Journal of Engineering Science and Technology
Vol. 7, No. 2 (2012) 219 - 231

HALL CURRENT EFFECT ON THERMOSOLUTAL INSTABILITY IN A VISCOELASTIC FLUID THROUGH POROUS MEDIUM

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Abstract

The thermosolutal instability in a layer of electrically conducting Oldroydian viscoelastic fluid in porous medium is studied to include the effect of Hall current. For the case of stationary convection, the Oldroydian fluid behaves like an ordinary Newtonian fluid. The magnetic field and the stable solute gradient are found to have stabilizing effects whereas Hall currents and medium permeability are found to have destabilizing effects on the system. Graphs have been plotted by giving numerical values to the parameters, to depict the stability characteristics. The sufficient conditions for the non-existence of overstability are also obtained.

Keywords: Thermosolutal instability, Oldroydian viscoelastic fluid, Hall current effect, Porous medium.

1. Introduction

The onset of convection in Newtonian fluids heated from below, under varying assumptions of hydrodynamics and hydromagnetics, has been treated by Chandrasekhar [1]. The effect of Hall currents on the thermal instability of a horizontal layer of conducting fluid has been studied by Gupta [2]. Veronis [3] has investigated the thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The heat and solute being two diffusing components, thermosolutal convection is the general term dealing with such phenomena.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamicists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of

Nomenclatures

a	Dimensionless wave number
g	Acceleration due to gravity, m/s^2
$\vec{g}$	Gravity field, m/s^2
$\vec{H}$	Magnetic field, G
$\vec{h}$	Perturbation in magnetic field, G
k	Wave number band, 1/m
k_x, k_y	Horizontal wave-numbers, 1/m
k_1	Medium permeability, m^2
M	Hall current number
n	Growth rate, 1/s
p	Fluid pressure, Pa
Q	Chandrasekhar number
$\vec{q}$	Velocity, m/s
R	Thermal Rayleigh number
S	Solute Rayleigh number
T	Temperature, K
t	Time, s
x	Wave number, 1/m

Greek Symbols

α	Coefficient of thermal expansion, 1/K
α'	Coefficient of analogous solvent expansion, 1/K
β	Uniform temperature gradient, K/m
β'	Uniform solute gradient, K/m
γ	Perturbation in concentration, Kg
ε	Medium porosity
η	Electrical resistivity, m^2/s
θ	Perturbation in temperature, K
κ	Thermal diffusivity, m^2/s
κ'	Solute diffusivity, m^2/s
λ	Stress relaxation time, s
λ_0	Strain retardation time, s
μ	Dynamic viscosity, kg/ms
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3

crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in a porous medium. A macroscopic equation which describes incompressible flow of a Newtonian fluid of viscosity μ through a macroscopically homogeneous and isotropic porous medium of permeability k_1 is the well known Darcy's equation. The usual viscous term in the equations of fluid motion is replaced by the resistance term $-(\mu/k_1)\,\vec{v}$, where $\vec{v}$ is the filter velocity of the fluid.

Bhatia and Steiner [4] have studied the problem of thermal instability of a viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing effect, in contrast to the stabilizing effect on Newtonian fluid. Bhatia and Steiner [5] have also considered the thermal instability of a Maxwell fluid in hydromagnetics and have found that the magnetic field has stabilizing effect on viscoelastic fluid, just as in case of Newtonian fluid. Sharma [6] studied the thermal instability of a layer of Oldroyd fluid acted on by a uniform rotation and found that the rotation has a destabilizing as well as stabilizing effect under certain conditions, in contrast to Maxwell fluid, where it has destabilizing effect [4]. Experimental demonstration by Toms and Strawbridge [7] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd fluid.

Sharma and Sunil [8] have studied the effect of suspended particles on the thermal instability of an Oldroydian viscoelastic fluid in porous medium. The magnetic field, suspended particles, viscoelasticity, and porous medium effects create oscillatory modes in the system. Sharma and Kumar [9] also studied the thermal instability of an Oldroydian viscoelastic fluid in porous medium. For stationary convection, the medium permeability is found to have a destabilizing effect and the sufficient conditions for non-existence of overstability are obtained. Bhatia and Mathur [10] have examined the Rayleigh-Taylor instability of two superposed viscoelastic Oldroydian fluids in a uniform vertical magnetic field through porous medium. The magnetic field stabilizes the unstable configuration for wave number band $k>k^*$ in which the system is unstable in the absence of the magnetic field. Kumar et al. [11] have examined the instability of the plane interface between two Oldroydian viscoelastic superposed fluids in the presence of uniform rotation and variable magnetic field in porous medium.

