The above problem belongs to the category of constrained nonlinear mixed-integer optimization problems because the number of redundancy n_i are the positive integer values and the component reliability r_i are the real values between 0 and 1.

The main task while solving the constraint optimization problem is to handle the constraints relating to the problem because the feasible solution of this problem is not easy to find due to presence of both types of constraints in the form of equalities as well as inequalities. The penalty function method, consisting of a sum of the objective and the constraints weighted by penalties, is one of the widely used and quite popular methods to handle the constraints relating to the problem. Despite the popularity of penalty functions, they have several drawbacks from which the main one is that there are too many parameters to be adjusted and finding the right combination of the same may not be easy. Also during that the search is very slow and there is no guarantee that the optima will be attained. To overcome this limitation, Deb [14] modified these algorithms by eliminating the penalty coefficients and the new modified objective function contained only the constraints violations as the additional term. The new objective function $F(x)$ was framed as follow:

$$F(x) = \begin{cases} f_w + \sum_{j=1}^{M} g_j(x) & \text{if } x \notin S \\ f(x) & \text{if } x \in S \end{cases} \tag{2}$$

The parameter f_w is the function value of the worst feasible solution in the population. Hence, the constrained problem (1) when converted into unconstrained problem with the objective function given in (2) is viewed as

$$\text{Maximize : } \quad F(x)$$
$$\text{subject to : } \quad x_k^l \le x_k \le x_k^u, \quad k = 1, 2, \ldots, m$$

where x_k is the set of decision variables. The unconstrained optimization problem thus obtained is solved by using PSO. For the sake of completeness we discuss the PSO technique in the next subsection.

Optimization using Particle Swarm Optimization

Particle Swarm Optimization (PSO), first introduced by Kennedy and Eberhart [15], is a stochastic global optimization technique. The algorithm models the behavior of a group of particles whose initial values are specified by a group of proposed random solutions called particles. These particles repeatedly search the environment of the problem to reach new solutions. The position and velocity of i^{th} particle at iteration 't' are specified by $x_i(t)$ and $v_i(t)$ respectively in the searching space. Each particle conserves its best position $pbest_i(t)$ and global best position *gbest*. Then velocity and position of particle i at iteration t+1 are respectively updated as follows

$$v_i(t+1) = w \times v_i(t) + c_1 \times rand \times (pbest_i(t) - x_i(t)) + \ldots . c_2 \times rand \times (gbest(t) - x_i(t)) \quad (3)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (4)$$

where c_1 and c_2 are the acceleration constants with positive values; $rand$ are random numbers between 0 and 1. The parameter w is the inertia weight factor which is decreased and varied linearly from initial (w_1) to final (w_2) with respect to iteration number [16].

The particle velocity in Eq. (3) is an important parameter because it determines the resolution about the solution regions. Furthermore it was necessary to set a control parameter v_{max} for the velocity that is unable to exceed this value. The choice of a too small value for v_{max} can cause very small updating of velocities and positions of particles at each iteration. Hence, the algorithm may take a long time to converge and face a problem of getting struck to local minima. To overcome these situations, Clerc and Kennedy [17] have proposed improved velocity update rules employing a constriction factor χ and accordingly the velocity update equation is

$$v_i(t+1) = \chi[v_i(t) + c_1 * rand * (pbest_i(t) - x_i(t)) + c_2 * rand * (gbest(t) - x_i(t)]$$

where $\chi = \dfrac{2}{|2 - \phi - \sqrt{\phi^2 - 4\phi}|}$ with $\phi = c_1 + c_2, \phi > 4$

Clerc and Kennedy [17] found that the system behavior could be controlled to have the following features: (i) the system does not diverge in a real value region and finally can converge; and (ii) the system can search different regions efficiently by avoiding premature convergence.

4. Case study: Pharmaceutical Plant

For illustration, we have taken an industrial problem of a pharmaceutical plant.

4.1. System description

The Pharmaceutical plant consists of various units viz. Weighing Machine, Sifter Machine, Mass Mixer, Granulator, Fluid Bed Dryer, Octagonal Blender, Rotary Compression Machine, Coating Machine, Air compressor, Strip Packing Machine. These subsystems are arranged in series. Initially different raw materials are weighed according to the master formula with the help of weighing machine. Then this mixture is placed into the Shifter. Shifter is used for sieving of raw material. After sieving, raw material is transferred to Mass Mixer for proper mixing and then granulation is done with the help of granulator, then these wet granules are too dried up with the help of Fluid Bed Dryer. After drying of granules they are shifted to Octagonal blender for lubrication then lubricated granules are compressed with the help of compression machine. Then coating of compressed tablets is done with the help of coating machine and hereafter coated tablets are ready for final packing. The reliability block diagram (RBD) of the pharmaceutical plant is shown in Fig. 1.

