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NEURAL STABLE ADAPTIVE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS WITHOUT USE OF A SUPERVISORY TERM IN THE CONTROL LAW

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Abstract

In this paper, a direct adaptive control scheme for a class of nonlinear systems is proposed. The architecture employs a Gaussian radial basis function (RBF) network to construct an adaptive controller. The parameters of the adaptive controller are adapted and changed according to a law derived using Lyapunov stability theory. The centres of the RBF network are adapted on line using the *k*-means algorithm. Asymptotic Lyapunov stability is established without the use of a supervisory (compensatory) term in the control law and with the tracking errors converging to a neighbourhood of the origin. Finally, a simulation is provided to explore the feasibility of the proposed neuronal controller design method.

Keywords: Feedback linearization, Adaptive control, *k*-means algorithm, Lyapunov stability, Radial basis function network.

1. Introduction

It is well known that neural networks (NN) are massively parallel computational models inspired by the structure of the human brain and are capable of learning highly complex and nonlinear mapping. It has been proven that artificial neural networks can approximate any nonlinear functions to any desired degree [1-3]. They are thought to be potentially powerful tools for nonlinear as well as linear control systems. There has consequently been considerable research on the development and application of neural networks over the last decade [3-5]. In control engineering Multilayered Perceptrons, MLP, and radial basis functions, RBF, are the most widely used neural networks. The first applications of NN in control did not include rigorous analysis of the stability [6, 7]. However, in control

Nomenclatures

A	Cross section area of the cylinder, cm^2
A_c	A matrix of size (n×n)
b	Control gain
b_c	Vector containing the control gain b
c_i	Centres of the basis function number i
$d(t)$	External disturbance
$de=\dot{e}$	Derivative of the error
e	Tracking error
$\underline{e}$	Error vector
$f(\underline{x})$	Nonlinear function
g	Universal gravitation, cm/s^2
int	Integer part
K	Constant gain vector
K^T	Transpose of vector K
k_i	Integral action of the regulator
k_p	Proportional action of the regulator
Lm_2	Reference signal, cm
L_2	Liquid level in Tank2, cm
MLP	Multilayered perceptrons
min	Minimum
NN	Neural network
n	Degree of the system
nr	Number of basis functions
P	Solution (n×n matrix) of the Lyapunov equation
PI	Proportional Integral regulator
P_n	Last column of $P(n\times1)$
Q	Positive diagonal symmetric (n×n) matrix
Q_1	Flow rate for Tank1, cm^3/s
Q_2	Flow rate for Tank2, cm^3/s
R^n	Real value set
r	Euclidean distance, cm
S	Cross section, cm^2
SISO	Single input single output
s	Laplace operator
sup	Supreme
$T1, T2, T3$	Tank1, Tank2, Tank3
t	Time, s
u^*	Optimal control law
$u(t)$	Input of the system (Control input)
$u(\underline{x},\underline{\theta})$	Approximation of the control input u
$u(\underline{x},\dot{\underline{\theta}})$	Approximation of the ideal control input u^*
V	Lyapunov function
$\dot{V}$	Derivative of the Lyapunov function
w	Minimum approximation error
w_1	The quantity, $-(bw + d)$
$x(t)$	State of the system

$\underline{x}(t)$	State vector
$y(t)$	Output of the system
$y_m(t)$	Reference signal
Greek Symbols	
δ	Positive constant
$\delta(t)$	gain belonging to the interval [0 1]
φ	Error between $\underline{\theta}$ and $\underline{\dot{\theta}}$
$\dot{\varphi}$	Derivative of φ
γ	Positive constant
λ_{Qmin}	minimum eigenvalue of Q
μ	Positive constant
μ_1	Outflow coefficient $\mu_1 = \mu_2 = \mu_3$
ν	Artificial (equivalent) input of the system
θ	Connection weight of the RBF network
$\underline{\theta}$	Vector of connection weights
θ^*	Ideal parameter (ideal connection weight)
$\underline{\theta}^*$	Ideal parameter vector
σ	Width of the Gaussian function
$\xi(\underline{x})$	Output of the basis function
Ω_e	Compact set
$\psi(r)$	Radial basis function

systems, it is important to have design methodologies that provide proofs of stability for the system. Several neural network adaptive control algorithms based on Lyapunov's stability theory have been proposed [8-10].

The advantage is that these adaptive laws guarantee the stability of the closed loop systems. Generally this is done in an adaptive control framework. Most works in adaptive control are based on the assumption of linear or simplified form of nonlinear mathematical models of systems to be controlled. In fact, adaptive control of linear systems and certain special classes of nonlinear systems has been well developed from the late 1970's to the 1990's. While adaptive control of general nonlinear systems still presents a challenge to control community. Nevertheless, mathematical models might not be available for many complex systems in practice, and the adaptive control problem of these systems is far from being satisfactorily resolved [11, 12]. Most of the adaptive controllers involve certain types of function approximators in their learning mechanism. Also, fuzzy logic systems are widely used for this purpose. Based on this, a great number of works on adaptive fuzzy control have been proposed [13-16, 18, 19], where the general approach is usually based on the feedback linearization technique as mentioned by Slotine [17]. The used fuzzy inference system is introduced for approximating part or all the components of the control law. In most cases however [13-16, 18], a complementary term, called a supervisory or a compensatory controller, is added to the output of the fuzzy inference system as a part of the control law in order to guarantee the global stability using the Lyapunov theory. The supervisory term plays the role of a robust controller. When the system is operating within the prescribed range, the supervisory controller is turned off. It is activated only if the system tends to go beyond the prescribed tolerance.

This work was built on the initial proposal [14-16] to construct an RBF direct adaptive control SISO nonlinear system instead of the fuzzy control system used in these papers but without use of a compensatory or a supervisory control term as done in these papers and our system contains also an external disturbance. Usually, in RBF based adaptive control, the online adaptation is concerned only with the connections weights.

The centres of the basis functions are fixed offline [9, 20]. In particular, the adaptation of both the centres and the connections weights is considered in [21] and in some other works. The main advantage of the RBF network is that their output depends linearly on the connections weights and thus the training becomes a linear optimisation problem. In this work, we propose to online adjust both the centres of the basis functions and the connections weights. The *k*-means algorithm [22] will be used on line for the centres adjustment.

