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INTERFERENCE REJECTION OF SIGNALS BY ADAPTIVE MINIMUM MEAN SQUARE ERROR CRITERION OVER RAYLEIGH FADING CHANNELS

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Abstract

Channel time-variation (or fading) is the major source of impairment in digital wireless communications. This occurs due to mobility of the user or of the objects in the propagation environment. The limited spectral bandwidth necessitates the use of resource sharing schemes between multiple users. As the transmission medium is shared between the users, this leads to interference between the users. Sharing of resource results in interference such as multiple access interference. This paper deals with methods to study and mitigate such interference considering Rayleigh fading channels. There are various classes of fading conditions. The use of CDMA is under active research as a viable alternative to TDMA and FDMA. Performance in this system is limited by narrowband and multiple access interference. Various methods are used to mitigate them. But here, linear MMSE detector is considered. MMSE technique results in interference rejection. Its adaptive form is applied to Rayleigh fading channels, which are reflective and nondispersive. It results into better results than before.

Keyword: MMSE (Minimum mean square error), SIR (signal to interference ratio), Rayleigh fading channels, CDMA (Code division multiple access), MAI (multiple access interference).

1. Introduction

Wireless communications is one of the most active areas of research for technology enhancement of current times. Videos, images, text and data can be transmitted with its development. As a result of its progress, the demand for transmitted power and bandwidth is increasing. But these two resources are severely limited in the deployment of modern wireless networks. So current

Nomenclatures

A	Received complex amplitude, V
$a_j[k]$	*k*th element of *j*th user's spreading sequence
$\hat{b}$	Detected bit
$\hat{b}_1(m)$	Differentially decoded bit
$b_{i,j}$	*i*th bit of *j*th user
C^N	Autocorrelation matrix of dimension *N*
$\bar{c}$	Canonical representation of MMSE linear detector
$d(m)$	Differentially encoded bit
E	Expected value
E_b	Bit energy, W
$e(m)$	Error signal
H	Hermitian transpose (Conjugate transpose)
$h_{k,l}$	Channel impulse response
$J(m)$	Mean square error
K	Number of users
L	Maximum channel order = $\max\{L_k\}$
L_k	Delay spread/bit interval
l	Available resolved multipath components
M	Number of data symbols per user
m	$\in\{0,1,\ldots, M\text{-}1\}$
N	Processing gain
N_0	Noise power, W
$n(m)$	Residual MAI plus noise, W
$n(t)$	Noise, W
$\tilde{n}(m)$	Residual MAI plus noise at the output of the MMSE filter
P_e	Probability of error
P_j	Power of *j*th user, W
Q_k	Number of paths for the *k*th signal
R	Covariance matrix
$r(t)$	Received signal, W
$\tilde{r}(m)$	MAI + noise,W
S	Set of all users signalling waveforms
SIR	Ratio of desired user signal power to sum of powers due to noise and multiple access interference
S	Set of all users signalling waveforms
s	Signature sequence of user
s_j	Spreading waveform
T	Interval of a bit, s
T_c	Duration of a chip, s
U_j	Power of *j*th user, W
u	Signal vector
v_j	Delay of *j*th user signal, s
w_1	Weight vector of MMSE detector
w_c	Carrier frequency, Hz
x_1	Autocorrelation matrix of dimension *K*, $\in C^K$
z	Detection statistic

Greek Symbols

α	Complex fading process
Γ	SNR (Signal to noise ratio)
γ	Fading parameter
θj	Phase of *j*th user carrier, deg.
$\mu_{k,r}$	Equivalent amplitude of *k*th user's signal at output, V
σ^2	Power spectral density
τ	Relative delay, s
$\psi(t)$	Chip waveform

Abbreviations

CDMA	Code division multiple access
FDMA	Frequency division multiple access
FIR	Finite impulse response
IIR	Infinite impulse response
MAI	Multiple access interference
MMSE	Minimum mean square error
MSE	Mean square error
MSIR	Maximum signal to interference ratio
SIR	Signal to interference ratio
TDMA	Time division multiple access

effort in recent years is aimed at developing new wireless capacity through the deployment of greater intelligence in wireless networks.

To obtain maximal benefit from these transmission techniques, advanced receiver signal processing techniques such as channel equalization, and multi-user detection to mitigate multiple access interference are deployed. Here, rejection of interference by MMSE criterion is used. It is applied to fading channels and is found to suppress narrowband and multiple access interference.