The Hall effect is likely to be important in many geophysical situations like Earth's molten core as well as in flows of laboratory plasma. Sherman and Sutton [12] have considered the effect of Hall current on the efficiency of a magneto-fluid dynamic generator. Oberoi and Devanathan [13] have investigated the effects of Hall phenomenon on the propagation of small amplitude waves taking compressibility into account. As the Hall current, solute gradient and viscoelastic effects are likely to be important in geophysical situations; a reconsideration of the thermal convection effects occurring in porous medium including these effects is certainly called for and is the object of the present paper.

2. Formulation of the Problem and Perturbation Equations

Let Γ_{ij}, τ_{ij}, e_{ij}, μ, λ, λ_o ($<\lambda$), p, δ_{ij}, q_i, x_i and d/dt denote respectively the stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the stress relaxation time, the strain retardation time, the isotropic pressure, the Kroneckor delta, the velocity vector, the position vector and the convective derivative. Then the Oldroydian viscoelastic fluid is described by the constitutive relations (1).

Relations of the type (1) were first studied by Oldroyd [14]. Oldroyd also showed that many rheological equations of state; of general validity, reduce to (1) when linearized. λ_0=0 yields the fluid to be Maxwellian whereas $\lambda=\lambda_0$=0 gives the Newtonian viscous fluid.

$$\left.\begin{aligned}&\Gamma_{ij} = -p\delta_{ij} + \tau_{ij}\\&\left(1+\lambda\frac{d}{dt}\right)\tau_{ij} = 2\mu\left(1+\lambda_0\frac{d}{dt}\right)e_{ij}\\&e_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j}+\frac{\partial q_j}{\partial x_i}\right)\end{aligned}\right\} \tag{1}$$

Here we consider an infinite horizontal layer of an Oldroydian viscoelastic fluid of depth d in a porous medium, heated and soluted from below and acted on by gravity force $\vec{g} = (0, 0, -g)$ and magnetic field $\vec{H} = (0, 0, H)$.

The equations of motion, continuity and heat conduction for a viscous, incompressible fluid heated from below are [1]

$$\rho\frac{d\vec{q}}{dt} = \rho\vec{X} - \nabla p + div\ \vec{\tau} \tag{2}$$

$$\nabla.\,\vec{q} = 0 \tag{3}$$

$$\rho\frac{d\vec{q}}{dt}(c_v T) = \frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) \tag{4}$$

where p, ρ, T, $\vec{q}$ and $\vec{X}$ denote, respectively, the fluid pressure, density, temperature, velocity and the external force acting on the fluid. Here k and c_v stand for the thermal conductivity and the specific heat at constant volume. The viscous dissipation term, being very small in magnitude, has not been included in Eq. (4). Since external forces are of non-electromagnetic origin (gravity) and of electromagnetic origin (Lorentz force per unit volume), equations of motion (2) may be rewritten as

$$\rho\frac{d\vec{q}}{dt} = -\nabla p + \rho\vec{g} + \frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H} + div\ \tau\,. \tag{5}$$

Using the constitutive relations (1) for an Oldroydian viscoelastic fluid and also using the fact that when fluid flows through a porous medium, the gross effect is represented by Darcy's law, the equations of motion and continuity for an Oldroydian viscoelastic fluid through porous medium become

$$\frac{\rho}{\varepsilon}(1+\lambda\frac{d}{dt})\frac{d\vec{v}}{dt} = \left(1+\lambda\frac{d}{dt}\right)\left[-\nabla p + \rho\vec{g} + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{H}\right)\times\vec{H}\right] - \frac{\rho\nu}{k_1}\left(1+\lambda_0\frac{d}{dt}\right)\vec{v} \tag{6}$$

$$\nabla.\vec{v} = 0 \tag{7}$$

where $\vec{v}$ is the filter velocity, ε is medium porosity and k_1 is the medium permeability. $\nu(=\mu/\rho)$ and μ_e stand for kinematic viscosity and magnetic permeability respectively. The fluid velocity $\vec{q}$ and the Darcian (filter) velocity $\vec{v}$ are connected by the relation $q = \frac{\vec{v}}{\varepsilon}$.

