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OPERATIONAL READINESS OF GLOBAL MOBILE SATELLITE COMMUNICATION SYSTEM UNDER PARTIAL AND COMPLETE FAILURE

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Abstract

The aim of the present paper is to study and examine the various reliability characteristics of a global mobile satellite communication System (GMSCS) with the help of mathematical modelling. GMSCS basically comprises of four subsystems; Space segment, Land earth stations, Telecommunication terminals and Mobile earth stations. The system under consideration can have three different modes of working: normal, partial and complete failure. The system is characterized by determination of probabilities being in 'up' and 'down' states at any instant. Integro-differential equations are derived for these probabilities by identifying the system at suitable regeneration epochs. Based on the assumption that the failure rates of units are distributed exponentially while the repair rates are distributed arbitrarily, different reliability measures like operational availability, reliability, mean time to failure and cost effectiveness have been computed with the principle of Laplace transforms and supplementary variable technique. Considerable attention is devoted to illustrate the results with numerical examples to highlight the important features of reliability measures of the system.

Keywords: Reliability, Distributions, Laplace transform, MTTF, Supplementary variable technique, Cost effectiveness.

1. Introduction

Real-time and embedded systems are now a central part of our lives. Reliable functioning of these systems is of paramount concern to the millions of users that depend on these systems every day. Unfortunately most embedded systems still fall

Nomenclatures

$D/D_t/D_x$	$\frac{d}{dt} / \frac{\partial}{\partial t} / \frac{\partial}{\partial x}$
L	Land earth stations
M	Mobile earth stations
M_p	Mean time to repair where $M_p = -\overline{S'}_p(0)$ $\overline{S}_p(s) = p(x)\exp\left\{-\int_0^x p(x)dx\right\}$ *where* $p = \mu_L, \mu, \mu_M, \eta, \mu_T, \xi, \mu_S$
P	Probabilities
$P_F(x,t)\,\Delta$	P\{the system is in failed state at time t and elapsed repair time lies between x and $x+\Delta$\}
$P_k(x,t)\,\Delta$	P\{the system is in degraded state at time t and elapsed repair time lies between x and $x+\Delta$\}, $k = 1, 2, 3, 4, 5, 6$
$P_O(t)$	P\{at time t the system is in state O\}
S	Space segment
T	Telecommunication terminals

Greek Symbols

η	General repair rate of MES, national and international network
λ_L	Failure rate of LES
λ_M	Failure rate of MES
λ_S	Failure rate of space segment
λ_T	Failure rate of national and international network
μ	General repair rate of MES
μ_L	General repair rate of LES
μ_S	General repair rate of space segment
μ_T	General repair rate of national and international network
ξ	General repair rate of LES, MES, national and international network

Abbreviations

GMSCS	Global mobile satellite communication system
LES	Land earth station
LT	Laplace transform
MES	Mobile earth station
MTTF	Mean time to failure

short of users expectation of reliability. Basically a system is a combination of elements forming a unitary whole i.e. there is a functional relationship between its components. The properties and behaviour of each component ultimately affect the properties of the system. Any system has a hierarchy of components that pass through the different stages of operations which can be either operational, failure or repairing. Failure doesn't mean that it will always be complete; it can be partial as well. But both of these types affect the performance of system and hence the reliability. Further, modern engineered complex systems are made up of a highly complex and sensitive set of electrical, mechanical and electronic components, Global Mobile Satellite Communication System (GMSCS) is not an exception to this. With the increasing load demand and requirement the utility company has to ensure very high availability of the system which leads to growing importance of the reliability study of the system [1]. The reliability of a system has been defined in a probabilistic way that the probability of a system is not failing during the period [0, t] [2, 3].

Keeping above facts in view the present paper proposes to mathematical model and analyse a real life GMSCS to compute the various reliability parameters. From the late 1950's, satellites have become a growing part of the world and now run many different applications such as broadcast television, mobile phones, credit card transaction, Internet etc.[4-6]. Quite a good number of studies have been carried out by [7-10] involving the concept of non-identical parallel units and studied their reliability behaviour.