2. Method

The MMSE linear detector under the effect of interfering data signals has a bank of filters matched to the pulse shape of all users followed by symbol-rate samplers and IIR digital filter as shown in Fig.1.

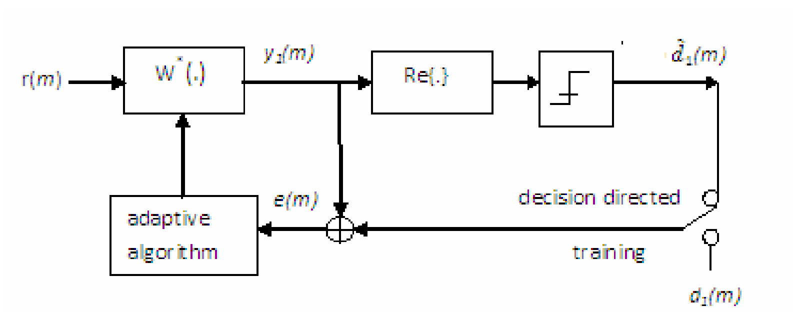

Fig. 1. Block Diagram of MMSE Receiver.

The channel output is sampled at the chip rate, and an N_tap adaptive FIR filter is used to minimize the mean squared error (MSE) between the transmitted and detected symbol. The detector can be implemented as an infinite-length fractionally spaced tapped-delay line. Even in the chip- and symbol-asynchronous situation, the N_tap MMSE detector has a far better performance than the matched filter.

The linear MMSE detector [2] can be described by a weight vector w_1 such that

$$w_1 \overset{\Delta}{=} m_1 \in C^N \tag{1}$$

designed to minimize MAI at the detector output [3]. The linear MMSE detector for user 1 is given by

$$m_1 = S\left(R + \sigma^2 \mid A \mid^{-2}\right)^{-1}$$

The linear detector must lie in the column space of S, therefore,

$$m_1 = Sx_1$$

for some $x_1 \in C^K$.

$$\| m_1 \|^2 = \left[\left(R + \sigma^2 \mid A \mid^{-2}\right)^{-1} R\left(R + \sigma^2 \mid A \mid^{-2}\right)^{-1}\right]_{1,1} \tag{2}$$

The linear MMSE detector output contains some residual MAI. Generally,

$$\| m_1 \| < \| d_1 \| \tag{3}$$

So that effects of ambient noise are reduced by the linear MMSE detector.

The received signal due to the jth user is given by the following expression as the sum of K simultaneous CDMA transmissions and additive white Gaussian noise.

$$r_j(t) = \sqrt{2U_j} \sum_{i=-\infty}^{\infty} b_{i,j} s_j\left(t - iT - v_j\right) \cos\left(w_c t + \theta_j\right) \qquad 1 \le j \le K \tag{4}$$

where $s_j(t)$ is the spreading waveform given by

$$s_j(t) = \sum_{k=0}^{N-1} a_j[k] \psi\left(t - kT_c\right) \tag{5}$$

There are three parameters, which are measures of the performance of the linear MMSE detector. These are Mean Square Error (MSE), Signal to Interference Ratio (SIR) and error probability. These can be defined as under [4]. The symbol b_1 can be detected given the received vector be

$$r = \sum_{j=1}^{L} b_j u_j + n \tag{6}$$

where $r \in R^M$. The set of interference vectors is

$$\left\{\sqrt{U_j} a_{0,j}, \sqrt{U_j} a_{-1,j}\right\}, \qquad j = 2, \ldots\ldots, K \tag{7}$$

The number of interference vectors L-1 can range from K-1 to 2(K-1). The detection algorithm has the form

$$\hat{b} = \operatorname{sgn}\left(c^T r\right) \tag{8}$$

where $c \in R^M$ is so chosen as to minimize the mean squared error.

$$\text{MSE} = E\left\{\left(c^T r - b\gamma\sqrt{U_s}\right)^2\right\} \tag{9}$$

The SIR is defined as the ratio of the desired user signal power to the sum of powers due to noise and multiple access interference at the output of filter c.

$$\text{SIR} = \frac{\left(c^T u_1\right)^2}{c^T \Gamma_c + \sum_{j=2}^{L}\left(c^T u_j\right)^2} \tag{10}$$