When the fluid flows through a porous medium, the equation of heat conduction is [15]

$$\left[\rho c_f \phi + \rho_s c_s (1-\phi)\right]\frac{\partial T}{\partial t} + \rho c_f (\vec{v}.\nabla) = k\nabla^2 T \tag{8}$$

An analogous solute concentration equation is

$$\left[\rho c'_f \phi + \rho_s c'_s (1-\phi)\right]\frac{\partial c}{\partial t} + \rho c'_f (\vec{v}.\nabla)c = k'\nabla^2 c \tag{9}$$

Using generalized Ohm's law to take account of Hall current

$$\vec{j} = \sigma\left(\vec{E} + \vec{v}\times\vec{B}\right) - \frac{c}{Ne}\vec{j}\times\vec{H} \tag{10}$$

and eliminating $\vec{E}$, $\vec{j}$, etc., the Oldroydian equations in terms of magnetic field become

$$\frac{\partial \vec{H}}{\partial t} = \frac{1}{\varepsilon}\nabla\times\left(\vec{v}\times\vec{H}\right) + \eta\nabla^2\vec{H} - \frac{c}{4\pi Ne}\nabla\times\left[\left(\nabla\times\vec{H}\right)\times\vec{H}\right] \tag{11}$$

and

$$\nabla.\vec{H} = 0 \tag{12}$$

Initially $\vec{v} = (0,0,0)$, $\rho = \rho(z)$, $p = p(z)$, $T = T(z)$, $c = c\,(z)$ and $\vec{H} = (0,0,H)$. Let $\delta\rho$, δp, θ, γ, $\vec{h}(h_x, h_y h_z)$ and $\vec{v}(u,v,w)$ denote respectively the perturbations in density ρ, pressure p, temperature T, solute concentration c, magnetic field $\vec{H}(0,0,H)$ and filter velocity (zero initially). Here k, k', α, α', $\beta\left(=\left|dt/dz\right|\right)$ and $\beta'\left(=\left|dc/dz\right|\right)$ stand for thermal diffusivity, solute diffusivity, thermal coefficient of expansion, an analogous solvent expansion, uniform temperature gradient and uniform solute gradient, respectively. Then, the linearized thermosolutal hydromagnetic perturbed equations of flow through porous medium (6)-(9), (11) and (12) following the Boussinesq approximation, become

$$\frac{\rho_0}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \vec{v}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \vec{g}\delta\rho + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H}\right] - \frac{\rho_0 \nu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\vec{v} \tag{13}$$

$$\nabla.\vec{v} = 0 \tag{14}$$

$$E\frac{\partial \theta}{\partial t} = \beta\, w + k\nabla^2\theta \tag{15}$$

$$E'\frac{\partial \gamma}{\partial t} = \beta' w + k'\nabla^2\gamma \tag{16}$$

$$\varepsilon\frac{\partial \vec{h}}{\partial t} = \nabla\times\left(\vec{v}\times\vec{H}\right) + \varepsilon\,\eta\,\nabla^2\,\vec{h} - \frac{c\varepsilon}{4\pi\,Ne}\nabla\times\left[\left(\nabla\times\vec{h}\right)\times\vec{H}\right] \tag{17}$$

$$\nabla.\vec{h} = 0 \tag{18}$$

where E = $\varepsilon + (1-\varepsilon)\left[\rho_s c_s/\rho_0 c_f\right]$ and ρ_0, c_f ; ρ_s, c_s stand for density and heat capacity of fluid and solid matrix, respectively. E' is an analogous solute parameter. Here c, η, N and e stand for speed of light, electrical resistivity, electron number density and charge of an electron, respectively. The equation of state is

$$\rho = \rho_0 \left[1 - \alpha(T - T_0) + \alpha'(c - c_0)\right] \tag{19}$$

where the suffix zero refers to values at the reference level $z = 0$ e.g. ρ_0, T_0 and c_0 stand for density, temperature and solute concentration at the lower boundary $z = 0$.