In the present study, the authors obtained reliability characteristics like Operational Readiness, Steady State Availabilities, Reliability Analysis, Mean Time to Failure (MTTF), Cost Analysis of the considered system with the help of Supplementary variable and Laplace transforms technique. In the proposed analysis, probabilistic considerations and limiting procedure of the considered system yielded the differential equations. Laplace transform (LT) is used to solve the differential equations. It is to be noted that while LT can be used to transform a differential equation into an algebraic equation, just like Fourier transform, LT introduces the initial condition at t = 0 explicitly. Furthermore, LT method has attractive feature to solve the linear integro-differential equation, as it obtains homogeneous and particular integral solution simultaneously and helps in finding both the transient and steady state components of the solution which are required in the present study. Hence LT has been used to tackle the present problem. At last some numerical examples have been taken to highlight the important reliability characteristics of the system. Figures 1 and 2 describe the block and transition diagram of the GMSCS.

System description

Global mobile satellite communications are specific communication systems for maritime, land and aeronautical applications. It enables connections between moving objects such as ships, vehicles and aircrafts and telecommunications subscribers through the medium of communications satellite, land earth stations and other landline telecommunication providers. Mobile satellite communications and technology have been in use for over two decades. Numerous schemes have been proposed to improve the efficiency of satellite communication networks [11]. Our existing system consists of:

i) The Space segment (**S**), the satellite and their ground support facilities with repeaters.
ii) The Land Earth Stations (LESs) (**L**) which provide an interface between the space segment and national or international telecommunication terminals (**T**).
iii) The Mobile Earth Stations (MESs) or terminals (**M**) which are located on ships, trucks etc.

Global mobile satellite communication system provides transactional, regional or global coverage from a constellation of satellites accessible with transportable terminals. Electromagnetic rays from the subscriber terminal are received via satellite by one of the LES providing the access to public service telephone networks. Satellite has at least two repeaters with one of them relaying messages from LESs to terminals and the other back.

The satellites are digital transponders that receive digital signals, reform the pulses and then retransmit them to ground stations. The ground stations perform billing and act as gateways to public switched telephone network and internet. Mismanagement, natural disasters, environmental conditions etc. are major factors in the failure of satellite and land earth station. Due to atmospheric attenuation noise signals, transmission latencies, weak receive signals like problems arise.

- System fails completely due to the failure of subsystem **S**.
- The subsystem **L** consists of one unit while subsystem **T** and **M** have dissimilar n and m units in parallel respectively.
- Initially at $t = 0$, all units are good.
- Failure of either unit of **T** or **M** brings the system to lesser efficiency (degrade).
- Failure of subsystem **L** brings **T** automatically to failure mode.
- Failures are statistically independent.
- The repair time of the units are assumed to be arbitrarily distributed.
- Repaired subsystem/unit(s) works like new.

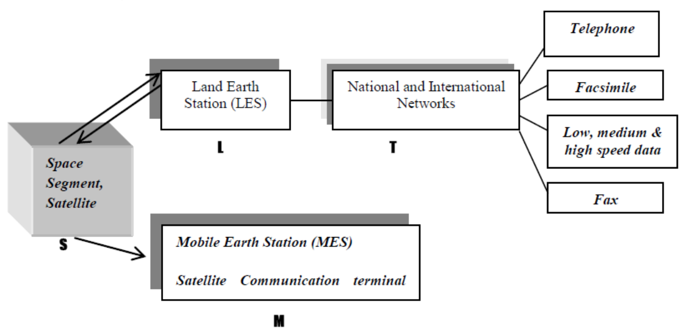

Fig. 1. Block Diagram.

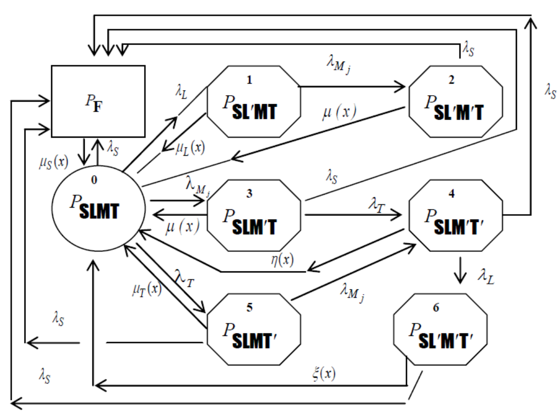

Fig. 2. Transition Diagram.

2. States of the System and Notations

Possible states of the system, constant failure rates, and general repair rates are given in Tables 1, 2, and 3 respectively. The notations used are given in the Nomenclatures list.

Table 1. States of the System.