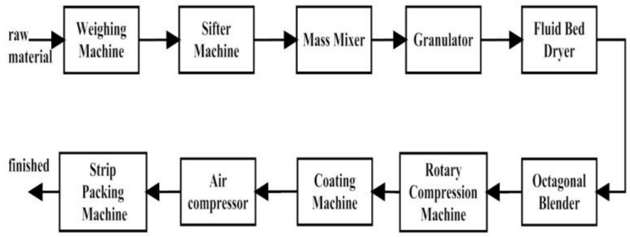

Fig. 1. RBD of the Pharmaceutical Plant.

4.2. Mathematical model of the RRAP

The goal of reliability engineering is to improve the system reliability. The reliability - redundancy optimizations are useful for system designs that are largely assembled and manufactured using off-the-shelf components, and also, have high reliability requirements. A reliability - redundancy optimization problem can be formulated with system reliability as the objective function or in the constraint set. In this work, the reliability - redundancy allocation problem of maximizing the system reliability subject to multiple nonlinear constraints can be stated as a nonlinearly mixed-integer programming model.

Let R_s be the system reliability of the plant, then the RRAP has been formulated for the above system by considering the system reliability as the objectives while the cost, weight and volume of the system as the constraints. Based on these objective and constraints, the nonlinear mathematical model of the system is viewed as

$$\text{Maximize} \quad R_s = \prod_{i=1}^{10}[1-(1-r_i)^{n_i}]$$

$$\text{subject to} \quad g_1(r,n) = \sum_{i=1}^{10} c(r_i)[n_i + \exp(n_i/4)] - C \le 0$$

$$g_2(r,n) = \sum_{i=1}^{10} w_i n_i \exp(n_i/4) - W \le 0$$

$$g_3(r,n) = \sum_{i=1}^{10} v_i n_i^2 - V \le 0$$

$$0.5 \le r_i \le 1-10^{-6} \qquad r_i \in [0,1]$$

$$1 \le n_i \le 5 \qquad n_i \in Z^+$$

where $c(r_i) = \alpha_i(-T/\ln(r_i))^{\beta_i}$ is the cost of the each component with reliability r_i at subsystem i; T is the operating time during which the component must not fail. The parameters β_i and α_i are the physical feature (shaping and scaling factor) of the cost - reliability curve of each component in stage i. The symbol v_i is the volume and w_i is the weight of each components at the stage i. The factor $\exp(n_i/4)$ accounts for the interconnecting hardware. V is the upper limit on the sum of the subsystems' products of volume and weight, C is the upper limit on the cost of the system, and W is the upper limit on the weight of the system. The input parametric values for the above system are given in Table 1.

Table 1. Data Used in the Given Plant.

Stage	$10^5\alpha_i$	β_i	v_i	w_i	V	C	W	T (hrs)
1	0.611360	1.5	4	9				
2	4.032464	1.5	5	7				
3	3.578225	1.5	3	5				
4	3.654303	1.5	2	9				
5	1.163718	1.5	3	9	289	553	483	1000
6	2.966955	1.5	4	10				
7	2.045865	1.5	1	6				
8	2.649522	1.5	1	5				
9	1.982908	1.5	4	8				
10	3.516724	1.5	4	6				

4.3. Computational results

The particles of the swarm for the Pharmaceutical plant uses the variable vectors $'n'$ and $'r'$. During the evolution process, the integer variable n_i is treated as real variables, and in evaluating the objective functions, the real values are transformed to the nearest integer values. The optimization method was implemented in Matlab (MathWorks) and the program was run on a T6400 @ 2GHz Intel Core(TM) 2 Duo processor with 2GB of Random Access Memory (RAM). In order to eliminate stochastic discrepancy, 25 independent runs were made involving 25 different initial trial solutions with population size 100 and maximum number of generation as 500. The other parameters are c_1=c_2=2.05 and inertia weight reduces gradually from w_1=0.9, w_2=0.4 with the number of iterations. The termination criterion has been set either to a maximum number of 500 generations or the order of relative error equal to 10^{-6}, whichever is achieved first. Based on these selected parameters, the optimal result of the PSO scheme for the system is shown in Table 2.

Table 2. Optimal Result for the Plant.