The connections weights are adapted and changed according to a law derived using Lyapunov stability theory. Asymptotic Lyapunov stability of the resulting closed loop system is established without the use of a compensatory or a supervisory term in the control law and with the tracking errors converging to a neighbourhood of the origin. This work is organised as follows: in section 2, the problem formulation is introduced, in section 3, the stability analysis is developed and the adaptive laws are derived, in section 4, the direct adaptive RBF controller is used in simulation to control some stable and unstable nonlinear systems to show the effectiveness of the proposed method.

2. Problem Formulation

Consider a non linear system that can be transformed into the following Slotine form [17]

$$x^{(n)} = f(x, \dot{x}, \ldots x^{(n-1)}) + b.u(t) + d(t), \qquad y(t) = x(t) \tag{1}$$

where $u(t) \in R$ and $y(t) \in R$ are the input and output of the system respectively, *f* is a unknown non linear function, *b* is a positive unknown bounded constant and *d(t)* is an external bounded disturbance. Assume that the state vector $\underline{x} = (x_1, x_2, \ldots, x_n)^T = (x, \dot{x}, \ldots x^{(n-1)})^T \in R^n$ is available for measurement. The control objective is to force the output *y* to follow a given bounded reference signal $y_m(t)$, under the constraints that all signals involved must be bounded. More specifically, determine a feedback control estimation $u(\underline{x}, \underline{\theta})$ of *u*, and all this is based on an RBF network. Determine also an adaptive law using Lyapunov theory for adjusting the parameters vectors $\underline{\theta}$ such that the following conditions are met

- The closed-loop system must be globally stable in the sense that all variables must be uniformly bounded.
- The tracking error $e = y - y_m$ should be as small as possible under the constraints in Eq. (1).

Define now the error vector as

$$\underline{e} = (e, \dot{e}, \ldots, e^{(n-1)})^T \in R^n \tag{2}$$

Step1: Choose u to cancel the nonlinearities in a nonlinear system so that the closed-loop dynamics is in a linear form, and guarantee tracking convergence based on a feedback linearization theory [17]. If the function f is known and the external disturbance d does not exist, and assuming b to be positive constant, then, from Eq. (1), the optimal control law is

$$u^* = \frac{1}{b}.(v - f(\underline{x})) \tag{3}$$

Step2: Choose the artificial input v (an equivalent input) as a simple linear pole-placement controller $v = y_m^{(n)} - K^T \underline{e}$ that provides guarantee about the stability of the overall system. The vector K is defined below.

Substituting Eq. (3) into Eq. (1), in order to cancel the nonlinearities and obtain the simple input-state relation

$$x^{(n)} = v \tag{4}$$

the vector K defined as

$$K = (k_0, k_1, ..., k_{n-1})^T \in R^n \tag{5}$$

is chosen so that the polynomial

$$s^n + k_{n-1}.s^{n-1} + ... + k_0 = 0 \tag{6}$$

has all its roots strictly in the left-half complex plane. Then the optimal control law is

$$u^* = \frac{1}{b}.(y_m^{(n)} - K^T.\underline{e} - f(\underline{x})) \tag{7}$$

based on $e = y - y_m$ then

$$e^{(n)} = y^{(n)} - y_m^{(n)} \tag{8}$$

Substituting Eq. (7) into Eq. (1), using Eq. (8) and based on $y = x$, see Eq. (1), yields

$$e^{(n)} + k_{n-1}.e^{(n-1)} + ... + k_0.e = 0 \tag{9}$$

This implies that $\lim_{t\to\infty} e(t) \to 0$ (exponentially stable dynamics), which is the main objective of control. Since f is unknown and the external disturbance d exists, the optimal control u^* of Eq. (7) can not be implemented. Our purpose is to design an RBF network with output $u(\underline{x}, \underline{\theta})$ to approximate this optimal control law.

3. The Direct Adaptive RBF Controller

3.1. The RBF network

The RBF network can be considered as a two-layer network with only one hidden layer. The output depends linearly on the weights, then, the training is simply a linear optimization problem [23]. More explicitly, the RBF network performs the transformation

$$f_r: R^n \longrightarrow R\text{, with : } u(\underline{x},\underline{\theta}) = \sum_{i=1}^{nr} \xi_i \theta_i = \underline{\theta}^T \xi(\underline{x})\text{ , } \xi_i = \psi(\|\underline{x} - c_i\|_2) \qquad (10)$$

$\underline{x}$ is the input vector, ψ is a non linear function called radial basis function, $\underline{\theta}$ are connections weights to be adapted (parameters) between the hidden layer and the output layer, c_i are centres of basis functions and nr is the number of basis functions. The most used basis function is the Gaussian function. However, it is shown in [21] that Gaussian basis function does have the best approximation property. This is the principle reason being the selection of Gaussian functions to characterize the membership function in this work.

$$\psi(r) = \exp(\frac{-r^2}{2.\sigma^2}) \qquad (11)$$

with $r = \|\underline{x} - c_i\|_2$, c_i is the vector of centres of the Gaussian function $\psi(r)$, σ is an associated constant to the function $\psi(r)$ and represents the width of the Gaussian function.

3.2. Training and centres placement in an RBF network

In this work, both the centres of the basis functions and the connections weights are online adjusted. The *k*-means algorithm [24] is used for the centres adjustment. The connections weights are adapted and changed according to a law derived using Lyapunov stability theory.

3.2.1. Centres adjustment

The *k*-means algorithm is an unsupervised training method for data clustering [22]. The most commonly used *k*-means clustering is the adaptive *k*-means clustering based on the Euclidien distance [24, 25]. Adaptive *k*-means clustering can be considered as a special case of the gradient descent algorithm where only the winning cluster is adjusted at each learning step. It consists in dividing the input space into k classes as follows

- Choose a number of classes (*k* basis functions in our case).
- Initialise the centres of basis functions.
- Compute the Euclidean distances between the vector of centres c_i of each basis function and the input vector $\underline{x}$, i.e.,

$$dist(i) = \|\underline{x} - c_i\|_2\text{ , } i=1 \text{ to } nr \qquad (12)$$

- Adjust the vector of centres c_i of the basis function corresponding to the minimum distance $dist(j) = \min\|\underline{x} - c_i\|_2$ using the following adaptation law [24]

$$c_j(t) = c_j(t-1) + \delta(t).\left(\underline{x}(t) - c_j(t-1)\right) \qquad (13)$$

where j indicates the nearest vector of centres $c_j(t)$ to the vector of data $\underline{x}(t)$ (or j is the index of the basis function which corresponds to the minimum

Euclidean distance $dist(j)$). Notice that, the centres and the data are written in terms of time t where $c_j(t-1)$ represents the centres location at the previous clustering step. The adaptation rate $\delta(t)$ is a gain belonging to the interval [0 1] and can be selected in a number of ways. Chen et al. [25] used an adaptation rate that is updated at each step and tending to zero as $t \to \infty$ according to

$$\delta(k) = \frac{\delta(t-1)}{\sqrt{1+\mathrm{int}(t/nr)}} \tag{14}$$

This law (14) was the suitable one for our work, where *t* is the time, *nr* is the number of basis functions, and *int* is the integer part of (t/nr).