States	Subsystem S	L	M	T	System state
O	O	O	O	O	O
1	O	F	O	O	D
2	O	F	F_j	O	D
3	O	O	F_j	O	D
4	O	O	F_j	F_i	D
5	O	O	O	F_i	D
6	O	F	F_j	F_i	D
F	F	O/F	O/F	O/F	F

O: Operable
D: Degraded
F: Failed
F_j: j[th] unit of M failed
F_i: i[th] unit of T failed

Table 2. Constant Failure Rates.

From	O				1	3	4	5	1,2,3, 4,5,6
To	1	3	5	F	2	4	6	4	F
Failure rate	λ_L	λ_{Mj}	λ_{Ti}	λ_S	λ_{Mj}	λ_{Ti}	λ_L	λ_{Mj}	λ_S

Table 3. General Repair Rates.

Repair rate	$\mu_L(x)$	$\mu(x)$	$\mu_M(x)$	$\eta(x)$	$\mu_T(x)$	$\xi(x)$	$\mu_S(x)$
From	1	2	3	4	5	6	F
To				O			

3. Basic Equations and Their Laplace Transform

Probabilistic considerations and limiting procedure yield the following integro-differential equations satisfying the model:

$$\begin{aligned}[D+\lambda_L+\lambda_S+\lambda_M+\lambda_T]P_O(t)=&\int\mu_L(x)P_1(x,t)dx+\int\mu(x)P_2(x,t)dx\\&+\int\mu_M(x)P_3(x,t)dx+\int\eta(x)P_4(x,t)dx\\&+\int\mu_T(x)P_5(x,t)dx+\int\xi(x)P_6(x,t)dx\\&+\int\mu_S(x)P_F(x,t)dx\end{aligned} \tag{1}$$

$$[D_x+D_t+\mu_L(x)+\lambda_S+\lambda_M]P_1(x,t)=0 \tag{2}$$

$$[D_x+D_t+\mu(x)+\lambda_S]P_2(x,t)=0 \tag{3}$$

$$[D_x+D_t+\mu_M(x)+\lambda_S+\lambda_T]P_3(x,t)=0 \tag{4}$$

$$[D_x+D_t+\eta(x)+\lambda_S+\lambda_L]P_4(x,t)=0 \tag{5}$$

$$[D_x+D_t+\mu_T(x)+\lambda_S+\lambda_M]P_5(x,t)=0 \tag{6}$$

$$[D_x+D_t+\xi(x)+\lambda_S]P_6(x,t)=0 \tag{7}$$

$$[D_x+D_t+\mu_S(x)]P_F(x,t)=0 \tag{8}$$

3.1. Boundary conditions

$$P_1(0,t) = \lambda_L P_O(t) \tag{9}$$

$$P_2(0,t) = \lambda_M P_1(t) \tag{10}$$

$$P_3(0,t) = \lambda_M P_O(t) \tag{11}$$

$$P_4(0,t) = \lambda_T P_3(t) + \lambda_M P_5(t) \tag{12}$$

$$P_5(0,t) = \lambda_T P_O(t) \tag{13}$$

$$P_6(x,t) = \lambda_L P_4(t) \tag{14}$$

$$P_F(0,t) = \lambda_S \left[P_O(t) + P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) + P_6(t)\right] \tag{15}$$

3.2. Initial conditions

We assume that the system, initially (at t = 0) is normal i.e. $P_o(0)$ = 1 state otherwise $P_i(0) = 0$ where i = 0, 1, 2, 3 4, 5 ,6 and F. (16)

Solving Eqs. (1) through (8) by supplementary variable technique and using initial and boundary conditions, one may obtain following transition state probabilities of the system:

$$\overline{P_O}(s) = \frac{1}{I(s)} \tag{17}$$

$$\overline{P_1}(s) = \lambda_L A(s) \frac{1}{I(s)} \tag{18}$$

$$\overline{P_2}(s) = \lambda_L \lambda_M A(s) B(s) \frac{1}{I(s)} \tag{19}$$

$$\overline{P_3}(s) = \lambda_M C(s) \frac{1}{I(s)} \tag{20}$$

$$\overline{P_4}(s) = \lambda_T \lambda_M E(s) \frac{1}{I(s)} \tag{21}$$

$$\overline{P_5}(s) = \lambda_T D(s) \frac{1}{I(s)} \tag{22}$$

$$\overline{P_6}(s) = \lambda_L \lambda_M \lambda_T E(s) F(s) \frac{1}{I(s)} \tag{23}$$