Parameters	Optimal values
$R_s = f(r,n)$	0.956021
(r_1, n_1)	(0.871922, 3)
(r_2, n_2)	(0.827480, 3)
(r_3, n_3)	(0.835569, 3)
(r_4, n_4)	(0.800000, 3)
(r_5, n_5)	(0.865663, 3)
(r_6, n_6)	(0.831345, 3)
(r_7, n_7)	(0.864687, 3)
(r_8, n_8)	(0.800000, 3)
(r_9, n_9)	(0.858897, 3)
(r_{10}, n_{10})	(0.832932, 3)
Slack (g_1)	0.053205
Slack (g_2)	13.025996
Slack (g_3)	10.00000

5. Conclusions

This paper illustrates the application of PSO to the reliability redundancy allocation problem of Pharmaceutical plant (situated in the northern part of India) where we maximize the system reliability with respect to several constraints as cost, weight and volume of the system. The constraints in the problem have been handled with the parameter free penalty method and the resulting problem is solved with one of the meta-heuristic technique named as particle swarm optimization. The decision variable corresponding to the each components of the system are reported which may be targeted so that system analyst/decision maker(s) may analysed their behaviour and improve the performance of the plant. The plant personnel /system experts may use the optimal results to allocate the system reliability and hence enhance the productivity of the system.

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References

1. Kuo, W.; and Prasad, V.R. (2000). An annotated overview of system-reliability optimization. *IEEE Transaction on Reliability*, 49(2), 176-187.
2. Hikita, M.; Nakagawa, Y.; and Harihisa, H. (1992). Reliability optimization of systems by a surrogate constraints algorithm. *IEEE Transactions on Reliability*, 41(3), 473-480.
3. Xu, Z.; Kuo, W.; and Lin, H.-H. (1990). Optimization limits in improving system reliability. *IEEE Transactions on Reliability*, 39(1), 51- 60.
4. Misra, K.B.; and Sharma, U. (1991). An efficient algorithm to solve integer programming problems arising in system reliability design. *IEEE Transactions on Reliability*, 40(1), 81-91.
5. Shi, D. (1987). A new heuristic algorithm for constrained redundancy - optimization in complex systems. *IEEE Transactions on Reliability*, R-36(5), 621-623.
6. Kuo, W.; Prasad, V.R.; Tillman, F. A.; and Hwang, C. (2001). *Optimal reliability design - Fundamentals and Applications*. Cambridge University Press, Cambridge, UK.
7. Tillman, F.A.; Hwang, C.L.; and Kuo, W. (1980). *Optimization of systems reliability*. New York: Marcel Dekker.
8. Yokota, T.; Gen, M.; Li, Y.; and Kim, C.E. (1996). A genetic algorithm for interval nonlinear integer programming problem. *Computers and Industrial Engineering*, 31(3-4), 913-917.
9. Hsieh, Y.-C.; Chen, T.-C.; and Bricker, D.L. (1998). Genetic algorithms for reliability design problems. *Microelectronics Reliability*, 38(10), 1599-1605.
10. Coit, D.W.; and Smith, A.E. (1996). Solving the redundancy allocation problem using a combined neural network/genetic algorithm approach. *Computers and Operations Research*, 23(6), 515-526.

11. Chen, T.-C. (2006). IAs based approach for reliability redundancy allocation problems. *Applied Mathematics and Computation*, 182(2), 1556-1567.
12. Coelho, L.S. (2009). An efficient particle swarm approach for mixed-integer programming in reliability-redundancy optimization applications. *Reliability Engineering and System Safety*, 94(4), 830-837.
13. Garg, D.; Kumar, K.; and Pahuja, G.L. (2010). Redundancy-allocation in pharmaceutical plant. *International Journal of Engineering Science and Technology*, 2(5), 1088-1097.
14. Deb, K. (2000). An efficient constraint handling method for genetic algorithms. *Computer Methods in Applied Mechanics and Engineering*, 186(2-4), 311-338.
15. Kennedy, J.; and Eberhart, R.C. (1995). Particle swarm optimization, in: *IEEE International Conference on Neural Networks*, Vol. IV, Piscataway, NJ, Seoul, Korea, 1942-1948.
16. Shi, Y.; and Eberhart, R.C. (1998). Parameter selection in particle swarm optimization. *Evolutionary programming VII: Lecture Notes in Computer Science*, 1447, 591-600.
17. Clerc, M.; and Kennedy, J.F. (2002). The particle swarm: explosion, stability, and convergence in a multi-dimensional complex space. *IEEE Transactions on Evolutionary Computation*, 6(1), 58-73.