The change in density $\delta\rho$ caused by the perturbations θ, γ in temperature and solute concentration, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma) \tag{20}$$

Equations (13)-(18) and (20) give

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 w - g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta-\alpha'\gamma) - \frac{\mu_e H}{4\pi\rho_0}\nabla^2\frac{\partial h_z}{\partial z}\right] = -\frac{\nu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\nabla^2 w \tag{21}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{1}{\varepsilon}\frac{\partial\zeta}{\partial t} - \frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}\right] = -\frac{\nu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\zeta \tag{22}$$

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z} - \frac{cH\varepsilon}{4\pi Ne}\frac{\partial\xi}{\partial z} \tag{23}$$

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)\xi = H\frac{\partial\zeta}{\partial z} + \frac{cH\varepsilon}{4\pi Ne}\nabla^2\frac{\partial h_z}{\partial z} \tag{24}$$

$$\left(E\frac{\partial}{\partial t} - \chi\nabla^2\right)\theta = \beta\, w \tag{25}$$

$$\left(E'\frac{\partial}{\partial t} - \chi'\nabla^2\right)\gamma = \beta' w \tag{26}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ denote the z-components of vorticity and current density, respectively .

The fluid is confined between the planes $z = 0$ and $z = d$ maintained at constant temperatures and solute concentrations. Since no perturbations in temperature and concentration are allowed and since normal component of the velocity must vanish on these surfaces, we have

$$w = 0,\ \theta = 0 \text{ and } \gamma = 0 \text{ at } z = 0 \text{ and } z = d \tag{27}$$

Here we consider the case of two free boundaries, and the medium adjoining the fluid is electrically non-conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres [16] and in certain geophysical situations where it is most appropriate, but it allows for an analytical solution. The condition of vanishing of tangential stresses at free surfaces implies

$$\frac{\partial^2 w}{\partial z^2} = 0 \text{ and } \frac{\partial\zeta}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = d \tag{28}$$

Moreover, $\xi = \left(\nabla\times\vec{h}\right)_z = 0$ and $\vec{h}$ is continuous at $z = 0$ and $z = d$ (29)

3. The Dispersion Relation

Here we assume the perturbations to be of the form

$$[w,\theta,\gamma,h_z,\zeta,\xi]=[W(z),\Theta(z),\Gamma(z),K(z),X(z)]\exp(\iota k_x x+\iota k_y y+nt) \tag{30}$$

where k_x k_y are horizontal wave numbers, $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number and n is a complex constant. Using the dimensionless variables

$$a=kd,\ \sigma=\frac{nd^2}{\nu},\ p_1=\frac{\nu}{\chi},\ p_2=\frac{\nu}{\eta},\ q=\frac{\nu}{\chi'},\ P_l=\frac{k_1}{d^2},\ x'=\frac{x}{d},\ y'=\frac{y}{d},\ z'=\frac{z}{d},$$

$$F=\frac{\lambda\nu}{d^2},\ F_0=\frac{\lambda_0\nu}{d^2}\ and\ D=d/dz,$$

and removing the dashes for convenience , Eqs. (21)-(26), with the help of (30) become

$$\left[\frac{\sigma}{\varepsilon}(1+F\sigma)+\frac{1}{P_l}(1+F_0\sigma)\right](D^2-a^2)W+(1+F\sigma)\frac{gd^2a^2}{\nu}(\alpha\Theta-\alpha'\gamma)-$$
$$(1+F\sigma)\frac{\mu_e Hd}{4\pi\rho_0\nu}(D^2-a^2)DK=0 \tag{31}$$

$$\left[\frac{\sigma}{\varepsilon}(1+F\sigma)+\frac{1}{P_l}(1+F_0\sigma)\right]Z=(1+F\sigma)\frac{\mu_e Hd}{4\pi\rho_0\nu}DX \tag{32}$$

$$\left[D^2-a^2-p_2\sigma_1\right]K=-\left(\frac{Hd}{\eta\varepsilon}\right)DW+\frac{cHd}{4\pi Ne\eta}DX \tag{33}$$

$$\left[D^2-a^2-p_2\sigma\right]X=-\left(\frac{Hd}{\eta\varepsilon}\right)DZ-\frac{cH}{4\pi Ne\eta d}(D^2-a^2)DK \tag{34}$$

$$\left[D^2-a^2-Ep_1\sigma\right]\Theta=-\left(\frac{\beta d^2}{\chi}\right)W \tag{35}$$

$$\left[D^2-a^2-E'q\sigma\right]\Gamma=-\left(\frac{\beta' d^2}{\chi'}\right)W \tag{36}$$

The boundary conditions (27)-(29), using expression (30), become

$W=D^2W=0$, $\Theta=0$, $\Gamma=0$, $DZ=0$, $X=0$, and h_x, h_y, h_z are continuous at $z=0,\ 1$ (37)