$$\overline{P_F}(s) = \lambda_S G(s) \frac{1}{I(s)} \tag{24}$$

where
$$\begin{aligned} I(s) = {} & s + \lambda_L + \lambda_M + \lambda_T + \lambda_S - \lambda_L \overline{S}_{\mu_L}(s + \lambda_S + \lambda_M) \\ & - \lambda_M \lambda_L A(s) \overline{S}_{\mu}(s + \lambda_S) - \lambda_M \overline{S}_{\mu_M}(s + \lambda_S + \lambda_T) \\ & - \{C(s) + D(s)\} \lambda_M \lambda_T \overline{S}_{\eta}(s + \lambda_S + \lambda_L) - \lambda_S H(s) \overline{S}_{\mu_S}(s) \\ & - \lambda_T \overline{S}_{\mu_T}(s + \lambda_S + \lambda_M) - \lambda_L \lambda_M \lambda_T E(s) \overline{S}_{\xi}(s + \lambda_S) \end{aligned}$$

4. Evaluation of Laplace Transforms of Up and Down State Probabilities

$$\overline{P}_{up}(s) = H(s) \times \frac{1}{I(s)} \tag{25}$$

$$\overline{P}_{down}(s)=\frac{1}{s}-\overline{P}_{up}(s) \tag{26}$$

where

$$A(s)=\frac{1-\overline{S}_{\mu_L}(s+\lambda_S+\lambda_M)}{s+\lambda_S+\lambda_M}\ ;\ B(s)=\frac{1-\overline{S}_{\mu}(s+\lambda_S)}{s+\lambda_S}\ ;\ C(s)=\frac{1-\overline{S}_{\mu_M}(s+\lambda_S+\lambda_T)}{s+\lambda_S+\lambda_T}\ ;$$

$$D(s)=\frac{1-\overline{S}_{\mu_T}(s+\lambda_S+\lambda_M)}{s+\lambda_S+\lambda_M}\ ;\ E(s)=\{C(s)+D(s)\}\times\frac{1-\overline{S}_{\eta}(s+\lambda_S+\lambda_L)}{s+\lambda_S+\lambda_L}\ ;$$

$$F(s)=\frac{1-\overline{S}_{\xi}(s+\lambda_S)}{s+\lambda_S}\ ;\quad H(s)=1+\lambda_L A(s)+\lambda_L\lambda_M A(s)B(s)+\lambda_M C(s)+\lambda_T\lambda_M E(s)+\lambda_T D(s)+\lambda_L\lambda_T\lambda_M E(s)F(s);$$

$$G(s)=H(s)\times\frac{1-\overline{S}_{\mu_S}(s)}{s}$$

5. Ergodic Behaviour of the System

Using Abel's lemma in Laplace transforms,

$$\lim_{s\to 0} s\bar{f}(s)=\lim_{t\to\infty} f(t)=f(say)$$

provided the limit on the right hand side exists, the time independent operational availability and non-availability are obtained as follows :

$$P_{up}=\left[1+\lambda_L A(0)+\lambda_L\lambda_M A(0)B(0)+\lambda_M C(0)+\lambda_T\lambda_M E(0)+\lambda_T D(s)+\lambda_L\lambda_T\lambda_M E(s)F(s)\right]\times\frac{1}{I'(0)} \tag{27}$$

$$P_{down}=\lambda_S H(0)\frac{M_{\mu_S}}{I'(0)} \tag{28}$$

where $I'(0)=\left[\frac{d}{ds}I(s)\right]_{s=0}$

It would be interesting to note that $P_{up}+P_{down}=1$

6. Special Cases

6.1. Constant repair rates

When repairs follow exponential time distribution: Setting $\overline{S}_p(s)=\frac{p}{s+p}$, where $p=\mu_L,\mu,\mu_M,\eta,\mu_T,\xi,\mu_S$ in Eqs. (25) and (26), one can get

$$\overline{P}_{up}(s)=H(s)\times\frac{1}{I(s)} \tag{29}$$

$$\overline{P}_{down}(s)=\frac{1}{s}-\overline{P}_{up}(s) \tag{30}$$

where

$$H(s)=1+\lambda_L A(s)+\lambda_L\lambda_M A(s)B(s)+\lambda_M C(s)+\lambda_T\lambda_M E(s)+\lambda_T D(s)+\lambda_L\lambda_T\lambda_M E(s)F(s)$$

