Journal of Engineering Science and Technology
Vol. 5, No. 1 (2010) 52 - 66

SIMPLY SUPPORTED BEAM RESPONSE ON ELASTIC FOUNDATION CARRYING REPEATED ROLLING CONCENTRATED LOADS

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Abstract

The response of a simply-supported beam on elastic foundation to repeated moving concentrated loads is obtained by means of the Fourier sine transformation. The cases of the response of the beam to loads of different and equal magnitude are studied. Numerical examples are given in order to determine the effects of various parameters on the response of the beam.

Keywords: Simply supported beam, Elastic Foundation, Repeated rolling concentrated loads.

1. Introduction

The group of problems on beams carrying moving loads has a long history in the literature of engineering mechanics, which have been reviewed by a number of authors. However, some discrepancies exist among existing papers, and some of the results are less than conclusive.

Volterra [1] describes a method for analyzing the response of railroad tracks to a moving concentrated load, and determines the maximum dynamic deflection and theoretical critical speed (1168 mph) which is more than ten times the critical speed determined experimentally by Inglis [2].

Inglis [2] studied the dynamic effects on railway bridges, produced by the action of locomotives and other moving loads. Inglis's study includes oscillations produced by stationary but alternating distributed loads, moving loads of constant magnitude, moving alternating force, moving alternating force associated with concentrated moving mass, the spring movement of a locomotive, and vector methods for computing oscillations due to alternating forces.

Nomenclatures

A_j, B_j, C_j, D_j	Constant coefficients of the series representation of the beam deflection
d	Distance between repeated moving concentrated loads
$\bar{d}$	Dimensionless distance parameter, $\bar{d} = d / l$
E	Young's modulus
f_i	Frequency of the forcing function, $f = v/2$
f_j	Fundamental frequency of the simple-supported beam
g	Acceleration of gravity
I	Area moment of inertia of a beam
j	Index 1, 2, 3, …
k_e	Elastic foundation modulus
l	Beam length
m	Mass of the beam per unit length
$P(x,t)$	Moving concentrated load
P_i	Constant magnitude of moving concentrated load, $i = 0, 1$
$\bar{P}$	Dimensionless parameter, $\bar{P} = P_i / mg$
t	Time
$\bar{X}$	Dimensionless parameter, $\bar{X} = x / l$
$\bar{X}_0$	Dimensionless load location parameter, $\bar{X}_0 = x_0 / l$
x	Spatial coordinate along the beam
x_0	Initial location of the load
$Y(j,t)$	Fourier sine finite integral transformation of function $y(x,t)$
$y(x,t)$	Beam deflection
$y_s(x)$	Static deflection produced by load P_i
$\bar{y}$	Dimensionless beam deflection, $\bar{y} = y(x,t)/(\delta_e)_{\max}$

Greek Symbols

α_e	Dimensionless elastic foundation parameter, $\alpha_e = k_e l^4 / EI$
$(\delta_e)_{max}$	Maximum static deflection of simple-supported beam on an elastic foundation due to load P_i
ν	Constant speed of the moving concentrated load
$(\nu_{cr})_j$	Critical speed
$\bar{v}$	Dimensionless speed parameter, $\bar{v} = (v_{cr})_1 / v$
ϕ	Angle, $\phi = j\pi d / l$
ϕ_D	Dynamic amplifications
ω	Circular frequency of the forcing function
ω_j	Fundamental circular frequency of the simple-supported beam

Nelson and Conover [3] have studied the stability of a simply supported beam on an elastic foundation carving a continuous series of equally spaced mass particles. The results were obtained using Floquet theory and stability curves are presented for one, five and ten mass particles.

The most notable experimental results in the area of moving loads are those presented by Ayre et al. [4]. The authors used an apparatus capable of moving loads of various weights with different velocities across both single-span and double–span beams. The single–span results were compared with the theoretical results of Schallenkamp [5] and found to be in very close agreement.

And also, Lee [6] studied the dynamic behaviour of an Euler beam traversed by a moving concentrated mass, he analyzed for the general case of a mass moving with a varying speed. The equation of motion in a matrix form was formulated using the Lagrangian approach and the assumed mode method.