3.2.2. Weights adaptation

In the following, the adaptation law for the connections weights of the RBF network is derived using Lyapunov synthesis approach. As mentioned in section 2 (Problem formulation), since *f* is unknown, and the external disturbance *d* exists, the optimal control u^* of Eq. (7) can not be implemented. Our purpose is then to design an RBF network with output $u(\underline{x},\underline{\theta})$ to approximate this optimal control law. Thus, replace the control input u in Eq. (1) by the RBF system with output $u(\underline{x},\underline{\theta})$, then Eq. (1) becomes

$$x^{(n)} = f(\underline{x}) + b.u(\underline{x},\underline{\theta}) + d \tag{15}$$

Now adding and subtracting $b\,u^*$ to Eq. (15), gives

$$x^{(n)} = f(\underline{x}) + bu(\underline{x},\underline{\theta}) + d + bu^* - bu^* \tag{16}$$

Substituting Eq. (7) into Eq. (16) yields

$$x^{(n)} = f(\underline{x}) + bu(\underline{x},\underline{\theta}) + d - bu^* + y_m^{(n)} - K^T.\underline{e} - f(\underline{x}) \tag{17}$$

thus

$$x^{(n)} - y_m^{(n)} = -K^T\underline{e} + b(u(\underline{x},\underline{\theta}) - u^*) + d \tag{18}$$

Based on $y = x$ in Eq. (1) and using Eqs. (2) and (8), Eq. (18) leads to the error system

$$\dot{\underline{e}} = A_c.\underline{e} + b_c[b.(u(\underline{x},\underline{\theta}) - u^*) + d] \tag{19}$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_0 & -k_1 & -k_2 & \cdots & -k_{n-2} & -k_{n-1} \end{bmatrix}, \quad b_c = \begin{bmatrix} 0 \\ 0 \\ . \\ 0 \\ 1 \end{bmatrix} \tag{20}$$

Let's now study the stability of the system in order to develop an adaptive law to adjust the parameter vector $\underline{\theta}$. Define the optimal parameters vector $\underline{\theta}^*$ as the parameters vector which corresponds to the best (optimal) approximator term $u(\underline{x},\underline{\theta}^*)$ of the optimal control signal u^* of (7). Then $\underline{\theta}^*$ is defined as

$$\underline{\theta}^* = \arg\min[\sup|w|] , \quad \text{with} \quad w = u(\underline{x},\underline{\theta}^*) - u^*(\underline{x}) \tag{21}$$

w is the minimum approximation error . Thus, the error Eq. (19) can be rewritten as

$$\underline{\dot{e}} = A_c.\underline{e} + b_c[b.(u(\underline{x},\underline{\theta}) - u(\underline{x},\underline{\theta}^*)) + b.w + d] \tag{22}$$

Based on Eq. (10) we have

$$u(\underline{x},\underline{\theta}) = \underline{\theta}^T.\xi_1(\underline{x}), \text{ and } u(\underline{x},\underline{\theta}^*) = \underline{\theta}^{*T}.\xi_1(\underline{x}) \tag{23}$$

let $\varphi = \underline{\theta} - \underline{\theta}^*$ and using Eq. (23), thus Eq. (22) becomes

$$\underline{\dot{e}} = A_c\underline{e} + b_c b\varphi^T\xi_1(\underline{x}) + b_c(bw + d) \tag{24}$$

Define the Lyapunov function candidate

$$V = \frac{1}{2}\underline{e}^T P\underline{e} + \frac{b}{2.\gamma}\varphi^T\varphi \tag{25}$$

where γ is a positive constant and P is a solution of the Lyapunov equation

$$A_c^T.P + PA_c = -Q \quad \text{with} \quad Q > 0 . \tag{26}$$

Differentiate V with respect to time

$$\dot{V} = \frac{1}{2}\underline{\dot{e}}^T P\underline{e} + \frac{1}{2}\underline{e}^T P\underline{\dot{e}} + \frac{b}{2\gamma}\dot{\varphi}^T\varphi + \frac{b}{2\gamma}\varphi^T\dot{\varphi} \tag{27}$$

using Eqs. (24) and (26), we have

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} + \underline{e}^T Pb_c b.\varphi^T\xi_1(\underline{x}) + \underline{e}^T Pb_c(b.w + d) + \frac{b}{\gamma}\varphi^T\dot{\varphi} \tag{28}$$

Let P_n be the last column of P, and using Eq. (20), resulted

$$\underline{e}^T Pb_c = \underline{e}^T P_n \tag{29}$$

Substituting Eq. (29) into Eq. (28), will obtain

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} + \frac{b}{\gamma}\varphi^T[\gamma.\underline{e}^T P_n\xi_1(\underline{x}) + \dot{\varphi}] + \underline{e}^T Pb_c(bw + d) \tag{30}$$

If the adaptive law is chosen as

$$\underline{\dot{\theta}} = -\gamma\underline{e}^T P_n\xi_1(\underline{x}) \tag{31}$$

This will result in

$$\frac{b}{\gamma}\varphi^T(\gamma.\underline{e}^T P_n \xi_1(\underline{x}) + \dot{\varphi}) = 0 \tag{32}$$

Using the fact that $\dot{\varphi} = \dot{\underline{\theta}} - \dot{\underline{\theta}}^* = \dot{\underline{\theta}}$, because the optimal parameters vector $\underline{\theta}^*$ is constant and obviously its derivative is zero, i.e., $\dot{\underline{\theta}}^* = 0$, then Eq. (30) becomes

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} + \underline{e}^T P b_c (bw + d) \tag{33}$$

or

$$\dot{V} = -\frac{1}{2}\underline{e}^T Q\underline{e} - \underline{e}^T P b_c w_1 \tag{34}$$

where

$$w_1 = -(bw + d) \tag{35}$$

As a summarization from the above development, the obtained adaptive law for the RBF network parameters vector $\underline{\theta}$ is

$$\dot{\underline{\theta}} = -\gamma.\underline{e}^T P_n \xi_1(\underline{x}) \tag{36}$$

The overall scheme of the direct RBF adaptive controller is shown in Fig. 1. The following theorem shows the properties of the direct adaptive RBF controller.