$$A(s)=\frac{1}{s+\mu_L+\lambda_S+\lambda_M}\ ;\ B(s)=\frac{1}{s+\mu+\lambda_S}\ ;\ C(s)=\frac{1}{s+\mu_M+\lambda_S+\lambda_T}\ ;$$

$$D(s)=\frac{1}{s+\mu_T+\lambda_S+\lambda_M}\ ;\ E(s)=\{C(s)+D(s)\}\times\frac{1}{s+\eta+\lambda_S+\lambda_L}\ ;$$

$$F(s)=\frac{1}{s+\xi+\lambda_S}\ ;\ G(s)=H(s)\times\frac{1}{s+\mu_S}\ ;$$

$$\begin{aligned}I(s)=\ & s+\lambda_L+\lambda_M+\lambda_T+\lambda_S-\lambda_L\frac{\mu_L}{s+\mu_L+\lambda_S+\lambda_M}\\ & -\lambda_M\lambda_L A(s)\frac{\mu}{s+\mu+\lambda_S}-\lambda_M\frac{\mu_M}{s+\mu_M+\lambda_S+\lambda_T}\\ & -\{C(s)+D(s)\}\lambda_M\lambda_T\frac{\eta}{s+\lambda_S+\lambda_L+\eta}-\lambda_T\frac{\mu_T}{s+\mu_T+\lambda_S+\lambda_M}\\ & -\lambda_L\lambda_M\lambda_T E(s)\frac{\xi}{s+\xi+\lambda_S}-\lambda_S H(s)\frac{\mu_S}{s+\mu_S}\end{aligned}$$

6.2. Non repairable system

If the system is non repairable then the probabilities will be independent of x and repair rates zero then the reliability function is given by

$$\begin{aligned}\overline{R}(s)=\ &\frac{1}{(s+\lambda_L+\lambda_M+\lambda_S+\lambda_T)}\Bigg[1+\frac{\lambda_L}{s+\lambda_S+\lambda_M}+\frac{\lambda_L\lambda_M}{(s+\lambda_S+\lambda_M)(s+\lambda_S)}\\ &+\frac{\lambda_M}{s+\lambda_S+\lambda_T}+\frac{\lambda_T}{s+\lambda_S+\lambda_M}+\frac{\lambda_L\lambda_M}{s+\lambda_S+\lambda_L}\left\{\frac{1}{s+\lambda_S+\lambda_T}+\frac{1}{s+\lambda_S+\lambda_M}\right\}\\ &+\frac{\lambda_L\lambda_M\lambda_T}{(s+\lambda_S)(s+\lambda_S+\lambda_L)}\left\{\frac{1}{s+\lambda_S+\lambda_T}+\frac{1}{s+\lambda_S+\lambda_M}\right\}\Bigg]\end{aligned}\tag{31}$$

where, $\overline{R}(s)$ is the Laplace transform of the reliability function.

- The reliability of the transit system is obtained as

$$\begin{aligned}\overline{R}(t)=\ & e^{-(\lambda_S+\lambda_T)t}+\frac{\lambda_M\lambda_L e^{-(\lambda_S+\lambda_L+\lambda_M+\lambda_T)t}}{(\lambda_L+\lambda_T)(\lambda_L+\lambda_T+\lambda_M)}-\frac{\lambda_L e^{-(\lambda_S+\lambda_M)t}}{(\lambda_L+\lambda_T)}\\ &+\frac{\lambda_L e^{-\lambda_S t}}{(\lambda_L+\lambda_T+\lambda_M)}+\frac{\lambda_M\left\{e^{-(\lambda_S+\lambda_T)t}-e^{-(\lambda_S+\lambda_L+\lambda_M+\lambda_T)t}\right\}}{(\lambda_L+\lambda_M)(\lambda_L+\lambda_T+\lambda_M)}\\ &+\frac{\lambda_M\lambda_T e^{-(\lambda_S+\lambda_L+\lambda_M+\lambda_T)t}}{(-\lambda_L+\lambda_T)}\left\{\frac{e^{(\lambda_S+\lambda_T)t}-1}{\lambda_M+\lambda_T}-\frac{e^{(\lambda_S+\lambda_T)t}}{\lambda_M+\lambda_L}\right\}\\ &+\frac{\lambda_M\lambda_T e^{-(\lambda_S+\lambda_L+\lambda_M+\lambda_T)t}}{(-\lambda_L+\lambda_M)}\left\{\frac{e^{(\lambda_S+\lambda_T)t}-1}{\lambda_M+\lambda_T}-\frac{e^{(\lambda_S+\lambda_T)t}-1}{\lambda_T+\lambda_L}\right\}\end{aligned}$$

