

ON THE PAPR REDUCTION IN OFDM SYSTEMS: A NOVEL ZCT PRECODING BASED SLM TECHNIQUE

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Abstract

High Peak to Average Power Ratio (PAPR) reduction is still an important challenge in Orthogonal Frequency Division Multiplexing (OFDM) systems. In this paper, we propose a novel Zadoff-Chu matrix Transform (ZCT) precoding based Selected Mapping (SLM) technique for PAPR reduction in OFDM systems. This technique is based on precoding the constellation symbols with ZCT precoder after the multiplication of phase rotation factor and before the Inverse Fast Fourier Transform (IFFT) in the SLM based OFDM (SLM-OFDM) Systems. Computer simulation results show that, the proposed technique can reduce PAPR up to 5.2 dB for $N=64$ (System subcarriers) and $V=16$ (Dissimilar phase sequences), at clip rate of 10^{-3} . Additionally, ZCT based SLM-OFDM (ZCT-SLM-OFDM) systems also take advantage of frequency variations of the communication channel and can also offer substantial performance gain in fading multipath channels.

Keywords: Peak to average power ratio, Zadoff-Chu matrix transform, Orthogonal frequency division multiplexing, SLM-OFDM.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission scheme that has become the technology of choice for next generation wireless and wireline digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrow band interference and frequency selective fading [1-3]. OFDM thwarts Inter Symbol Interference (ISI) by inserting a Guard Interval (GI) using a Cyclic Prefix (CP) and moderates the frequency selectivity of the Multi Path (MP) channel with a simple equalizer [4]. OFDM is widely adopted in various communication standards

Nomenclatures

A	Precoding matrix
$E [.]$	Expectation
j	Imaginary number, $\sqrt{-1}$
k	Precoder index
L	ZC sequence length
N	System's subcarriers
r	Any integer relatively prime to L
q	Any integer
V	Dissimilar phase sequences

like Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Wireless Local Area Networks (WLAN), Wireless Metropolitan Area Networks (WMAN), Wireless Personal Area Networks (WPAN) and even in the beyond 3G Wide Area Networks (WAN) etc.

However, among others, the Peak to Average Power Ratio (PAPR) is still one of the major drawbacks in the transmitted OFDM signal [5]. For zero distortion of the OFDM signal, the RF High Power Amplifier (HPA) must not only operate in its linear region but also with sufficient back-off. Thus, HPA with a large dynamic range are required for OFDM systems. These amplifiers are very expensive and are major cost components. Thus, if we reduce the PAPR it not only means that we are reducing the cost of OFDM system and reducing the complexity of A/D and D/A converters, but also increasing the transmit power, thus, for same range improving received SNR, or for the same SNR improving range.

The literature is replete with a large number of PAPR reduction techniques. Among them, schemes like constellation shaping [6], phase optimization [7], nonlinear companding transforms [8], Tone Reservation (TR) and Tone Injection (TI) [9, 10], clipping and filtering [11], Partial Transmit Sequence (PTS) [12], precoding based techniques [13-15], Selected Mapping (SLM) [16-18], Precoding based Selected Mapping (PSLM) [19], and phase modulation transform [20-22] are popular. The precoding based techniques, however, show great promise as they are simple linear techniques to implement.

Wang and Tellambura [11] proposed a soft clipping technique which preserves the phase and clips only the amplitude. They also put a lot of effort to characterize the performance and discover some properties to simplify the job. However, the PAPR gain is only estimated by simulations and is limited to a specific class of modulation technique. Han and Lee [12] proposed a PAPR reduction technique based on Partial Transmit Sequence technique in which they divide the frequency bins into sub blocks and then they multiply each sub-block with a constant phase shift. Choosing the appropriate phase shift values reduces PAPR. The most critical part of this technique is to find out the optimal phase value combination and in this regard they also proposed a simplified search method and evaluated the performance of the proposed technique.

Min and Jeoti [14] presented a Zadoff-Chu precoding based PAPR reduction technique. This technique is efficient, signal independent, distortionless and it

does not require any optimization algorithm. In addition, this precoding based PAPR reduction technique does not require power increase and side information to be sent for receiver.