Moreover, Seong-Min Kim [7] studied the vibration and stability of an infinite Bernoulli–Euler beam resting on a Winkler-type elastic foundation when the system is subjected to a static axial force and a moving load with either constant or harmonic amplitude variations. Formulations were developed in the transformed field domains of time and moving space.

The object of this study is to obtain the response of a simply – supported beam on an elastic foundation to repeated moving concentrated load by means of the finite Fourier sine transformation.

2. Formulation of the Governing Equations

Now, to illustrate how one would obtain the response of a simply–supported beam on an elastic foundation to repeated moving concentrated loads, the beam shown in Fig. 1 is considered. As indicated in Fig. 1, the loads are *d* distance apart, $0 \le d < l$, and move with constant speed v from left to right.

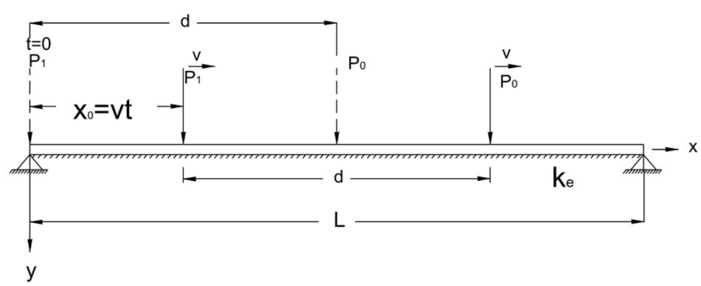

Fig. 1. Simply-Supported Beam on a Elastic Foundation Carrying Repeated Moving Concentrated Loads.

The response and its time rate of change of a simply–supported beam on an elastic foundation to a moving concentrated load have been given as [8,9]:

$$y(x,t) = \frac{P_0}{2\pi^2 ml} \sum_{j=1}^{\infty} \left[\frac{\sin 2\pi jft - (jf / f_j) \sin 2\pi f_j t}{f_j^2 - (jf)^2} \right] \sin \frac{j\pi x}{l} \tag{1}$$

and

$$\frac{\delta y(x,t)}{\delta t}=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{2\pi jf\left(\cos 2\pi jft-\cos 2\pi f_j t\right)}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{2}$$

respectively.

The instant that the first load P_0 is d distance from the left support, the Eqs. (1) and (2) become

$$y\left(x,\frac{d}{v}\right)=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{\sin 2\pi jf\frac{d}{v}-\left(jf/f_j\right)\sin 2\pi f_j\frac{d}{v}}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{3}$$

$$\left.\frac{\delta y(x,t)}{\delta t}\right|_{t=\frac{d}{v}}=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{2\pi jf\left(\cos 2\pi jf\frac{d}{v}-\cos 2\pi f_j\frac{d}{v}\right)}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{4}$$

If the time t is measured from this instant, the differential equation governing the transverse motion of the beam may be written in the form of

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+m\frac{\partial^2 y(x,t)}{\partial t^2}+k_e y(x,t)=P_1\delta(x-vt)+P_0\delta[x-(vt+d)],0\le t\le\frac{l-d}{v} \tag{5}$$

with the boundary and initial conditions respectively becoming

$$y(x,t)=0;\ \frac{\partial^2 y(x,t)}{\partial t^2}=0 \text{ at } x=0 \text{ and } x=l \tag{6}$$

and

$$\left.y(x,t)\right|_{t=0}=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{\sin 2\pi jf\frac{d}{v}-\left(jf/f_j\right)\sin 2\pi f_j\frac{d}{v}}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{7}$$

$$\left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0}=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{2\pi jf\left(\cos 2\pi jf\frac{d}{v}-\cos 2\pi f_j\frac{d}{v}\right)}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{8}$$

Equation (5) together with conditions (6), (7) and (8) will be solved by the method of finite Fourier transformation. A knowledge of even–order derivatives at the boundaries calls for a finite sine transform defined by [9]

$$Y(j,t)=\int_0^l y(x,t)\sin\frac{j\pi x}{l}dx \tag{9}$$

$$y(x,t)=\frac{2}{t}\sum_{j=1}^{\infty}Y(j,t)\sin\frac{j\pi x}{l} \tag{10}$$

where $Y(j,t)$ is the transform of the original $y(x,t)$.