$$+\frac{\lambda_M\lambda_L\lambda_T e^{-(\lambda_S+\lambda_L+\lambda_M+\lambda_T)t}}{(-\lambda_L+\lambda_M)}\left[\frac{1}{-\lambda_L}\left\{\frac{e^{(\lambda_S+\lambda_T)t}-1}{\lambda_M+\lambda_T}-\frac{e^{(\lambda_M+\lambda_L+\lambda_T)t}-1}{\lambda_M+\lambda_L+\lambda_T}\right\}\right.$$
$$\left.-\frac{1}{-\lambda_M}\left\{\frac{e^{(\lambda_T+\lambda_L)t}-1}{\lambda_T+\lambda_L}-\frac{e^{(\lambda_M+\lambda_L+\lambda_T)t}-1}{\lambda_M+\lambda_L+\lambda_T}\right\}\right]$$
$$+\frac{\lambda_M\lambda_L\lambda_T e^{-(\lambda_S+\lambda_L+\lambda_M+\lambda_T)t}}{(-\lambda_L+\lambda_T)}\left[\frac{1}{-\lambda_L}\left\{\frac{e^{-(\lambda_M+\lambda_T)t}-1}{\lambda_M+\lambda_T}-\frac{e^{-(\lambda_M+\lambda_L+\lambda_T)t}-1}{\lambda_M+\lambda_L+\lambda_T}\right\}\right.$$
$$\left.-\frac{1}{-\lambda_T}\left\{\frac{e^{(\lambda_M+\lambda_L)t}-1}{\lambda_M+\lambda_L}-\frac{e^{(\lambda_M+\lambda_L+\lambda_T)t}-1}{\lambda_M+\lambda_L+\lambda_T}\right\}\right] \tag{32}$$

- The mean time to failure (MTTF) is given as under

$$\text{MTTF}=\int R(t)dt=\frac{1}{a}+\left\{\frac{1}{\lambda_S+\lambda_M}-\frac{1}{\lambda_T+\lambda_L}\right\}+\frac{\lambda_M\lambda_L}{(\lambda_T+\lambda_L)\cdot a\cdot b}$$
$$+\frac{\lambda_L}{b\cdot\lambda_S}-\frac{\lambda_L}{\lambda_T+\lambda_L}\cdot\frac{1}{\lambda_S+\lambda_M}+\frac{\lambda_M}{\lambda_L+\lambda_M}\left\{\frac{1}{\lambda_S+\lambda_T}-\frac{1}{a}\right\}$$
$$+\frac{\lambda_T\lambda_M}{\lambda_T-\lambda_L}\left\{\frac{1}{\lambda_T+\lambda_M}\left(\frac{1}{\lambda_S+\lambda_L}-\frac{1}{a}\right)-\frac{1}{\lambda_L+\lambda_M}\left(\frac{1}{\lambda_S+\lambda_T}-\frac{1}{a}\right)\right\}$$
$$+\frac{\lambda_T\lambda_M}{\lambda_M-\lambda_L}\left\{\frac{1}{\lambda_T+\lambda_M}\left(\frac{1}{\lambda_S+\lambda_L}-\frac{1}{a}\right)-\frac{1}{\lambda_L+\lambda_T}\left(\frac{1}{\lambda_S+\lambda_M}-\frac{1}{a}\right)\right\}$$
$$+\frac{\lambda_T\lambda_M\lambda_L}{\lambda_T-\lambda_L}\left[-\frac{1}{\lambda_L}\left\{\frac{1}{\lambda_T+\lambda_M}\left(\frac{1}{\lambda_L+\lambda_S}-\frac{1}{a}\right)-\frac{1}{b}\left(\frac{1}{\lambda_S}-\frac{1}{a}\right)\right\}\right.$$
$$\left.+\frac{1}{\lambda_T}\left\{\frac{1}{\lambda_L+\lambda_M}\left(\frac{1}{\lambda_T+\lambda_S}-\frac{1}{a}\right)-\frac{1}{b}\left(\frac{1}{\lambda_S}-\frac{1}{a}\right)\right\}\right]$$
$$+\frac{\lambda_T\lambda_M\lambda_L}{\lambda_M-\lambda_L}\left[-\frac{1}{\lambda_L}\left\{\frac{1}{\lambda_T+\lambda_M}\left(\frac{1}{\lambda_L+\lambda_S}-\frac{1}{a}\right)-\frac{1}{b}\left(\frac{1}{\lambda_S}-\frac{1}{a}\right)\right\}\right.$$
$$\left.+\frac{1}{\lambda_M}\left\{\frac{1}{\lambda_L+\lambda_T}\left(\frac{1}{\lambda_M+\lambda_S}-\frac{1}{a}\right)-\frac{1}{b}\left(\frac{1}{\lambda_S}-\frac{1}{a}\right)\right\}\right] \tag{33}$$