Lim et al. [17] proposed a selected mapping (SLM) technique for PAPR reduction. In this technique, they multiply the constellation with a phase rotated sequence to reduce the PAPR. Liang and Ouyang [18] also proposed a low complexity an SLM technique in which they rotate the bins by one of the phase sequences and then select the sequence with lower PAPR for transmissions. The main emphasis of the paper is on method of generating the time domain results and IFFT is not performed on every possible phase rotation. Han and Lee [23] and Jiang and Wu [24] presented an excellent survey of PAPR reduction techniques like clipping and filtering, coding, PTS, SLM, interleaving, TR, TI and ACE. In addition they also provide mathematical analysis of the distribution of PAPR in OFDM systems.

In this paper, we present a novel ZCT precoding based SLM technique for PAPR reduction in OFDM systems. In the proposed system we applied ZCT based precoder after the multiplication of phase rotation factor and before the IFFT in the SLM based OFDM (SLM-OFDM) system. This paper is organized as follows. Section 2 describes the basics of the OFDM system and PAPR reduction, In Section 3 we present the proposed system model for PAPR reduction, and Section 4 presents computer simulation results and section 5 describe the conclusion.

2. OFDM System and PAPR

2.1. OFDM conventional

Figure 1 shows the block diagram of an OFDM system. Baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N . We can write the complex vector of size as $X = [X_0, X_1, X_2, \dots, X_{N-1}]^T$, X is then passed through the IFFT block. The complex baseband OFDM signal with N subcarriers can be written as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{n}{N} k}, n = 0, 1, 2, \dots, N-1 \tag{1}$$

The PAPR of OFDM signal in Eq. (1) can be written as:

$$PAPR = \frac{\max |x_n|^2}{E[x_n]^2} \tag{2}$$

where $E[.]$ denotes expectation and the Complementary Cumulative Distribution Function (CCDF) for an OFDM signal can be written as:

$$P(PAPR > PAPR_o) = 1 - (1 - e^{-PAPR_o})^N \tag{3}$$

where $PAPR_o$ is the clipping level. This equation can be read as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_o$.

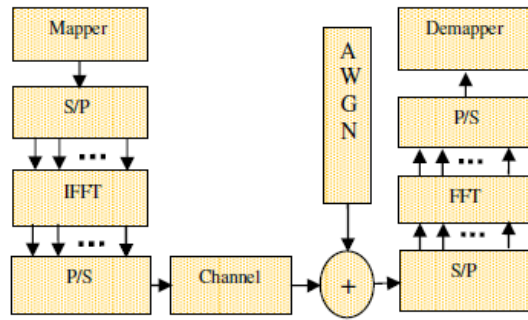


Fig. 1. Block Diagram of Conventional OFDM System.

2.2. Zadoff-Chu sequences

Zadoff-Chu sequences are class of poly phase sequences having optimum correlation properties. Zadoff-Chu sequences have an ideal periodic autocorrelation and constant magnitude. According to Popovic’ [25], the Zadoff-Chu sequences of length N can be defined as:-

$$a_n = \begin{cases} e^{\frac{j2\pi}{N}\left(\frac{k^2}{2}+qk\right)}, & \text{for } N \text{ even} \\ e^{\frac{j2\pi}{N}\left(\frac{k(k+1)}{2}+qk\right)}, & \text{for } N \text{ odd} \end{cases} \quad (4)$$

where $k = 0, 1, 2, \dots, N-1$, q is any integer, r is any integer relatively prime to N and $j = \sqrt{-1}$.

2.3. Zadoff-Chu matrix transform (ZCT)

The kernel of the ZCT is defined in Eq. (5). The ZCT, A , of size $N = L \times L$ is obtained by reshaping the ZC sequence with $k = m + lL$ as hereunder:

$$A = \begin{bmatrix} a_{00} & a_{01} & \dots & a_{0(L-1)} \\ a_{10} & a_{11} & \dots & a_{1(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(L-1)0} & a_{(L-1)1} & \dots & a_{(L-1)(L-1)} \end{bmatrix} \quad (5)$$

Here m is the row variable and l the column variable. In other words, the $N=L^2$ point long ZC sequence fills the kernel of the matrix column-wise.