Multiplying each term of Eq. (5) by $\sin\frac{j\pi x}{l}$ and then integrating with respect to x between 0 and l, and taking into account the boundary conditions (6), one obtains the finite sine transform of Eq. (5) as

$$EI\frac{\pi^4 j^4}{l^4}Y(j,t)+m\frac{d^2Y(j,t)}{dt^2}+k_eY(j,t)=P_1\sin\frac{j\pi vt}{l}+P_0\sin\frac{j\pi(vt+d)}{l} \tag{11}$$

Now, denote the circular frequencies of the beam by

$$\omega_j^2=\frac{EI\pi^4 j^4+k_el^4}{l^4m} \tag{12}$$

the corresponding fundamental frequencies by

$$f_j=\frac{\omega_j}{2\pi}=\frac{1}{2\pi l^2}\left[\frac{EI\pi^4 j^4+k_el^4}{m}\right]^{1/2} \tag{13}$$

and the circular frequencies of the forcing function by

$$\omega=\frac{j\pi v}{l}. \tag{14}$$

Using above notation Eq. (11) becomes

$$\frac{d^2Y(j,t)}{dt^2}+\omega_j^2Y(j,t)=\frac{P_1}{m}\sin\omega t+\frac{P_0}{m}\sin(\omega+\phi)t \tag{15}$$

or

$$\frac{d^2Y(j,t)}{dt^2}+\omega_j^2Y(j,t)=\left[\frac{P_1}{m}+\frac{P_0}{m}\cos\phi\right]\sin\omega t+\frac{P_0}{m}\sin\phi\cos\omega t \tag{16}$$

where

$$\phi=\frac{j\pi d}{l}. \tag{17}$$

The homogenous solution of Eq. (16) is given by

$$Y_H(j,t)=A_j\sin\omega_jt+B_j\cos\omega_jt \tag{18}$$

Assume a particular solution of the form

$$Y_P(j,t)=C_j\sin\omega t+D_j\cos\omega t \tag{19}$$

Substituting Eq. (19) into (16) equating coefficients of $\sin\omega t$ and $\cos\omega t$ yield

$$C_j=\frac{P_1+P_0\cos\phi}{m\left(\omega_j^2-\omega^2\right)},\ j=1,2,3,... \tag{20}$$

and

$$D_j = \frac{P_0 \sin\phi}{m\left(\omega_j^2 - \omega^2\right)}, \; j = 1,2,3,... \tag{21}$$

Hence, the complete solution of Eq. (16) becomes

$$Y(j,t) = A_j \sin\omega_j t + B_j \cos\omega_j t + \frac{P_1 + P_0 \cos\phi}{m\left(\omega_j^2 - \omega^2\right)} \sin\omega t + \frac{P_0 \sin\phi}{m\left(\omega_j^2 - \omega^2\right)} \cos\omega t \tag{22}$$

The response $y(x,t)$ is obtained by writing the inverse transform of Eq. (22),

$$y(x,t) = \frac{2}{l} \sum_{j=1}^{\infty} \sin\frac{j\pi x}{l} \left[A_j \sin\omega_j t + B_j \cos\omega_j t + \frac{P_1 + P_0 \cos\phi}{m\left(\omega_j^2 - \omega^2\right)} \sin\omega t + \frac{P_0 \sin\phi}{m\left(\omega_j^2 - \omega^2\right)} \cos\omega t \right] \tag{23}$$

Coefficients A_j and B_j are to be determined from initial conditions (7) and (8).