Theorem

Consider the nonlinear plant (1) with the control law $u = u(\underline{x}, \underline{\theta})$ given by Eq. (23) and updating law given by Eq. (36) for the parameters vectors $\underline{\theta}$, then, the overall scheme guarantees that

i) The tracking error $\underline{e}(t)$ converges to a compact set Ω_e defined by

$$\Omega_e = \left\{ \underline{e} : /|\underline{e}| \le \sqrt{\frac{\delta}{\mu}} \right\} \tag{37}$$

where δ and μ are two positive constants

ii) if w_1 in Eq. (34) is squared integrable, that is $\int_0^\infty \|w_1(t)\|^2 dt < \infty$, then $\lim_{t\to\infty}\|e(t)\| = 0$.

Proof of the theorem

The following Barbalat's lemma is used to proof the part ii) of the theorem. Barbalat's lemma [15, 17]

if $\underline{e}(t) \in L_2$ (squared integrable, i.e., $\int_0^\infty \|\underline{e}(t)\|^2 dt < \infty$)), and $\underline{e}(t) \in L_\infty$ (bounded), and $\dot{\underline{e}}(t) \in L_\infty$ (bounded), then $\lim_{t\to\infty}\|e(t)\| = 0$. Let us now start with the proof of the first part of the theorem

i) Let $\lambda_{Q\min}$ be the minimum eigenvalue of Q then, from Eq. (34), that gives

$$\dot{V} \le -\frac{1}{2}\lambda_{Q\min}\|\underline{e}\|^2 - \underline{e}^T P b_c w_1 \tag{38}$$

which can be written as

$$\dot{V} \le -\frac{1}{2}\lambda_{Q\min}\|\underline{e}\|^2 - \underline{e}^T P.b_c.w_1 + \frac{1}{2}\|\underline{e}\|^2 - \frac{1}{2}\|\underline{e}\|^2 + \frac{1}{2}\|Pb_c w_1\|^2 - \frac{1}{2}\|Pb_c w_1\|^2$$
$$= -\frac{\lambda_{Q\min}-1}{2}\|\underline{e}\|^2 + \frac{1}{2}\|Pb_c w_1\|^2 - \frac{1}{2}\|(\underline{e} + Pb_c w_1)\|^2 \tag{39}$$

which can be simplified to

$$\dot{V} \le -\frac{\lambda_{Q\min}-1}{2}\|\underline{e}\|^2 + \frac{1}{2}\|Pb_c w_1\|^2 \tag{40}$$

because the term $\frac{1}{2}\|(\underline{e} + Pb_c w_1)\|^2$ is greater than or equal to 0.

Choose Q such that $\lambda_{Q\min} > 1$ because it is determined by the designer. It follows that

$$\dot{V} \le -\mu\|\underline{e}\|^2 + \delta \tag{41}$$

where $\mu = \frac{\lambda_{Q\min}-1}{2}$, and $\delta = \frac{1}{2}\|Pb_c w_1\|^2$ (42)

From Eq. (41) It can be concluded that $\dot{V} < 0$ if $\|\underline{e}\| > \sqrt{\frac{\delta}{\mu}}$. The compact set is defined as

$$\Omega_e = \left\{ \underline{e} : /|\underline{e}| \le \sqrt{\frac{\delta}{\mu}} \right\} \tag{43}$$

From Eq. (35), we have $w_1 = -(b.w + d)$ where the minimum approximation error w in Eq. (21) can be made arbitrarily small by using an appropriate number of radial basis functions approximators [1, 2]. The constant control gain b and the disturbance d are assumed to be bounded. Hence the quantity w_1 is bounded, and based on this, from Eq. (42), δ is bounded, thus the set Ω_e in Eq. (43) is bounded.

Now $\dot{V}$ is negative as long as $\underline{e}(t)$ is outside the compact set Ω_e, according to Lyapunov stability theory, it can be concluded that the error $\underline{e}(t)$ is bounded and will converge to Ω_e.

ii) Integrating both sides of Eq. (40), we obtain

$$[V(t) - V(0)] \le -\frac{\lambda_{Q\min}-1}{2}\int_0^t \|\underline{e}(\tau)\|^2 d\tau + \frac{1}{2}\|Pb_c\|^2 . \int_0^t \|w_1(\tau)\|^2 d\tau \tag{44}$$

Then

$$\int_0^t \|\underline{e}(\tau)\|^2 d\tau \le \frac{2}{\lambda_{Q\min}-1}[V(0)-V(t)] + \frac{1}{\lambda_{Q\min}-1}\|Pb_c\|^2 . \int_0^t \|w_1(\tau)\|^2 d\tau \qquad (45)$$

This will result in

$$\int_0^t \|\underline{e}(\tau)\|^2 d\tau \le \frac{1}{\lambda_{Q\min}-1}[2(\|V(0)\| + \|V(t)\|) + \|Pb_c\|^2 . \int_0^t \|w_1(\tau)\|^2 d\tau] \qquad (46)$$

As shown by Wang [15], this implies that if $w_1 \in L_2$ (i.e., squared integrable), then from Eq. (46) $\underline{e}(t) \in L_2$, and based on the conclusion above, according to Lyapunov stability theory, $\underline{e}(t)$ is bounded. On the other hand from Eq. (24) $\dot{\underline{e}}(t) \in L_\infty$ (bounded) because all elements of its right hand side are bounded. Using Barbalat's lemma mentioned above, it can be concluded that $\lim_{t\to\infty}\|e(t)\| = 0$.

Remark

In the above developments, global stability results are provided using Lyapunov theory without use of the compensatory (or a supervisory) control term in addition to the control law as usually done in most cases as mentioned in the introduction.

3.3. Design of the direct adaptive RBF controller

From the above analysis, the design of the direct RBF adaptive controller can be summarized in the following steps

Step 1: Off-line computations

Define the number of basis functions with centres uniformly cover the domain of data variation for the RBF network.