Eliminating Θ, Z, Γ, X and K between Eqs. (31)-(36), we obtain

$$\frac{\left\{\frac{\sigma}{\varepsilon}(1+F\sigma)+\frac{1}{P_l}(1+F_0\sigma)\right\}^2}{(1+F\sigma)}\left[(D^2-a^2)(D^2-a^2-Ep_1\sigma)(D^2-a^2-E'q\sigma)(D^2-a^2-p_2\sigma)^2\right]W$$

$$+\frac{Q}{\varepsilon}\left[\left\{(D^2-a^2-Ep_1\sigma)(D^2-a^2-E'q\sigma)(D^2-a^2)\right\}\left\{2\left(\frac{\sigma}{\varepsilon}[1+F\sigma]+\frac{1}{P_l}[1+F_0\sigma]\right)\right.\right.$$

$$\left(D^2 - a^2 - p_2\sigma\right) + \frac{Q}{\varepsilon}\left(1 + F\sigma\right)D^2 \Big\}\Big] D^2W + M \frac{\left(\frac{\sigma}{\varepsilon}\left[1 + F\sigma\right] + \frac{1}{P_l}\left[1 + F_0\sigma\right]\right)^2}{\left(1 + F\sigma\right)} \Big[\left(D^2 - a^2\right)^2$$

$$\left(D^2 - a^2 - Ep_1\sigma\right)\left(D^2 - a^2 - E'q\sigma\right)\Big] D^2W - \Big[\Big\{Ra^2\left(D^2 - a^2 - E'q\sigma\right) - Sa^2\left(D^2 - a^2 - Ep_1\sigma\right)\Big\}$$

$$\left\{\left(\frac{\sigma}{\varepsilon}\left[1 + F\sigma\right] + \frac{1}{P_l}\left[1 + F_0\sigma\right]\right)\left(D^2 - a^2 - p_2\sigma\right)^2 + \frac{Q}{\varepsilon}\left(1 + F\sigma\right)\left(D^2 - a^2 - p_2\sigma\right)D^2\right.$$

$$\left. + M\left(\frac{\sigma}{\varepsilon}\left[1 + F\sigma\right] + \frac{1}{P_l}\left[1 + F_0\sigma\right]\right)\left(D^2 - a^2\right)D^2\right\}\Bigg] W = 0 \tag{38}$$

Here,

R = $\dfrac{g\alpha\beta d^4}{\nu\chi}$, is the thermal Rayleigh number,

S = $\dfrac{g\alpha'\beta' d^4}{\nu\chi'}$, is the analogous solute Rayleigh number ,

Q = $\dfrac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$, is the Chandrasekhar number, and

$M = \left(\dfrac{cH}{4\pi Ne\eta}\right)^2$, is a non-dimensional number accounting for the Hall currents.

Using the boundary conditions (37), it can be shown with the help of Eqs. (31)-(36) that all the even order derivatives of W vanish at the boundaries, and hence the proper solution of (38) characterizing the lowest mode is

$$W = W_0 \sin \pi z \quad , \tag{39}$$

where W_0 is constant. Substituting (39) in Eq. (38) and letting

$a^2 = \pi^2 x$, $R_1 = \dfrac{R}{\pi^4}$, $S_1 = \dfrac{S}{\pi^4}$, $Q_1 = \dfrac{Q}{\pi^2}$, $\iota\sigma_1 = \dfrac{\sigma}{\pi^2}$ and $P = \pi^2 P_l$ we obtain the dispersion relation

$$R_1 x = \left[\frac{(1 + x)(1 + x + \iota Ep_1\sigma_1)}{\left(1 + \iota\pi^2 F\sigma_1\right)}\left(\frac{\iota\sigma_1}{\varepsilon}\left[1 + \iota\pi^2 F\sigma_1\right] + \frac{1}{P}\left[1 + iF_0\pi^2\sigma_1\right]\right)^2 \left\{(1 + x + ip_2\sigma_1)^2 + \right.\right.$$

$$M(1 + x)\Big\} + \frac{Q_1}{\varepsilon}(1 + x)(1 + x + iEp_1\sigma_1)\Big\{2(1 + x + ip_2\sigma_1)\left(\frac{i\sigma_1}{\varepsilon}\left[1 + i\pi^2 F\sigma_1\right] + \frac{1}{P}\left[1 + i\pi^2 F_0\sigma_1\right]\right)$$