From solution (23)

$$y(x,0) = \frac{2}{l} \sum_{j=1}^{\infty} \left[B_j + \frac{P_0 \sin\phi}{m\left(\omega_j^2 - \omega^2\right)} \right] \sin\frac{j\pi x}{l} \tag{24}$$

and

$$\left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0} = \frac{2}{l} \sum_{j=0}^{\infty} \left[\omega_j A_j + \frac{(P_1 + P_0 \cos\phi)\omega}{m\left(\omega_j^2 - \omega^2\right)} \right] \sin\frac{j\pi x}{l} \tag{25}$$

Using Eqs. (7), (8), (24) and (25) one obtains the coefficients A_j and B_j as

$$A_j = \frac{\omega/\omega_j}{m\left(\omega_j^2 - \omega^2\right)} \left[P_0 \left(\cos\omega\frac{d}{v} - \cos\omega_j \frac{d}{v} \right) - P_0 \cos\phi - P_1 \right] \; j = 1,2,3,... \tag{26}$$

and

$$B_j = \frac{P_0}{m\left(\omega_j^2 - \omega^2\right)} \left[\sin\omega\frac{d}{v} - \frac{\omega}{\omega_j} \sin\omega_j \frac{d}{v} - \sin\phi \right] \; j = 1,2,3,.... \tag{27}$$

Introducing Eqs. (26) and (27) in Eq. (23) and using Eqs. (12), (13), (14), and (17), the response of the beam takes the form

$$y(x,t) = \frac{P_0}{2\pi^2 ml} \sum_{j=1}^{\infty} \frac{\sin\frac{j\pi x}{l}}{f_j^2 - (jf)^2} \left[\frac{P_1}{P_0} \sin 2\pi jft + \sin\left(\frac{j\pi d}{l} + 2\pi jft \right) \right.$$
$$\left. - \frac{jf}{f_j} \left(\frac{P_1}{P_0} \sin 2\pi f_j t + \sin 2\pi f_j \left(t + \frac{d}{v} \right) \right) \right]. \tag{28}$$

Substitution of $t = \frac{x_0}{v}$ and $v = 0$ into Eq. (18) yield corresponding static solution as

$$y_s(x) = \frac{2P_0 l^3}{EI} \sum_{j=1}^{\infty} \frac{\sin \frac{j\pi x}{l}}{(j\pi)^4 + \frac{k_e l^4}{EI}} \left[\frac{P_1}{P_0} \sin \frac{j\pi x_0}{l} + \sin\left(\frac{j\pi d}{l} + \frac{j\pi x_0}{l} \right) \right] \tag{29}$$

For the loads of equal magnitude, $P_0 = P_1$, the dynamic and static response given by Eqs. (28) and (29) become

$$y(x,t) = \frac{P_0}{2\pi^2 ml} \sum_{j=1}^{\infty} \frac{\sin \frac{j\pi x}{l}}{f_j^2 - (jf)^2} \left[\sin 2\pi jft + \sin\left(\frac{j\pi d}{l} + 2\pi jft \right) - \frac{jf}{f_j} \left(\sin 2\pi f_j t + \sin 2\pi f_j \left(t + \frac{d}{v} \right) \right) \right] \tag{30}$$

and

$$y_s(x) = \frac{2P_0 l^3}{EI} \sum_{j=1}^{\infty} \frac{\sin \frac{j\pi x}{l}}{(j\pi)^4 + \frac{k_e l^4}{EI}} \left[\sin \frac{j\pi x_0}{l} + \sin\left(\frac{j\pi d}{l} + \frac{j\pi x_0}{l} \right) \right] \tag{31}$$

respectively.

When the distance between the two loads is zero, $d = 0$, solutions (30) and (31) respectively reduce to

$$y(x,t) = \frac{P_0}{\pi^2 ml} \sum_{j=1}^{\infty} \frac{\sin \frac{j\pi x}{l}}{f_j^2 - (jf)^2} \left[\sin 2\pi jft - \frac{jf}{f_j} \sin 2\pi f_j t \right] \tag{32}$$

and

$$y_s(x) = \frac{4P_0 l^3}{EI} \sum_{j=1}^{\infty} \frac{\sin \frac{j\pi x}{l}}{(j\pi)^4 + \frac{k_e l^4}{EI}} \sin \frac{j\pi x_0}{l} \tag{33}$$

which corresponds to the case of a simply-supported beam on an elastic foundation carrying a moving concentrated load whose magnitude is double that of the one given in [9].