- Specify the parameters $k_0,...,k_{n-1}$ for the RBF network such that all roots of $s^n + k_{n-1}.s^{n-1} + ...+ k_1.s^1 + k_0 = 0$ are in the open left-half plane.
- Specify a positive definite $n \times n$ matrix Q, where n is the degree of the system.
- Solve the Lyapunov Eq. (26) to obtain a symmetric $P > 0$.
- Select a positive scalar values γ.
- give initial values to the parameters vector (connection weights) $\underline{\theta}$ of the RBF network (controller) $u(\underline{x},\underline{\theta}) = \underline{\theta}^T \xi_1(\underline{x})$.

In this work, both the centres of the basis functions and the connections weights are online adjusted. The *k*-means algorithm [24] is used for the centres adjustment. The connections weights are adapted and changed according to a law derived using Lyapunov stability theory.

Step 2: On-line adaptation

- Apply the feedback control law Eq. (23), i.e., $u(\underline{x},\underline{\theta}) = \underline{\theta}^T .\xi_1(\underline{x})$ (the output of the RBF network) to the plant (1).

- Use the adaptive law Eq. (36) to adjust the controller parameter vector (controller connections weights) $\underline{\theta}$.
- Use the *k*-means algorithm described in section 3.2.1 to adjust the centres of the radial basis functions for the RBF network.

4. Simulation Results

In this paper the direct adaptive RBF controller as depicted in Fig. 1 was applied to control the level in a Three Tank System (Example 1), a nonlinear unstable system (Example 2) and a two dimensional non linear system (Example 3).

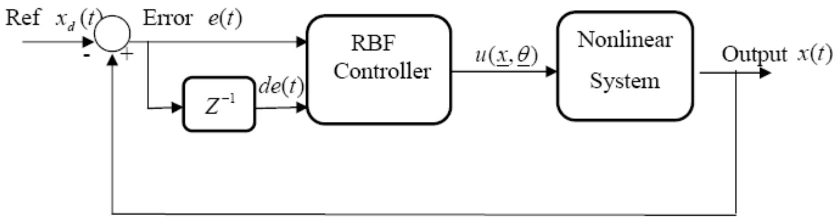

Fig. 1. The Direct RBF Adaptive Controller.

4.1. Example 1

In this first example, we apply the performance of our proposed RBF adaptive system to control the level in a Three Tank System and compare its behaviour to a proportional integral PI controller. The Three-Tank System [26] is a benchmark process widely used for modelling and control strategies for nonlinear systems. The nonlinear controlled system as depicted by Fig. 2 consists of three plexiglass cylinders T1, T2 and T3 with identical cross-sectional area A which are interconnected in series by two connecting pipes. The liquid leaving T2 is collected in a reservoir from which pumps 1 and 2 (driven by DC motors) supply tanks T1 and T2 with flow rates Q1 and Q2. All three tanks are equipped with piezo-resistive pressure transducer for measuring the level of the liquid (L_1, L_2 and L_3 in cm). The tanks are coupled by two connecting cylindrical pipes with a cross section S and an outflow coefficient $\mu_1 = \mu_3$. The nominal outflow is located at tank T2, it also has a circular cross section of S and an outflow coefficient μ_2. The connecting pipes and the tanks are additionally equipped with manually adjustable valves and outlets for the purpose of simulating clogs as well as leaks. In this example, the Three Tank System as a SISO system was considered, i.e., we will be interested to control the level L_2 in the tank T2 by the flow rate Q2.

The dynamic equation describing the SISO Three Tank System [26] is as follows

$$A\frac{dL_2}{dt} = Q_2 - \mu_2 S\sqrt{2gL_2} \qquad (47)$$

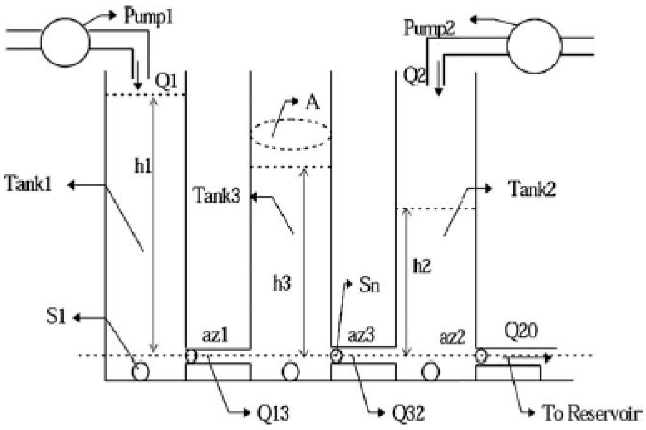

Fig. 2. The Structure of the Three Tank System.

where,

S = 0.5 cm^2, $\mu_2 = \mu_3 = 0.4896$, A = 154 cm^2, g = 9.81×100 cm/s^2 is the universal gravitation and $y = x = L_2$ is the level in Tank2. The reference signal will be $xm_1 = y_m = Lm_2$.

The other parameters are chosen as $\gamma = 0.07$, step size $dt = 1$, $k_0 = 1$ in order to have all roots of $.s + k_0 = 0$ in the open left-half plane, choose Q in Eq. (26) as $Q = 25 > 0$ **,** where the minimum eigenvalue of Q, i.e., $\lambda_{Q\min} = 25 > 1$ which will satisfy the transition from Eqs. (40) to (46) for $\lambda_{Q\min}$ in the proof of the theorem. Then $P = 12.5$ was obtained by solving Eq. (26). The RBF network has five radial basis functions. The controller parameters $\underline{\theta}$ are initialised to random values in the interval [0 1]. The centres of the basis functions are uniformly distributed in the interval [aa 14aa], where $aa = 1.13$ is a constant. The RBF network has two inputs $\underline{x} = [e \;\; de]$ with $e = y - y_m$ is the error, and de is the variation of error. The used basis functions are Gaussian functions under the form of (11) with a width $\sigma = 5$. The initial condition ($x(0) = L_2(0) = 0$ cm) is used in the simulation. For the first case ($0 \le t < 900$ s), the external disturbance d in Eq. (1) will not be taken in consideration. For all cases of simulation, the used PI parameters are k_p=7.5 (proportional action) and k_i=6.5 (integral action). Simulation results are shown in Figs. 3 and 4, where the corresponding results to the RBF controller are in dotted while those corresponding to the PI controller are in continuous and the reference signal is in dashed. Figure 3 shows the evolution of the level L_2 in Tank2. Figure 4 shows the corresponding control input. The first time interval of Fig. 3 shows that the system output (level in Tank2) has got the reference rapidly than with the PI controller and in the second and third time intervals, the response has less overshoot.