2.4. ZCT precoding based OFDM (ZCT-OFDM) system

Figure 2 shows the ZCT precoded OFDM system. In this system, the kernel of the ZCT acts as a column-wise precoding matrix A of dimension $N=L \times L$ and it is applied to constellations symbols before the IFFT to reduce the PAPR.

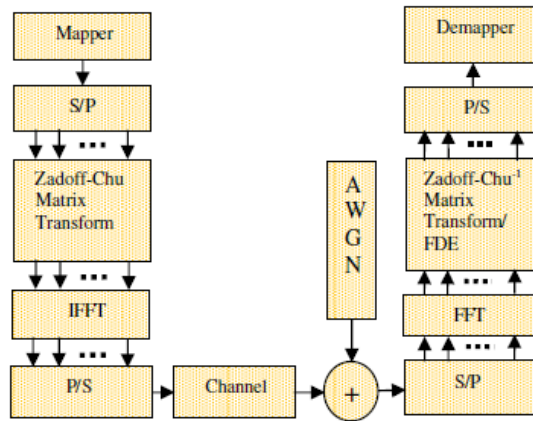


Fig. 2. Block Diagram of ZCT Precoding Based OFDM System.

In the ZCT precoded OFDM systems, the baseband modulated data is passed through S/P converter which generates a complex vector of size L that can be written as $X = [X_0, X_1, X_2, \dots, X_{L-1}]^T$. Then ZCT precoding is applied to this complex vector which transforms this complex vector into new vector of same length L . This new vector, of length L transformed by ZCT precoding can be written as: $Y=AX= [Y_0, Y_1, Y_2, \dots, Y_{L-1}]^T$, where A is a precoder matrix of size $L \times L$ and Y_m can be written as:-

$$Y_m = \sum_{l=0}^{L-1} a_{m,l} X_l, m = 0, 1, 2, \dots, L-1 \tag{6}$$

where $a_{m,l}$ means m^{th} row and l^{th} column of precoder matrix. Expanding Eq. (6) by using column wise reshaping $k=m+lL$ and putting $q=0$ and $r=1$ in Eq. (4) we get:

$$Y_m = \sum_{l=0}^{L-1} e^{j\frac{\pi(m+lL)^2}{L^2}} X_l = e^{j\frac{\pi m^2}{L^2}} \sum_{l=0}^{L-1} X_l e^{j\pi^2} e^{j\frac{2\pi ml}{L}} \tag{7}$$

where $m = 0, 1, 2, \dots, L-1$. Equation (7) represents the ZCT precoded constellations symbols. The complex baseband ZCT-OFDM signal with L subcarriers can be written as:-

$$x_n = \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} Y_m e^{j2\pi \frac{n}{L} m}, n=0, 1, 2, \dots, L-1 \tag{8}$$

The PAPR of ZCT-OFDM signal in Eq. (8) can be written as:-

$$PAPR = \frac{\max_{n=0,1,\dots,N-1} [x_n]^2}{\frac{1}{M} \sum_{n=0}^{N-1} [x_n]^2} \tag{9}$$

2.5. SLM based OFDM (SLM-OFDM) system

The SLM is a PAPR reduction technique which is based on phase rotations. In SLM-OFDM system, a set of V different data blocks are formed at the transmitter representing the same information and a data block with minimum PAPR is selected for transmission. Figure 3 shows the block diagram of SLM-OFDM system.

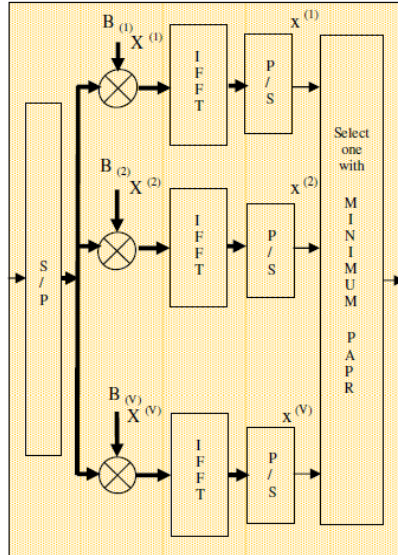


Fig. 3. Block Diagram of SLM Based OFDM System.