Keeping the condition of b_1=1, the error probability can be evaluated as

$$P_\theta\left(b_1\right) = Q\left(\frac{c^T u_1 + \sum_{j=2}^{K} b_j \left(c^T u_j\right)^2}{\left(c^T \Gamma_c\right)^{1/2}}\right) \tag{11}$$

where

$$Q(x) = (2\pi)^{-1/2} \int_x^\infty e^{-t^2/2} dt \tag{12}$$

The MMSE solution for c satisfies

$$\Lambda c = \left(1 - c^T u_1\right) u_1 \tag{13}$$

where $\Lambda = \sum_{j=2}^{L} u_j u_j^T + \Gamma$

All solutions to Eq. (13) minimize the MSE. The received power

$$E = 1 - c^T u_1 \tag{14}$$

$$\text{MSIR} = c^T u_1 / \left(1 - c^T u_1\right) \tag{15}$$

Now MMSE receiver will be subjected to fading channels. The performance of the MMSE receiver in a general fading channel that is frequency-selective is evaluated. Multipath fading can cause significant degradation in performance of the MMSE receiver. So, the receiver should have knowledge of the fading parameters from each path of all users' transmissions [5].

Assuming a fading multipath channel, each received signal takes on the form

$$R_k(t) = \sum_{r=1}^{Q_k} \sqrt{U_{k.r}}\, \gamma_{k.r}(t) S_k\left(t - \tau_{k,r}\right) \tag{16}$$

The MMSE receiver takes the signal at complex base band and passes it through a chip matched filter and samples the output of that filter at the chip rate and synchronous with the reception of the desired user's first path N-chip samples are stored for each symbol received and together these chip samples from the received vector for the mth symbol

$$r(m) = \left(r_{mN}, r_{mN+1}, \ldots\ldots, r_{(m+1)N-1}\right)^T \tag{17}$$

$$r_i = \int \psi\left(t - iT_c\right) R(t)\, dt \tag{18}$$

The MMSE receiver filters this received vector with a finite impulse response discrete filter characterized by the N-element tap weight vector $w(m)$. During each symbol interval, the MMSE receiver forms

$$z(m) = w^H(m) r(m) \tag{19}$$

The MMSE receiver is operated in a coherent manner, where decisions are made as

$$\hat{d}_1(m) = \mathrm{sgn}\left(\mathrm{Re}\left[z(m)\right]\right) \tag{20}$$

If the transmitted data bits are differentially encoded, the coherently decoded data can be differentially decoded to form

$$\hat{b}_1(m) = \hat{d}_1(m)\hat{d}_1(m-1) \tag{21}$$

To avoid difficulties faced on a fading channel, data decisions are formed according to

$$\hat{b}_1(m) = \mathrm{sgn}\left(\mathrm{Re}\left[z(m) z^*(m-1)\right]\right) \tag{22}$$

The tap weights of the MMSE filter are chosen to minimize the mean-squared error

$$J(m)=E[|e(m)|^2] \tag{23}$$

$$J(m)=E[|d_1(m)- z(m)|^2] \tag{24}$$

$$\mathrm{MSE} = E\left\{\left(c^T r - b\gamma\sqrt{U_s}\right)^2\right\} \tag{25}$$

The tap weight vector which minimizes this mean squared error is given by

$$w(m) = R^{-1}(m) p(m) \tag{26}$$

Here,

$$R(m) = E\left[r(m) r^H(m)\right] \tag{27}$$

and

$$p(m) = E\left[d_1^*(m) r(m)\right] \tag{28}$$

Now, factor γ has been introduced for analysis of the effect of fading on the system performance. It is defined as under

$$\alpha = \gamma\sqrt{\frac{E_b}{N_0}} \tag{29}$$

The received vector is specified as

$$r(m) = \sum_{k=1}^{K}\sum_{r=1}^{Q_k}\sqrt{\frac{U_{k,r}}{U_1}}\gamma_{k,r}(m)\left[d_k(m - L_{k,r} - 1)c_k^L(NT_C - \mu_{k,r}) + d_k(m - L_{k,r})c_k^R(\mu_{k,r})\right] + n(m) \tag{30}$$

where

$$\left.\begin{aligned} L_{k,r} &= \frac{\tau_{k,r}}{T_s} \\ \mu_{k,r} &= \tau_{k,r} - L_{k,r}T_s \end{aligned}\right\} \tag{31}$$