Also, in order to check the ability of our controller against perturbations, the external disturbance d in Eq. (1) will be taken in consideration in the interval ($900 \le t \le 1200$ s), where the disturbances on the level in Tank2 are introduced as follows: create a clogging, i.e., closing the nominal outflow valve of tank T2 with degree of 50% at time $t = 900$ s. In other words, in tank T2, the cross section S of the nominal outflow valve will take the value $S = 0.5/2$ cm^2 at $t = 900$ s instead of the nominal value $S = 0.5$ cm^2. Thus, the system equation (47) will be rewritten as

$$A\frac{dL_2}{dt} = Q_2 - \mu_2 (S/2)\sqrt{2gL_2} \tag{48}$$

which can be written as

$$A\frac{dL_2}{dt} = Q_2 - \mu_2 S\sqrt{2gL_2} + \mu_2 (S/2)\sqrt{2gL_2} \tag{49}$$

Based on the non perturbed system equation (47), the perturbed new system equation (49) was compared with the global system equation in (1), then the external disturbance d can be expressed as

$$d = \mu_2 (S/2)\sqrt{2gL_2} \tag{50}$$

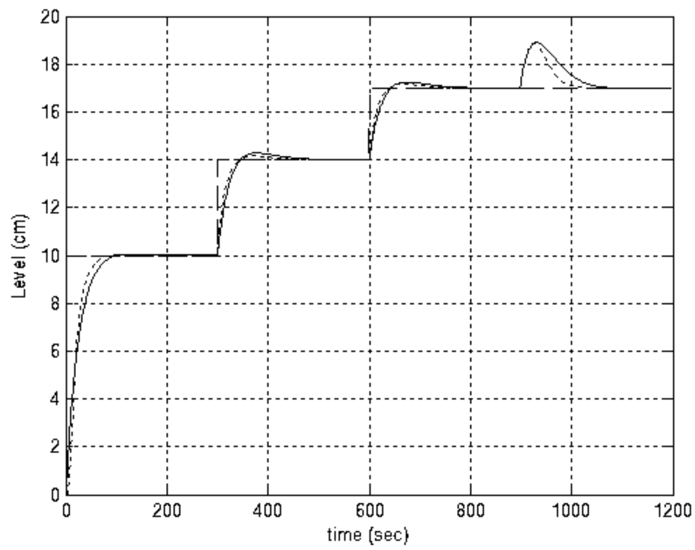

Fig. 3. The Level in Tank T2 with the RBF Controller (…) and with the PI Controller (-).

Clearly, from Eq. (50), the external disturbance d is bounded. In this case, simulation results are shown in the remaining time interval ($900 \le t \le 1200$ s) of the same previous Figs. 3 and 4. Clearly, the disturbances are suppressed rapidly with our RBF controller than with the PI controller. As a concluding remarks, from these figures, the proposed RBF controller was able to stabilise the level of the liquid in tank T2 at each interval and also was able to eliminate disturbances introduced through the outflow pipe of tank T2 in a better manner than with the PI controller, confirming also the robust property of the RBF system without the use of the supervisory term in the control law as discussed in the introduction.

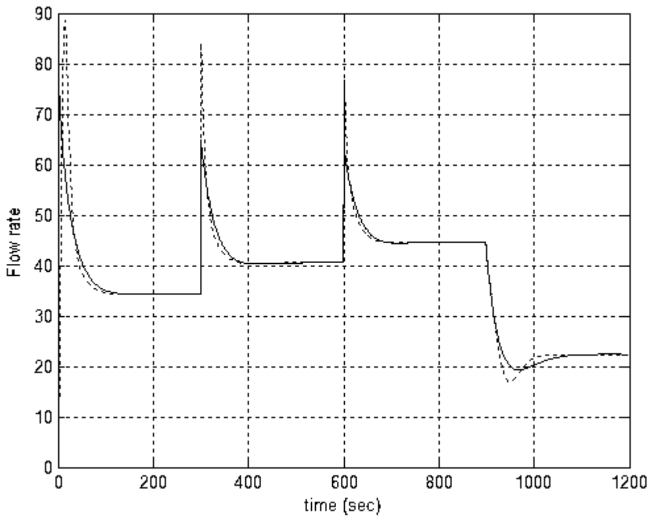

Fig. 4. The Control Signals of the RBF Controller (…) and of the PI controller (-).

4.2. Example 2

In this example, the direct adaptive RBF controller was applied to regulate to the origin an unstable system where the external disturbance *d* in Eq. (1) will not be taken in consideration as used in [14, 15]

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t) \tag{51}$$

From Eq. (51), it is clear that $\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} > 0$ for $x(t) > 0$, and $\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} < 0$ for $x(t) < 0$. The initial condition is $x(0) = 1$. According to the steps of the design procedure given in section 3.3, choosing first the number of radial basis functions. In control application the number of radial basis function is usually chosen between four and ten. Here, five radial basis functions are chosen. The centres of the basis functions are uniformly distributed in the interval [-2 2]. Since the degree of the system is $n = 1$, the error polynomial is $s + k_0 = 0$, we set $k_0 = 2.2$, so that its root is in the open left-half plane, choosing also Q = 12. Based on this, we obtain: $A_c = -k_0 = 2.2$ (see Eq. (20)), and by solving Eq. (26) we obtain P = 2.7273. Other choices have been tried; this last value has given a satisfactory transient performance. The step size for the system is 0.2, and the weights adaptation step is set to γ= 2.2. Smaller values give slower adaptations and higher values produce faster adaptation with a risk of instability. The parameters θ_i are all initialised to 0. Figure 5 shows the system state $x(t)$ and the desired position $y_m(t)$. From this figure it is clear that the proposed RBF direct adaptive control could regulate the plant to the origin. Figure 6 shows the corresponding control input $u(t)$. Clearly both the state and the control signal are bounded. Compared with the result in [14, 15], a good improvement on our system performance is observed, especially the response time (2.3 s in our system and 8 s in [14] and 11 s in [15]).