Every data block is multiplied by V dissimilar phase sequences, each of length N , $B^{(v)} = [b_{v,0}, b_{v,1}, b_{v,2}, \dots, b_{v,N-1}]^T, v=1, 2, \dots, V$ which results in the changed data blocks. Now suppose the altered data block for the v^{th} phase sequence is given by $X^{(v)} = [X_0 b_{v,0}, X_1 b_{v,1}, X_2 b_{v,2}, \dots, X_{N-1} b_{v,N-1}]^T, v=1, 2, \dots, V$. Each X_n^v can be defined as:

$$X_n^v = X_n b_{v,n}, (1 \leq v \leq V) \tag{10}$$

The SLM-OFDM signal with N subcarriers can be written as:-

$$x_n^{(v)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k^v e^{j2\pi \frac{n}{N} k}, n=0, 1, 2, \dots, N-1 \tag{11}$$

where $v=1, 2, 3, \dots, V$. The PAPR of SLM-OFDM signal in Eq. (11) can be written as:

$$PAPR = \frac{\max |x_n^{(v)}|^2}{E[|x_n^{(v)}|^2]} \tag{12}$$

Amongst the tailored data blocks $x^{(v)}= 1, 2, \dots, V$, the data block with least PAPR is chooses for the transmission and the information about selected phase sequence must be sent to the receiver as a side information. At the receiving end, the operation is performed in the reverse order to recover the actual data block.

3. Proposed Model

ZCT Precoded SLM Based OFDM (ZCT-SLM-OFDM) System

Figure 4 shows the block diagram of the proposed ZCT precoded SLM-OFDM system.

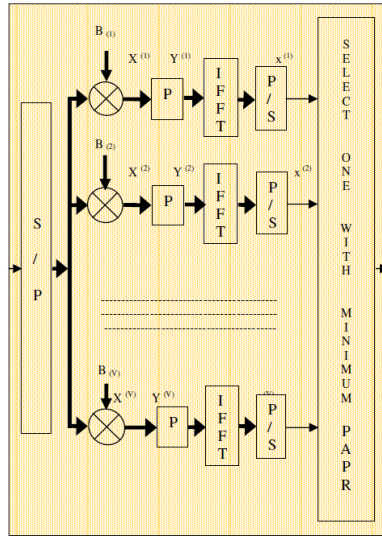


Fig. 4. Block Diagram of ZCT Precoding Based SLM-OFDM System.

Suppose data stream after serial to parallel conversion is $X = [X_0, X_1, X_2, \dots, X_{L-1}]^T$ and each data block is multiplied by V dissimilar phase sequences, each length equal to L , $B^{(v)} = [b_{v,0}, b_{v,1}, b_{v,2}, \dots, b_{v,L-1}]^T$, $v=1, 2, \dots, V$ which results in the altered data blocks. Now suppose the altered data block for the v^{th} phase sequence is given by $X^{(v)} = [X_0 b_{v,0}, X_1 b_{v,1}, X_2 b_{v,2}, \dots, X_{L-1} b_{v,L-1}]^T$, $v=1, 2, 3, \dots, V$. Each X_n^v can be defined as:

$$X_l^v = X_l b_{v,l}, (1 \leq v \leq V) \tag{13}$$

Now we pass the signal given in Eq. (13) through our ZCT precoder and the resultant signal Y_m^v can be written as:

$$Y_m^v = \sum_{l=0}^{L-1} a_{m,l} X_l^v, k=0, 1, 2, \dots, L-1 \tag{14}$$

where $a_{m,l}$ means m^{th} row and l^{th} column of the precoding ZCT matrix. The signal in Eq. (14) after performing the IFFT can be written as:

$$x_n^{(v)} = \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} Y_m^v e^{j2\pi \frac{m}{L} n}, n=0, 1, 2, \dots, L-1 \tag{15}$$

where $v = 1, 2, \dots, V$ the PAPR of ZCT-SLM-OFDM signal in Eq. (15) can be written as:

$$PAPR = \frac{\max |x_n^{(v)}|^2}{E \left[|x_n^{(v)}|^2 \right]} \tag{16}$$

The ZCT precoding based SLM-OFDM technique needs V IFFT operations and the information bits required as side information for each data block is $\lceil \log_2 V \rceil$. ZCT precoding based SLM-OFDM technique is applicable for any number of subcarriers and all types of modulation techniques. The PAPR reduction for ZCT-SLM-OFDM system depends on the number of phase sequences V , the output data with lowest PAPR is selected by the transmitter for transmissions. This technique can reduce more PAPR if we increase the value of V , but with the increase in the value of V the computational complexity is also increased. Thus, the value of V should be chosen carefully.