The fading processes do not change over the duration of a symbol. The carrier phase has been absorbed into the fading processes. Let

$$\Lambda_k(l) = \{r \mid L_{k,r} = l\} \tag{32}$$

and

$$h_{k,l}(m) = \sum_{r\in\Lambda_k(l)}\sqrt{\frac{U_{k,r}}{U_1}}\gamma_{k,r}(m)c_k^R(\mu_{k,r}) + \sum_{r\in\Lambda_k(l-1)}\sqrt{\frac{U_{k,r}}{U_1}}\gamma_{k,r}(m)c_k^L(NT_C - \mu_{k,r}) \tag{33}$$

$$r(m) = \sum_{k=1}^{K}\sum_{l=0}^{L_{k,\max}+1} d_k(m-l)h_{k,l}(m) + n(m) \tag{34}$$

where $L_{k,max}$ is the maximum value that $L_{k,r}$ takes on.

If the delay spread of the channel is less than one symbol interval, then $L_{1,max}$=0 and the received vector can be written as

$$r(m) = d_1(m)h_{1,0}(m) + d_1(m-1)h_{1,1}(m) + \tilde{r}(m) \tag{35}$$

where the first term represents the desired signal, the second term is (ISI) intersymbol interference and $\tilde{r}(m)$ is the combination of multiple access interference and noise.

Next, the adaptive receiver will be considered under flat fading channels. The tap weights of the receiver take care of the fading process and thus probability of error is reduced because the adaptive nature of the MMSE criterion takes into account the destructive or constructive addition of the signal and thus analyses error in a better fashion as shown below.

In the case of a flat fading channel model, the received signal vector becomes [6]

$$r(m) = d_1(m)\gamma_1(m)c_1 + \tilde{r}(m) \quad (36)$$

The second term represents noise due to a finite observation model and has to be neglected. The first part is the desired signal.

For slow fading case, the fading process is invariant over the interval of observation. The MMSE detector is then [7]

$$z(m) = w^H r(m) \quad (37)$$

$$w = c + x \quad (38)$$

The MMSE tap weights are given by [7]

$$w(m) = R^{-1}(m)p(m) \quad (39)$$

where

$$p(m) = h_{1,0}(m) \quad (40)$$

$$R(m) = E\left[\tilde{r}(m)\tilde{r}^H(m)\right] \quad (41)$$

A separate filter can be used for each path of the desired path. Thus Q_1 MMSE filters can be created with the modified error signals

$$e_q(m) = d_1(m)\gamma_{1,q}(m) - w_q^H(m)r(m) \quad (42)$$

Thus the MMSE tap weights are given by

$$w_q = R^{-1}c_1^R\left(\mu_{1,q}\right) \quad (43)$$

The code aided MMSE receiver is based on the decomposition of the linear detector as

$$\bar{c} = s + x \quad (44)$$

and $w = c + x$ (45)

where s is the signature sequence of the user and the other is the orthogonal and adaptive component. It is under the condition

$$s^T x = 0 \quad (46)$$

The detector c can be found by the method of Lagrange multipliers. Let

$$L(c) = MSE - 2\gamma\left(s^T c - 1\right) \quad (47)$$

and

$$\Delta_c L = 0 \quad (48)$$

Therefore

$$c = \left(U_s + \gamma\right)R^{-1}s \quad (49)$$

$$w_q = c_1^R\left(\mu_{1,q}\right) + x_q \quad (50)$$

$$c^R = \left(U_s^R + \gamma\right)R^{-1}s^R \tag{51}$$

This leads to the solution

$$w_q = R^{-1}\left(U_s^R + \gamma\right)R^{-1}s^R\left[\left[\left(U_s^R + \gamma\right)R^{-1}s^R\right]^T R^{-1}\left[\left(U_s^R + \gamma\right)R^{-1}s^R\right]\right]C_R^T c_1^R\left(\mu_{1,q}\right) \tag{52}$$

The performance of this system can be evaluated by parameter probability of error. The output of the MMSE filter is

$$z(m) = v_0 d_1(m) + v_1 d_1(m-1) + \tilde{n}(m) \tag{53}$$

where

$$\left.\begin{aligned} v_i &= w^H(m)h_{1,i}(m) \\ \tilde{n}(m) &= w^H(m)\tilde{r}(m) \end{aligned}\right\} \tag{54}$$

Here, $\tilde{n}(m)$ is the residual MAI plus noise at the output of the MMSE filter.