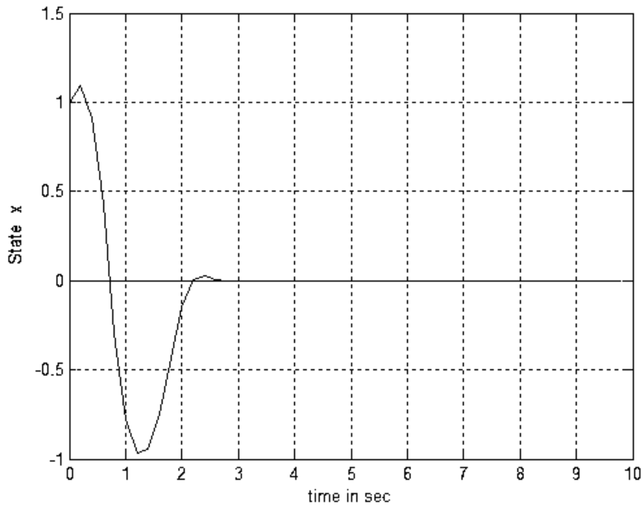

Fig. 5. The System State $x(t)$ and the Desired Position $y_m(t)$.

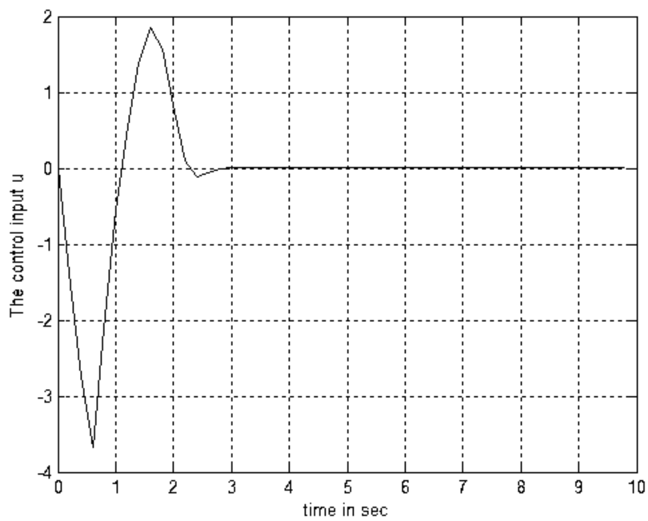

Fig. 6. The Control Input $u(t)$.

4.3. Example 3

In this example, consider a two dimensional non linear system controlled in [9]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 4.\left(\frac{\sin(4.\pi.x_1)}{\pi.x_1}\right).\left(\frac{\sin(\pi.x_2)}{\pi.x_2}\right)^2 + u(t). \tag{52}$$

The direct adaptive RBF controller is applied to control the system state $x_1(t)$ to track a desired trajectory which is specified as the output of a second order with a bandwidth driven by a unity amplitude, 0.5 mean, square wave [9] as depicted in Fig. 7. Five radial basis functions are chosen. The centres of the basis functions are uniformly distributed in the interval [-0.5 3]. Since the degree of the system is n = 2 the error polynomial is $s^2 + k_1.s + k_0 = 0$, setting $k_0 = 2$ and $k_1 = 3$, so that all their roots are in the open left-half plane. The step size for the system is $dt = 0.0165$, the step size for the weights adaptation law is set to γ = 75. Choosing $Q = diag(10,10) > 0$, then by solving Eq. (26), it results

$$P = \begin{bmatrix} 12.5 & 2.5 \\ 2.5 & 2.5 \end{bmatrix} \tag{53}$$

The parameters θ_i are initialised to zero. Figure 7 shows the system state $x_1(t)$ and the desired position $y_m(t)$. It is clear from this figure that the system state $x_1(t)$ tracks the desired trajectory $y_m(t)$ perfectly in comparison with the result in [9]. Figure 8 shows the corresponding control input $u(t)$. Figure 9 shows the corresponding velocity of the system $x_2(t)$ and the desired velocity $\dot{y}_m(t)$.

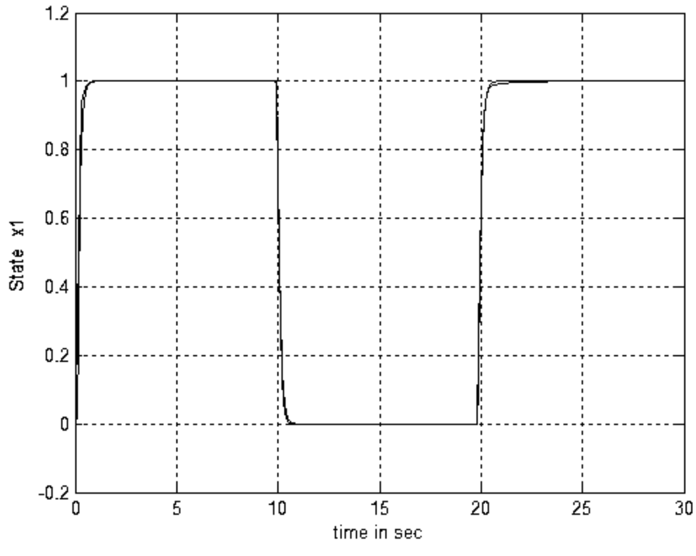

Fig. 7. The System State $x_1(t)$ and the Desired Position $y_m(t)$.

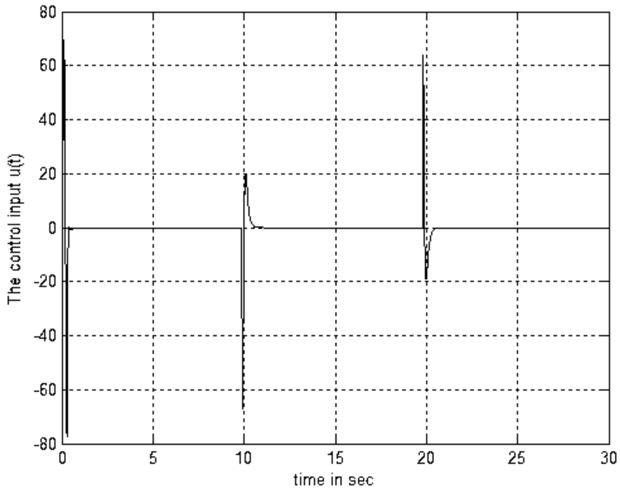

Fig. 8. The Control Input $u(t)$.

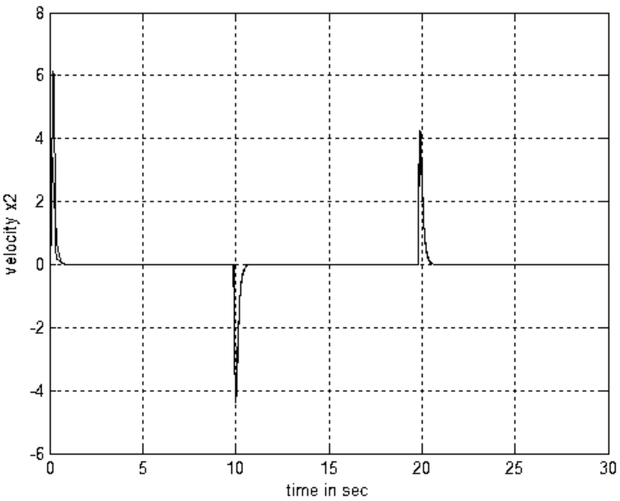

Fig. 9. The Velocity of the System $x_2(t)$ and the Desired Velocity $\dot{y}_m(t)$.