$$+ \frac{Q_1}{\varepsilon}\left(1 + i\pi^2 F\sigma_1\right)\Big\}\Big]\left[\left(\frac{i\sigma_1}{\varepsilon}\left[1 + i\pi^2 F\sigma_1\right] + \frac{1}{P}\left[1 + i\pi^2 F_0\sigma_1\right]\right)\Big\{(1 + x + ip_2\sigma_1)^2 + M(1 + x)\Big\}\right.$$

$$\left. + \frac{Q_1}{\varepsilon}\left(1 + i\pi^2 F\sigma_1\right)\left(1 + x + ip_2\sigma_1\right) + S_1 x \frac{\left(1 + x + iEp_1\sigma_1\right)}{\left(1 + x + iE'q\sigma_1\right)}\right]^{-1} \tag{40}$$

4. The Stationary Convection

For stationary convection, σ = 0 and the dispersion relation (40) reduces to

$$R_1=\left(\frac{1+x}{x}\right)\frac{\left(\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right)^2+\frac{M(1+x)}{P^2}}{\frac{1+x}{P}+\frac{Q_1}{\varepsilon}+\frac{M}{P}}+S_1 \tag{41}$$

and the Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid. In order to investigate the effects of Hall current, stable solute gradient and medium permeability, we examine the behaviour of dR_1/dM, dR_1/dS_1 and dR_1/dP, analytically. Equation (41) yields

$$\frac{dR_1}{dM}=-\left(\frac{1+x}{x}\right)\frac{Q_1\left(\frac{1+x}{P}+\frac{Q_1}{\varepsilon}\right)}{\left(\frac{1+x+M}{P}+\frac{Q_1}{\varepsilon}\right)^2} \tag{42}$$

which is negative. The Hall current, therefore, has a destabilizing effect on the thermosolutal convection in porous medium.

It is evident from Eq. (41), that

$$\frac{dR_1}{dS_1}=+1 \tag{43}$$

implying thereby that stable solute gradient has a stabilizing effect on the thermosolutal convection in porous medium .

Equation (41) also yields

$$\frac{dR_1}{dP}=-\frac{(1+x)}{xP^2}\frac{\left(\frac{1+x}{P^2}\right)(1+x+M)^2+\frac{2Q_1(1+x)}{\varepsilon P}+\frac{Q_1^{\,2}}{\varepsilon^2}(1+x-M)}{\left(\frac{1+x+M}{P}+\frac{Q_1}{\varepsilon}\right)^2} \tag{44}$$

which is negative if $1+x>M$. The condition $1+x>M$ is met for all wave numbers and the Hall current parameter $M<<1$. The medium permeability therefore has a destabilizing effect on thermosolutal convection in porous medium.

We now examine the dispersion relation (41) numerically. We have plotted the Rayleigh number R_1 versus wave number x for various values of M, S_1, and P and for fixed value of dimensionless parameter $\varepsilon = 0.5$.

In Fig. 1, R_1 is plotted against wave number x for P=50, Q_1=10, ε=0.5, S_1=10 and M=10, 20, 30. The Rayleigh Number decreases with the increase in the Hall current parameter showing its destabilizing effect on the system.

In Fig. 2, R_1 is plotted against wave number x for P=50, Q_1=10, ε=0.5, M=10, and S_1=10, 20, 30. It is clear that the stable solute gradient postpones the onset of convection as the Rayleigh number increase with the increase in stable solute gradient parameter.

Figure 3 represents the plot of R_1 versus x for Q_1=10, M=0.5, ε=0.5, S_1=10, and P=10, 20, 30. As the value of the medium permeability parameter increases, the corresponding value of the Rayleigh number decreases, showing its destabilizing effect on the system.

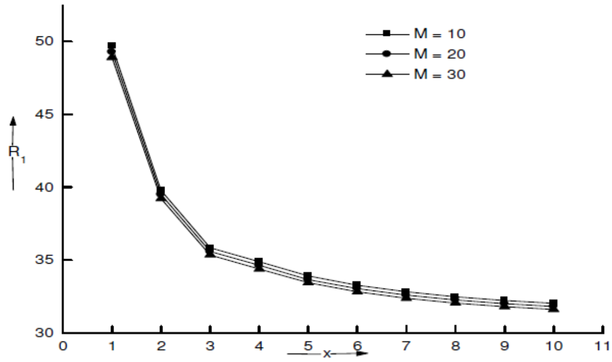

Fig. 1. Variation of Rayleigh Number, R_1, with Wave-Number, x, for P = 50, Q_1 = 10, ε = 0.5, S_1 = 10 and M = 10, 20, 30.