For the case when the distance between the loads is equal to the beam length, that is one load is leaving the beam as the other begins to travel onto the beam at the left end and if the time *t* is remeasured from this instant, the differential equation governing the transverse motion of the beam may be written in the form

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + m \frac{\partial^2 y(x,t)}{\partial t^2} + k_e y(x,t) = P_1 \delta(x - vt), \; 0 \le t \le \frac{l}{v} \tag{34}$$

with the boundary and initial conditions becoming

$$y(x,t)=0;\ \frac{\partial^2 y(x,t)}{\partial x^2}=0 \text{ at } x=0 \text{ and } x=l, \tag{35}$$

and

$$y(x,t)\Big|_{t=0}=-\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{(jf/f_j)^2\sin 2\pi f_j \frac{l}{v}}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{36}$$

$$\frac{\partial y(x,t)}{\partial t}\Bigg|_{t=0}=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\left[\frac{2\pi jf\left((-1)^j-\cos 2\pi f_j \frac{l}{v}\right)}{f_j^2-(jf)^2}\right]\sin\frac{j\pi x}{l} \tag{37}$$

respectively.

Transforming (34) in accordance with (9) and using (12), (13), (14) and boundary conditions (35) give

$$\frac{d^2 Y(j,t)}{dt^2}+\omega_j^2 Y(j,t)=\frac{P_1}{m}\sin\omega t. \tag{38}$$

The homogeneous solution of Eq. (38) is given by

$$Y_H(j,t)=A_j\sin\omega_j t+B_j\cos\omega_j t. \tag{39}$$

Assume a particular solution of the form

$$Y_P(j,t)=C_j\sin\omega t. \tag{40}$$

Substituting (40) into (38), and equating coefficients of $\sin\omega t$ and $\cos\omega t$ yield

$$C_j=\frac{P_1}{m\left(\omega_j^2-\omega^2\right)}, \qquad j=1,2,3,\ldots. \tag{41}$$

Thus, the complete solution of Eq. (38) become

$$Y(j,t)=A_j\sin\omega_j t+B_j\cos\omega_j t+\frac{P_1}{m\left(\omega_j^2-\omega^2\right)}\sin\omega t. \tag{42}$$

The response $y(x,t)$ is obtained by writing the inverse transform of (42),

$$y(x,t)=\frac{2}{l}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{l}\left[A_j\sin\omega_j t+B_j\cos\omega_j t+\frac{P_1}{m\left(\omega_j^2-\omega^2\right)}\sin\omega t\right]. \tag{43}$$

Coefficients A_j and B_j are to be determined from initial conditions (36) and (37). From Eq. (43)

$$y(x,0)=\frac{2}{l}\sum_{j=1}^{\infty}B_j\sin\frac{j\pi x}{l}, \tag{44}$$

and

$$\left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0}=\frac{2}{l}\sum_{j=1}^{\infty}\left[\omega_j A_j+\frac{P_1\omega}{m\left(\omega_j^2-\omega^2\right)}\right]\sin\frac{j\pi x}{l}. \tag{45}$$

Using Eqs. (36), (37), (44), and (45) one obtains the coefficients A_j and B_j as

$$A_j=\frac{\omega/\omega_j}{m\left(\omega_j^2-\omega^2\right)}\left[P_0\left((-1)^j-\cos\omega_j\frac{l}{v}\right)-P_1\right],\ j=1,2,3,... \tag{46}$$

And

$$B_j=-\frac{P_0\omega/\omega_j}{m\left(\omega_j^2-\omega^2\right)}\sin\omega_j\frac{l}{v}, \qquad j=1,2,3,\ldots. \tag{47}$$

Introducing (46) and (47) into (43) and recalling (12) and (13) the response of the beam takes the form

$$y(x,t)=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\frac{\sin\frac{j\pi x}{l}}{f_j^2-(jf)^2}\left\{\frac{P_1}{P_0}\sin 2\pi jft-\frac{jf}{f_j}\left[-(-1)^j+\frac{P_1}{P_0}\right]\sin 2\pi f_j t+\sin 2\pi f_j\left(t+\frac{l}{v}\right)\right\}. \tag{48}$$