4. Computer Simulations

Extensive simulations in MATLAB^(R) have been performed in order to evaluate the PAPR performance of the proposed ZCT precoding based SLM-OFDM system. To show the effect of our proposed ZCT-SLM-OFDM system we considered QPSK modulation technique with 10^5 random OFDM blocks. We also compared our results with ZCT-OFDM system, SLM-OFDM systems and conventional OFDM systems with $N = 64, 128, 256$ and 512 .

Figure 5 shows the CCDF comparisons of PAPR of ZCT-SLM-OFDM system with ZCT-OFDM system, SLM-OFDM systems and conventional OFDM systems, with $N = 64$ for QPSK modulation. At clip rate of 10^{-3} , the PAPR is to 10.3 dB, 7.6 dB, 7.4 dB, 6.5 dB, 6.0 dB and 5.2 dB, for Conventional OFDM systems, SLM-OFDM systems with $V=4$, ZCT-OFDM System, SLM-OFDM systems with $V=16$, ZCT-SLM-OFDM system with $V=4$, and ZCT-SLM-OFDM system with $V=16$ respectively.

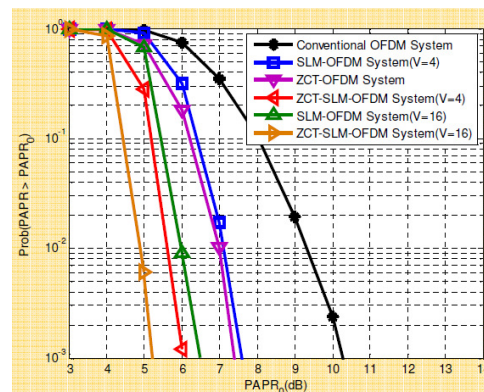


Fig. 5. CCDF of the PAPR of Conventional OFDM System, ZCT-OFDM System, SLM-OFDM System and ZCT-SLM-OFDM System, with System subcarriers $N=64$ for QPSK Modulation.

Figure 6 shows the CCDF comparisons of PAPR of ZCT-SLM-OFDM system with ZCT-OFDM system, SLM-OFDM systems and conventional OFDM systems, with $N = 128$ for QPSK modulation. At clip rate of 10^{-3} , the PAPR is to 10.5 dB, 8.1 dB, 7.5 dB, 6.8 dB, 6.3 dB and 5.3 dB, for Conventional OFDM systems, SLM-OFDM systems with $V=4$, ZCT-OFDM System, SLM-OFDM

systems with $V=16$, ZCT-SLM-OFDM system with $V=4$, and ZCT-SLM-OFDM system with $V=16$ respectively.

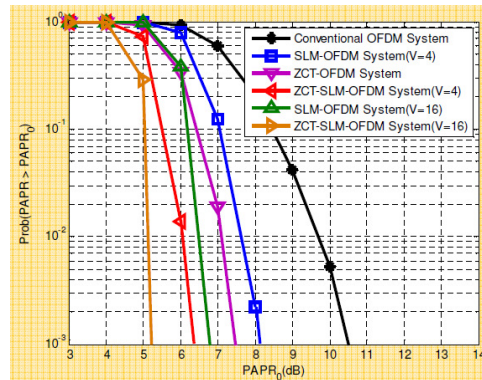


Fig. 6. CCDF of the PAPR of Conventional OFDM System, ZCT-OFDM System, SLM-OFDM System and ZCT-SLM-OFDM System, with System Subcarriers $N=128$ for QPSK Modulation.