where $a=\lambda_S+\lambda_T+\lambda_L+\lambda_M$ and $b=\lambda_T+\lambda_L+\lambda_M$

7. Cost Effectiveness of the System

Assuming that the service facility is always available, it remains busy from time‘t’ during (0, t]. Let C_1 and C_2 are revenue cost per unit time and service cost per unit time respectively, then the total expected cost $G(t)$ during the interval (0, t] is given by

$$
\begin{aligned}
G(t) &= C_1 \cdot \int_0^t P_{up}(t)dt - C_2 t \\
&= C_1 \Big[e^{-1.5826\,t} \{-0.2406688\ \cos(0.5828\,t) - 0.375736\ \sin(0.5828\,t)\} \\
&\quad - e^{-0.4432\,t} \{-1.2973\ \cos(0.5276\,t) + 0.65192\ \sin(0.5276\,t)\} \\
&\quad + e^{-0.3692} \{-4.543311\ \cos(0.2289\,t) + 0.34775\ \sin(0.2289\,t)\} \\
&\quad + 6.081232 \Big] - C_2 t
\end{aligned}
\tag{34}
$$

8. Numerical Computations

- **Analysis of operational readiness**

Setting
$\mu_L = \mu_M = \mu_T = \mu_S = 1;\ \mu = \eta = \xi = 0.8;$
$\lambda_L = 0.02;\quad \lambda_M = 0.02;\quad \lambda_S = 0.01;\quad \lambda_T = 0.03$
in Eq. (25) the operational readiness of the system is obtained as

$$
\begin{aligned}
P_{up}(t) &= 0.1619e^{-1.5826t}\cos(0.5828)t + 0.7349e^{-1.5826t}\sin(0.5828)t \\
&\quad - 0.9189e^{-0.4432t}\cos(0.5276)t - 0.3955e^{-0.4432t}\sin(0.5276)t \\
&\quad + 1.7570e^{-0.3692t}\cos(0.2289)t + 0.91158e^{-0.3692t}\sin(0.2289)t
\end{aligned}
$$

Putting $t = 0, 1, 2 \ldots$ in above equation, one can get results as illustrated in Fig. 3.

- **Reliability analysis**

Setting $\lambda_S = 0.01,\quad \lambda_L = 0.02,\quad \lambda_T = 0.03,\quad \lambda_M = 0.01$
in Eq. (32). By putting different values of t such as 5, 5.5, 6,...., one can obtain the output as shown in Fig. 4.

- **MTTF Analysis**

Setting $\lambda_L = 0.02$, $\lambda_T = 0.015$, $\lambda_M = 0.01$ in Eq. (33) and put $\lambda_S = 0.001$, 0.002, ... Figure 5 exhibits the variation of MTTF for different values of satellite failure rate.

- **Cost analysis**

Setting $C_1 = 1$ and $t = 0, 2, 4, \ldots$ in Eq. (34), the variation in costs for different service costs, $C_2 = 0.1$, 0.2 and 0.3 can easily be seen in Fig. 6.

9. Results and Discussion

Figure 3 reveals that the operational readiness of the system decreases as time passes away. Figure 4 shows that the reliability of the system decreases with passage of time. One can observe from Fig. 5 that MTTF decreases with increase in satellite failure rate. Critical examination of Fig. 6 yields that initially cost of the system increases in general with time but later on it decreases.