For the loads of equal magnitude, $P_0=P_1$, the response given by Eq. (48) becomes

$$y(x,t)=\frac{P_0}{2\pi^2 ml}\sum_{j=1}^{\infty}\frac{\sin\frac{j\pi x}{l}}{f_j^2-(jf)^2}\left\{\sin 2\pi jft-\frac{jf}{f_j}\left[1-(-1)^j\sin 2\pi f_j t+\sin 2\pi f_j\left(t+\frac{l}{v}\right)\right]\right\}. \tag{49}$$

3. Results and Discussion

In this section numerical examples are treated to illustrate the procedure, and the effects of some parameters are investigated.

For the purpose of further discussion it is convenient to introduce the dimensionless parameters as:

$$\overline{P}=\frac{P_0}{mgl};\qquad \alpha_e=\frac{k_e l^4}{EI};\qquad \overline{x}_0=\frac{x_0}{l};\qquad \overline{x}=\frac{x}{l}$$

$$y = \frac{y(x,t)}{(\delta_e)_{max}}; \qquad \bar{v} = \frac{(V_{er})_1}{v}; \ \bar{d} = \frac{d}{l}$$

where α_e is the elastic foundation parameter, $\bar{v}$ is the speed parameter, $\bar{x}_0$ is the load parameter, and $\bar{d}$ is the distance parameter.

To validate the method, a simply supported beam without an elastic foundation $\alpha_e = 0$ subjected to a concentrated force $\bar{d} = 0$ moving with constant velocity, is analyzed and the results are compared with those from the existing finite element analysis of Lin and Trethewey [10].

Results for the dynamic amplification factors ϕ_D, defined as the ratio of the maximum dynamic and static deflections at the centre of the beam, are computed and compare in Table 1, for different values of t/τ, where τ denotes the travelling time of the force moving from the left end of the beam to the right end, while t denotes the time after the moving load enters the beam from the left end.

Table 1. Dynamic Amplifications, ϕ_n .

t/τ	This study	Lin and Trethewey [10]
0.1	1.040	1.053
0.5	1.250	1.252
1.0	1.710	1.705
1.234	1.740	1.730
1.5	1.710	1.704
2.0	1.550	1.550

It is seen that present results have quit well agreement with the results of the others and thereby confirm the validity of the proposed procedure.

Then, in order to investigate the dependence of the beam response on elastic foundation, speed, load location, and distance parameters, two concentrated loads of $\bar{P} = 9.0$ are moved at various speeds over a beam of 30’ in length for different values of the elastic foundation and distance parameters.

Figures 2 through 10 show the dependence of the beam response on the distance parameter, $\bar{d}$, for the values of α_e= 0.0, 100, 1000, $\bar{v} = 2.0$, 4.0 and 256.

It can be seen from the present results that within the range of values that have been considered, an increase in speed parameter results in the increase of dynamic deflections, and also a decrease in distance parameter also results in the increase of dynamic deflections. Furthermore, by increasing elastic foundation parameter both the fundamental frequencies and the critical speeds of the beam increase.

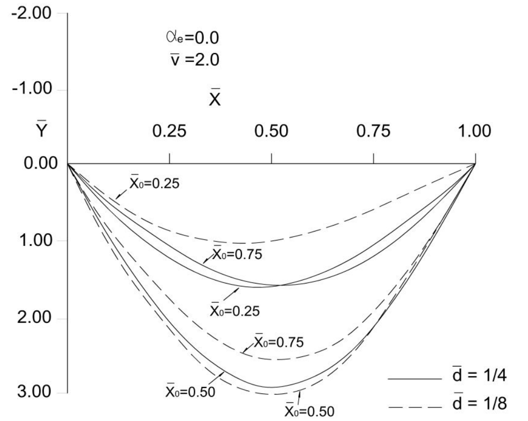

Fig. 2. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 0.0, \bar{v} = 2.0, \overline{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