Figure 7 shows the CCDF comparisons of PAPR of ZCT-SLM-OFDM system with ZCT-OFDM system, SLM-OFDM systems and conventional OFDM systems, with $N = 256$ for QPSK modulation. At clip rate of 10^{-3} , the PAPR is to 10.8 dB, 8.4 dB, 7.8 dB, 7.5 dB, 6.3 dB and 5.5 dB, for Conventional OFDM systems, SLM-OFDM systems with $V=4$, ZCT-OFDM System, SLM-OFDM systems with $V=16$, ZCT-SLM-OFDM system with $V=4$, and ZCT-SLM-OFDM system with $V=16$ respectively.

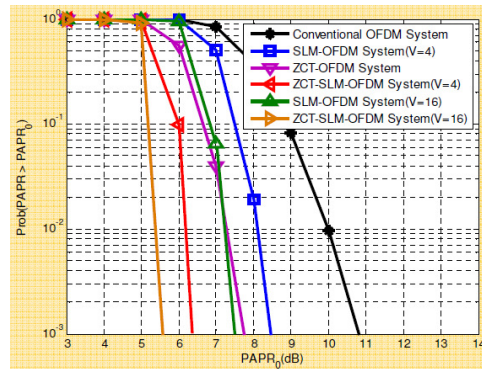


Fig. 7. CCDF of the PAPR of Conventional OFDM System, ZCT-OFDM System, SLM-OFDM System and ZCT-SLM-OFDM System, with System Subcarriers $N=256$ for QPSK Modulation.

Figure 8 shows the CCDF comparisons of PAPR of ZCT-SLM-OFDM system with ZCT-OFDM system, SLM-OFDM systems and conventional OFDM systems, with $N = 512$ for QPSK modulation. At clip rate of 10^{-3} , the PAPR is to 11.1 dB, 9.0 dB, 7.8 dB, 7.8 dB, 7.1 dB and 6.1 dB, for Conventional OFDM systems, SLM-OFDM systems with $V=4$, ZCT-OFDM System, SLM-OFDM

systems with $V=16$, ZCT-SLM-OFDM system with $V=4$, and ZCT-SLM-OFDM system with $V=16$ respectively.

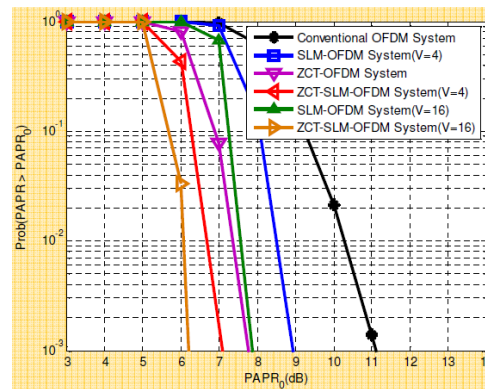


Fig. 8. CCDF of the PAPR of Conventional OFDM System, ZCT-OFDM System, SLM-OFDM System and ZCT-SLM-OFDM System, with System Subcarriers $N=512$ for QPSK modulation.

Table 1 presents the PAPR analysis of the ZCT-SLM-OFDM system, the ZCT-OFDM System, the SLM-OFDM systems and the conventional OFDM systems, at clip rate of 10^{-3} . It can be seen that our proposed ZCT-SLM-OFDM technique shows better PAPR reduction (PAPR gain) as compare to ZCT-OFDM System, SLM-OFDM systems and conventional OFDM systems.

Table 1. PAPR Analysis, At Clip Rate of 10^{-3} .

Systems	PAPR in dB			
	$N=64$	$N=128$	$N=256$	$N=512$
Conventional OFDM System	10.3	10.5	10.8	11.1
SLM-OFDM System ($V=4$)	7.6	8.1	8.4	9.0
ZCT-OFDM System	7.4	7.5	7.8	7.8
SLM-OFDM System ($V=16$)	6.5	6.8	7.5	7.8
ZCT-SLM-OFDM System ($V=4$)	6.0	6.3	6.3	7.1
ZCT-SLM-OFDM System ($V=16$)	5.2	5.3	5.5	6.1

5. Conclusions

In this paper, we proposed a novel ZCT precoding based SLM-OFDM technique to reduce the high PAPR generated by multi carrier modulation in the OFDM systems. Our proposed technique can reduce more PAPR if we increase the value of V , but with the increase in the value of V the computational complexity is also increased so, the value of V should be chosen carefully. Additionally, this technique is efficient, signal independent, distortionless, it does not require any complex optimization.

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