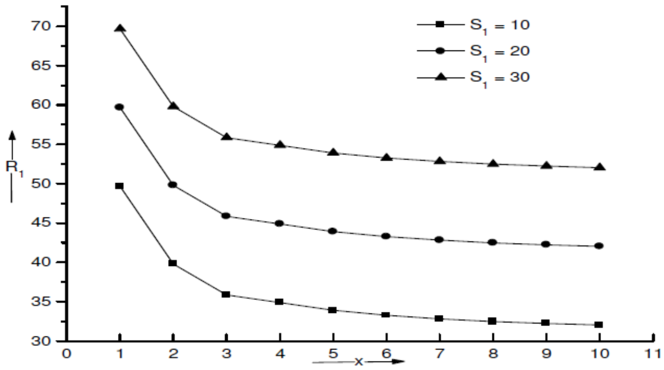

Fig. 2. Variation of Rayleigh Number, R_1, with Wave-Number, x, for P = 50, Q_1 = 10, ε = 0.5, M = 10, and S_1 = 10, 20, 30.

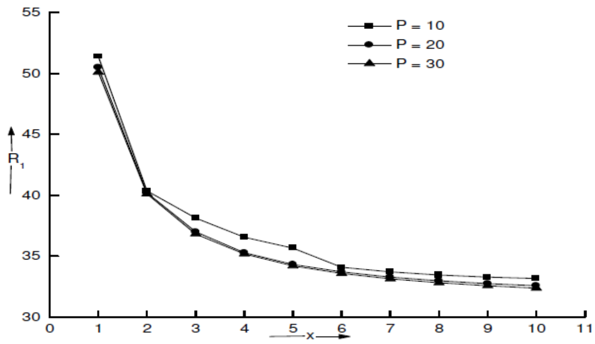

Fig. 3. Variation of Rayleigh Number, R_1, with Wave-Number, x, for Q_1 = 10, M = 0.5, ε = 0.5, S_1 = 10, and P = 10, 20, 30.

Thus, the stable solute gradient postpones the onset of convection while Hall current and medium permeability hasten the onset of convection.

5. The Overstable Case

Here we discuss the possibility as to whether instability may occur as overstability. Equating real and imaginary parts of Eq. (40) and eliminating R_1 between them, we obtain

$$A_6 c_1^6 + A_5 c_1^5 + A_4 c_1^4 + A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0 \tag{45}$$

where $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_6 = -\frac{\pi^6 F^3 p_2^4 E' q b}{\varepsilon^2}\left[EE'qp_1\left(\frac{1}{\varepsilon} - \frac{\pi^2 F_0}{P}\right) + \frac{\pi^2 F b}{\varepsilon}\left(Ep_1 - E'q\right)\right] \tag{46}$$

$$A_0 = \frac{1}{P^2}\left[\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right) + \frac{\pi^2 F_0}{P}\right]b^8 + \left[\frac{2}{P}\left(\frac{1}{\varepsilon} - - \frac{\pi^2 F}{P}\right)\left(M + \frac{Q_1}{\varepsilon}\right) + \frac{2\pi^2 F_0}{P^2}\left(\frac{M}{P} + \frac{Q_1}{\varepsilon}\right) + \right.$$
$$\left.\frac{2Q_1\pi^2 F_0}{P^2}\left(\frac{1}{\varepsilon} + \frac{M}{P}\right) + \frac{Ep_1}{P^3}\right]b^7 + \left[\left\{\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right) + \frac{\pi^2 F_0}{P}\right\}\left\{\frac{S_1(b-1)}{P} + \frac{M^2}{P^2} + \frac{Q_1^2}{\varepsilon^2} + \frac{2Q_1 M}{\varepsilon P}\right\} + \right.$$
$$\left.\frac{Ep_1}{P^2}\left(\frac{3Q_1}{\varepsilon} + \frac{M}{P}\right)\right]b^6 + \left[\frac{2Q_1^2}{\varepsilon^2 P}\left(Ep_1 - p_2\right) + \frac{2Q_1 M}{\varepsilon P}\left(Ep_1 - E'q\right) + \frac{Q_1^2\pi^2 M}{\varepsilon^2 P}\left(F - F_0\right) + \right.$$
$$\frac{Q_1^2}{\varepsilon^2}\left(\frac{Ep_1}{P} - \frac{M}{\varepsilon}\right) + \frac{Ep_1 M}{P^3} + \frac{Q_1 M p_2}{\varepsilon P^2} + \frac{2Q_1 E'q}{\varepsilon\ P^2} +$$
$$\left.\frac{S_1(b-1)}{P}\left\{\frac{2\pi^2 F_0 M}{P} + 2M\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right) + \frac{2p_2}{P} + \frac{2Q_1}{\varepsilon^2} + \frac{2Q_1\pi^2 F_0}{\varepsilon} + \frac{E'q}{P}\right\}\right]b^5 +$$
$$\left[\left\{\frac{Q_1^2\pi^2 F}{\varepsilon^2} + \frac{2Q_1}{\varepsilon P^2}\left(p_2 + E'q\right) + \frac{M}{P^2}\left(Ep_1 - E'q\right)\right\}S_1(b-1) + \frac{Q_1^3}{\varepsilon^3}\left(Ep_1 - p_2\right) + \right.$$
$$\left.\frac{Ep_1 M^2}{P^3} + \frac{Q_1^2 Ep_1 M}{\varepsilon^2 P} + \frac{Q_1 Ep_1 M}{\varepsilon P^2}\right]b^4 + \left[\left\{\frac{Ep_1 M}{P^2} + \frac{Q_1^2 E'q}{\varepsilon^2}\right\}S_1(b-1)\right]b^3 = 0 \tag{47}$$