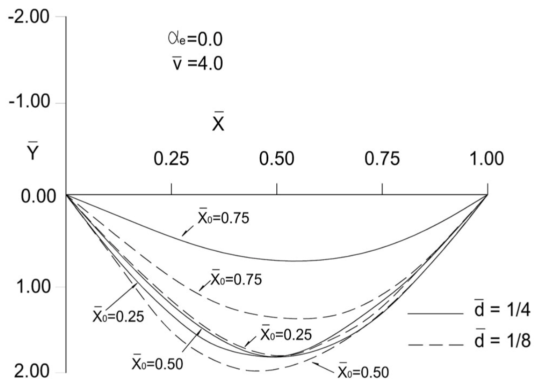

Fig. 3. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 0.0, \bar{v} = 4.0, \overline{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

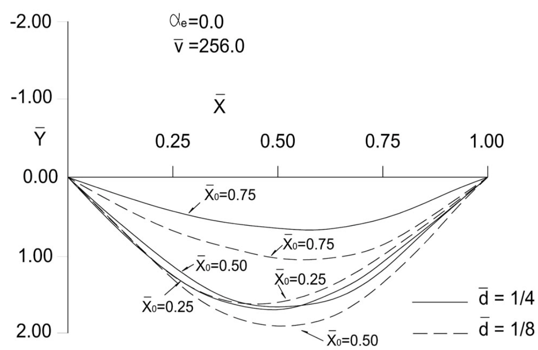

Fig. 4. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 0.0, \bar{v} = 256.0, \overline{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

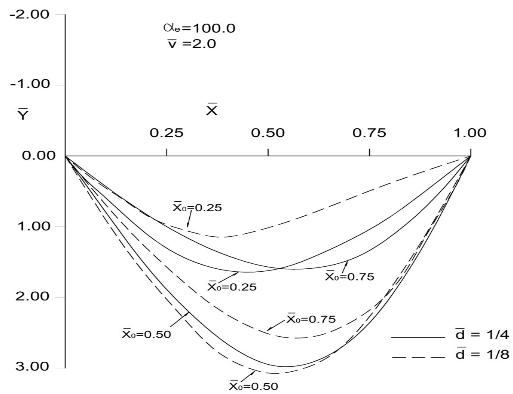

Fig. 5. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 100.0, \bar{v} = 2.0, \bar{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

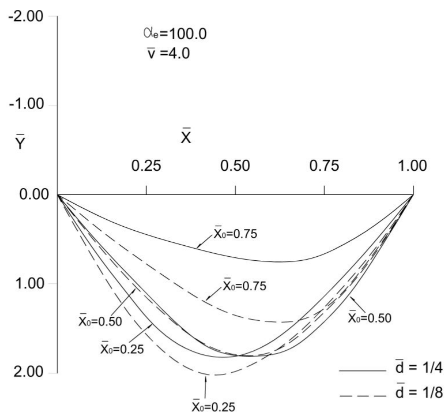

Fig. 6. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 100.0, \bar{v} = 4.0, \bar{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

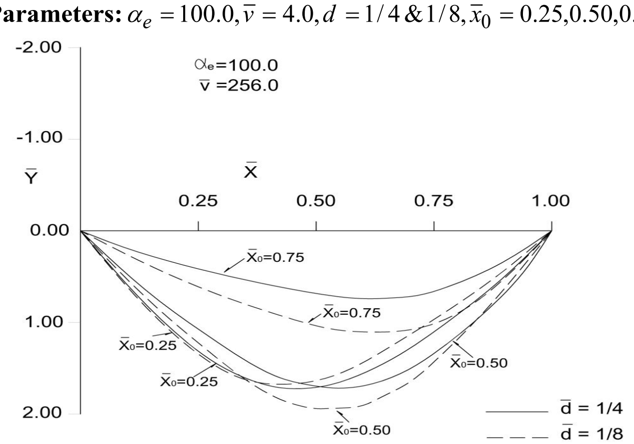

Fig. 7. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 100.0, \bar{v} = 256.0, \bar{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