Now, in order to check the ability of our controller against perturbations, considering the fallowing in Eq. (1): assuming that the external disturbance d is different from zero ($d(t) \neq 0$), and the control gain b is taken as a non unity gain ($b \neq 1$). Based on the work in [9] where the term

$$b(\underline{x}(t)) = 2 + \sin[3\pi(x_1(t) - 0.5)] \tag{54}$$

is taken as a non unity gain and $d(t)$ is zero in his second case. In our work, as the control gain b is constant, assuming

(i) $b = 2$ which is the first term of Eq. (54).

(ii) $d(t) = \sin[3\pi(x_1(t) - 0.5)]$, which is the second term of Eq. (54).

From above, it is clear that d and $b(\underline{x}(t))$ are bounded. Figure 10 shows that the system state $x_1(t)$ could track the desired trajectory $y_m(t)$ perfectly with a very small overshot with comparison to the previous case (without a perturbation). Figures 11 and 12 show respectively the corresponding control input $u(t)$ and the velocity of the system $x_2(t)$ with the desired velocity $\dot{y}_m(t)$. From these figures, the robust property and smoothness of our RBF adaptive controller are confirmed without the use of the supervisory term in the control law.

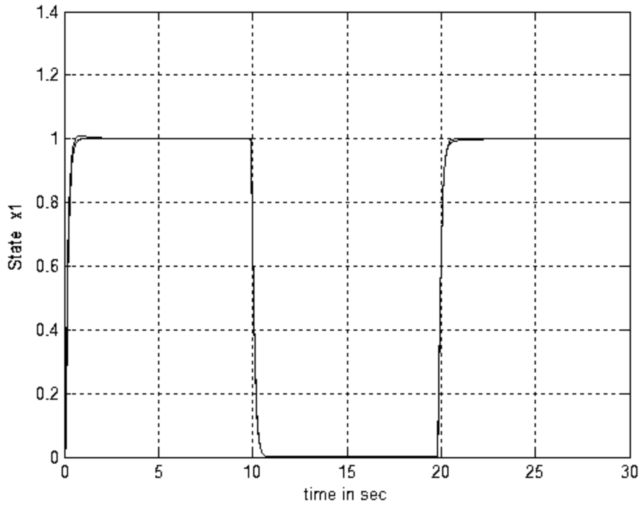

Fig. 10. The System State $x_1(t)$ and the Desired Position $y_m(t)$ in Perturbation Case.

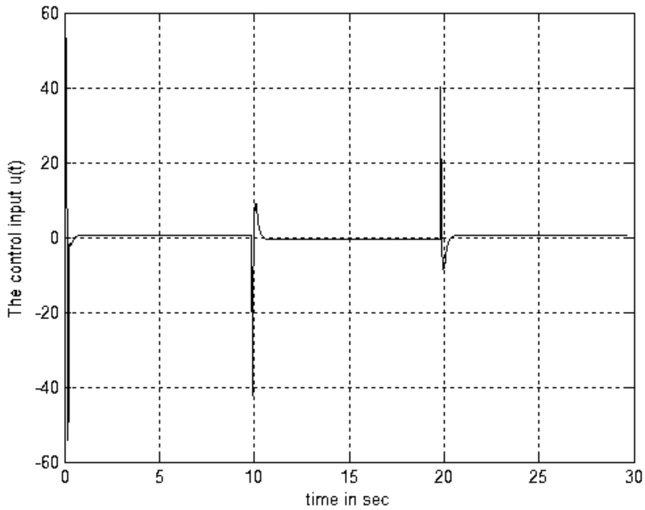

Fig. 11. The Control Input $u(t)$ in Perturbation Case.

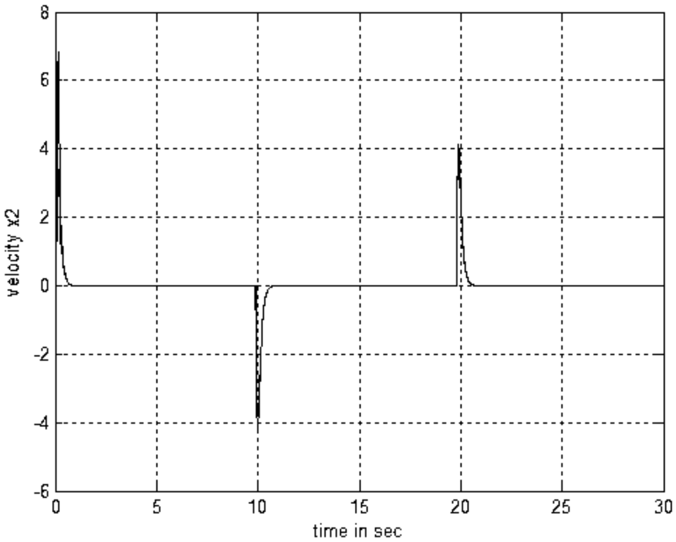

Fig. 12. The Velocity of the System $x_2(t)$ and the Desired Velocity $\dot{y}_m(t)$ in Perturbation Case.

5. Conclusions

In this paper, a stable direct adaptive control scheme for a class of unknown nonlinear dynamic systems was developed. For this purpose an on-line RBF (Radial Basis Function) neural network system was used to approximate the ideal control signal. Both the centres of the basis functions and the connections weights in the RBF network were online adjusted. The *k*-means algorithm was used for the centres adjustment, and the connections weights are adapted and changed according to a law derived using Lyapunov stability theory. The proposed method could guarantee the stability of the resulting closed-loop system in the sense that all signals involved were uniformly bounded. All this was achieved without the use of the supervisory term in the control law. Finally, the stable direct adaptive RBF controller was used to control the level in a Three Tank System (Example 1) an nonlinear unstable system (Example 2) and a two dimensional non linear system (Example 3). The results were encouraging in spite of the presence of disturbances confirming the robust and smoothing capability of our RBF system.

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