The six values of c_1, σ_1 being real are positive. The product of the roots $(= A_0 / A_6)$ is positive. A_6 is negative, if

$$\frac{1}{\varepsilon} > \frac{F_0\pi^2}{p}\ and\ Ep_1 > E'q \tag{48}$$

and A_0 is positive , if

$$\frac{1}{\varepsilon} > \frac{F\pi^2}{P}, Ep_1 > p_2, Ep_1 > E'q, F > F_0\ and\ \frac{Ep_1}{P} > \frac{M}{\varepsilon} \tag{49}$$

The inequalities (48) and (49) imply that the sufficient conditions for non-existence of overstability are

$$Ep_1 > E'q,\ \frac{1}{\varepsilon} > \frac{F\pi^2}{P},\ \frac{1}{\varepsilon} > \frac{F_0\pi^2}{P},\ F > F_0\ \text{, and}$$

$$Ep_1 > \max\left(p_2, \frac{MP}{\varepsilon}\right), \text{ i.e.,}$$

$$\frac{E\nu}{\chi} > \max\left[\frac{\nu}{\eta}, \left(\frac{cH}{4Ne}\right)^2 \frac{k_1}{d^2\varepsilon}\right], \text{ and}$$

$$E'\chi < E\chi', \lambda < \frac{k_1}{\varepsilon\nu}, \lambda_0 < \frac{k_1}{\varepsilon\nu}, F > F_0.$$

But $F > F_0$, *as* $\lambda > \lambda_0$, therefore, the sufficient conditions for non-existence of overstability becomes

$$\frac{E\nu}{\chi} > \max\left[\frac{\nu}{\eta}, \left(\frac{cH}{4Ne}\right)^2 \frac{k_1}{d^2\varepsilon}\right], \text{ and } E'\chi < E\chi', \lambda < \frac{k_1}{\varepsilon\nu}.$$

These are, therefore, the sufficient conditions for the non–existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

6. Conclusions

The effect of Hall current on thermosolutal instability in a layer of electrically conducting Oldroydian viscoelastic fluid in porous medium is considered in the present paper. The investigation of thermosolutal instability is motivated by its interesting complexities as a double diffusion phenomena as well as its direct relevance to geophysics and astrophysics. The main conclusions from the analysis of this paper are as follows:

- For the case of stationary convection, the Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid.
- It is observed for the case of stationary convection that the stable solute gradient has stabilizing effects whereas Hall currents and medium permeability have destabilizing effects on the system.
- It is also observed from the Figs. 1-3 that the stable solute gradient postpones the onset of convection while Hall current and medium permeability hasten the onset of convection.
- The conditions $E'\chi < E\chi'$, $\lambda < \frac{k_1}{\varepsilon\nu}$ are the sufficient conditions for the non-existence of overstability, the violation of which does not necessary imply the occurrence of overstability.

Acknowledgment

The authors wish to express their sincere thanks to the learned referees of the journal, whose critical and helpful comments have significantly improved the article in its present form.

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