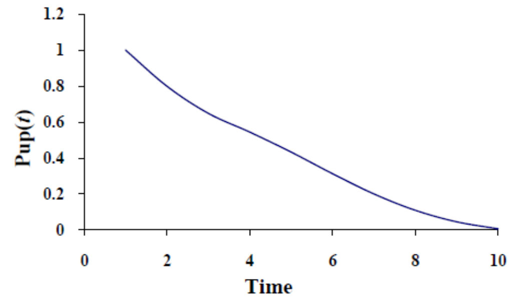

Fig. 3. Operational Readiness vs. Time.

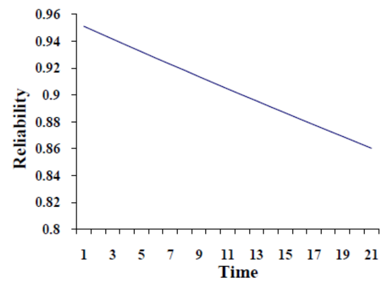

Fig. 4. Reliability vs. Time.

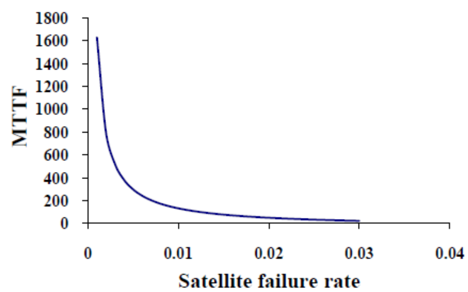

Fig. 5. MTTF vs. Satellite Failure Rate.

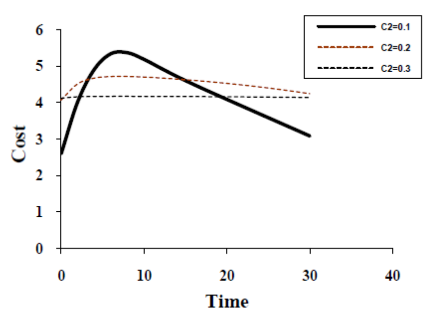

Fig. 6. Cost vs. Time.

10. Conclusions

In this paper, the operational readiness of global mobile satellite communication system is discussed using mathematical modelling approach. Also the comparative study of the reliability with time, MTTF analysis with satellite failure rate and variation of costs with respect to time is presented. The proposed method has the advantages of modelling and analysing system reliability in a more flexible and more intelligent manner.

The field of Global mobile satellite communication system is undoubtedly very vast and completely open for research at this moment. Our contribution is merely a step forward and an effort to explore such important technological field with viable implementation technique.

References

1. Cappelle, B.; and Kerre, E.E. (1993). On a possibilistic approach to reliability theory. *Second International Symposium on Uncertainty Modeling and Analysis*, 415-418.
2. Bazovsky, I. (2004). *Reliability Theory and Practice.* Dover Publications, Mineole, New York.
3. Birolini, A. (2007). *Reliability engineering: Theory and practice.* (5th Ed.), Springer.
4. Chen, T-H.; Lee, W-B.; and Chen, H-B. (2009). A self-verification authentication mechanism for mobile satellite communication systems. *Computers and Electrical Engineering*, 35(1), 41-48.
5. Egan, B.L. (1997). The role of wireless communications in the global information infrastructure. *Telecommunication Policy*, 21(5), 357-385.
6. Thomas, R.W.; Raines, R.A.; Baldwin, R.O.; and Temple, M.A. (2002). Performance analysis of multicast algorithms for mobile satellite communication networks. *Computer Communications*, 25(11-12), 1085-1093.
7. Dhillon, B.S.; and Viswanath, H.C. (1991). Reliability analysis of a non-identical unit parallel system with common cause failures. *Microelectronics Reliability*, 31(2-3), 429-441.
8. Azaron, A.; Katagiri, H.; Kato, K.; and Sakawa, M. (2007). A multi-objective discrete reliability optimization problem for dissimilar-unit standby systems. *OR Spectrum*, 29(2), 235-257.
9. Azaron, A.; Katagiri, H.; Kato, K.; and Sakawa, M. (2006). Reliability evaluation of multicomponent cold-standby redundant systems. *Applied Mathematics and Computations*, 173(1), 137-149.
10. Cappelle, B.; and Kerre, E. (1995). A general possibilistic framework for reliability theory. *LNCS*, *Advances in Intelligent Computing- IPMU* '94, 311-317.
11. Balasekar, S.; and Ansari, N. (1993). Adaptive map configuration and dynamic routing to optimise the performance of satellite communication network. *IEEE Global Telecommunication Conference including a Communications Theory Mini-Conference*, 2, 986-990.