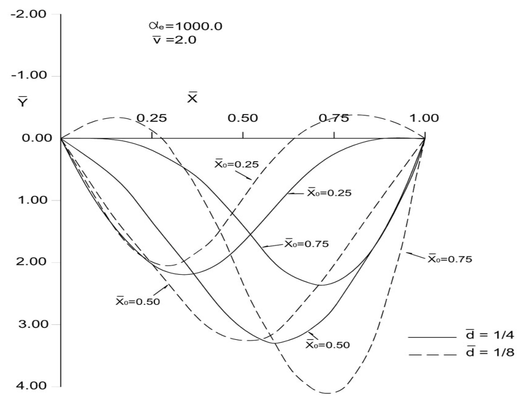

Fig. 8. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 1000.0, \bar{v} = 2.0, \bar{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

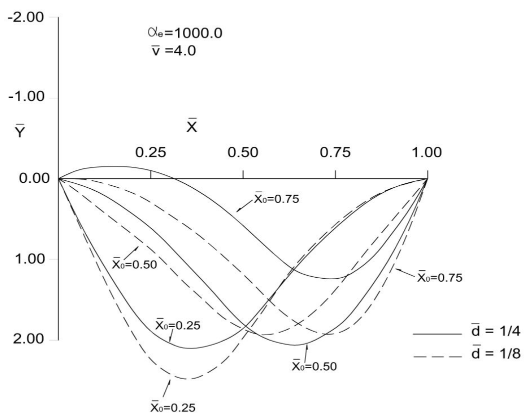

Fig. 9. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 1000.0, \bar{v} = 4.0, \bar{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

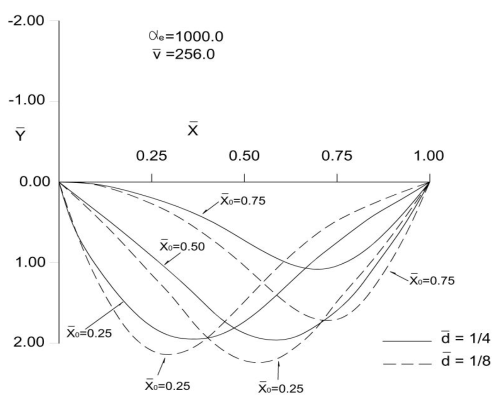

Fig. 10. The Dependence of the Beam Response on Distance.
Parameters: $\alpha_e = 1000.0, \bar{v} = 256.0, \bar{d} = 1/4 \,\&\, 1/8, \bar{x}_0 = 0.25, 0.50, 0.75$.

4. Conclusions

The response of a simply-supported beam on elastic foundation to repeated moving concentrated load by means of the Fourier sine transformation has been presented in this paper. This technique will be attractive for treating beams on an elastic foundation under moving loads, and it can be extended to treat railway track structures. The effects of some important parameters, such as the foundation stiffness, distance parameter, the travelling speed and also the cases of the response of the beam to loads of different and equal magnitude have been studied. Numerical examples are given in order to determine the effects of various parameters on the response of the beam and the major results have been taken in this study can be expressed as:

- Within the range of values considered an increase in speed parameter results in the increase in dynamic deflections.
- Given all other parameters, a decrease in distance parameter also results the increase in dynamic deflections.
- A comparison of Figs. 2 through 10 reveals that, as the speed parameter increases, the dynamic deflections approach those for the corresponding static ones and become symmetric for $\bar{x}_0 = 0.25$; $\bar{x}_0 = 0.50$ when $\bar{d} = 1/4$ as expected.
- It is readily apparent that the effect of increasing elastic foundation parameter is an increase to both the fundamental frequencies and the critical speeds of the beam.

This study contains useful contributions to the literature on moving loads problems particularly relation to transportation systems; therefore, the technique and the findings can be useful in practical applications such as railway track design.

5. Future Work

Future work will be aimed to investigate of unbounded beam response to a moving body on elastic foundation.

Acknowledgments

The authors would also like to express their gratitude to Saeid Kalami for his assistance, and help during all phases of this research endeavour.

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