The probability of error is

$$P_{e/\tilde{q}} = \frac{1}{2 + 2\left[\left(U_s + \gamma\right)R^{-1}s\right]^H R^{-1}\left(U_s + \gamma\right)R^{-1}s} \tag{55}$$

The performance of adaptive MMSE receiver has further been evaluated in form of various graphs in terms of P_e, number of users, MSE and signal power by solving the equations given above.

3. Results and Discussion

Figure 2 shows the plot between mean square error and signal power for the particular case of fading parameter of 0.1. It is seen that MSE decreases asymptotically with P_s in accordance with Eq. (9). This implies that the difference between the estimated and transmitted bit will decrease as the signal power increases as the effect of noise also decreases during the transmission of the signal if the signal power is less.

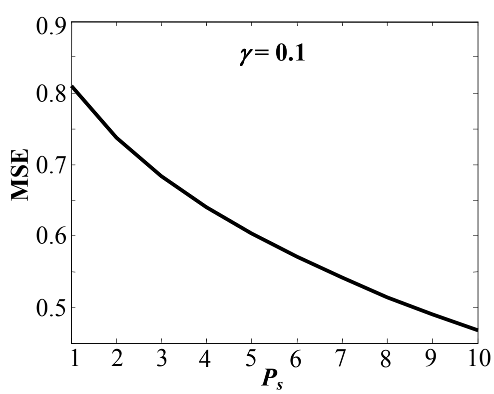

Fig. 2. Mean Square Error vs. Signal Power for Fading Parameter γ =0.1.

Figure 3 shows the plot between the mean square error and the signal power for lesser fading parameter. It is observed that the error decreases with decrease in fading effect. The lesser the constructive and destructive cancellation of the multipath signals, lesser is the mean square error. Hence, the bits are detected with more accuracy.

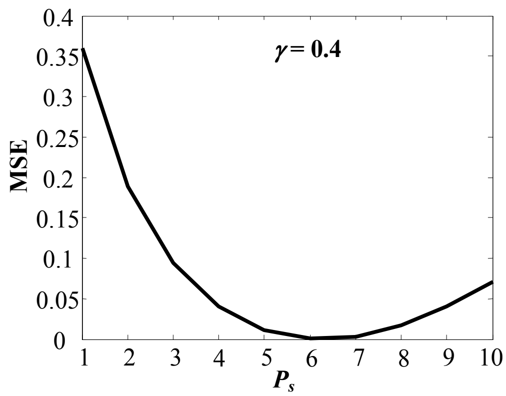

Fig. 3. Mean Square Error vs. Signal Power for Fading Parameter γ =0.4.

Figure 4 shows that the mean square error increases with increase in the signal or interfering signal power for decrease in the fading effect. This is so because if the effect of fading is lessened to the extent of its not existing, it becomes a linear detector with fading not coming in picture.

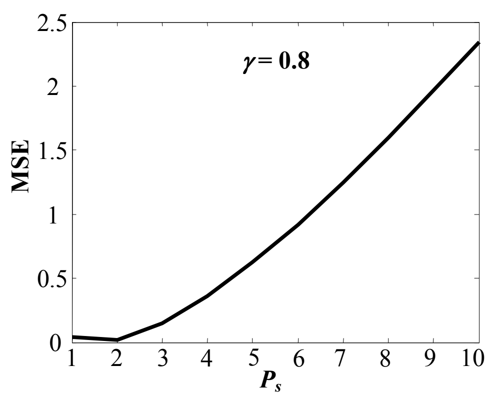

Fig. 4. Mean Square Error vs. Signal Power for Fading Parameter γ =0.8.

Figure 5 shows how the probability of error changes with the number of users for adaptive MMSE applied to fading. The received power from each interfering user is a log-normally distributed random variable with mean same as the desired signal and whose standard deviation is 1.5 db. Instantaneous values of the received powers vary due to the fading processes. It is observed that the proposed adaptive MMSE has lesser probability of error for the same number of users.

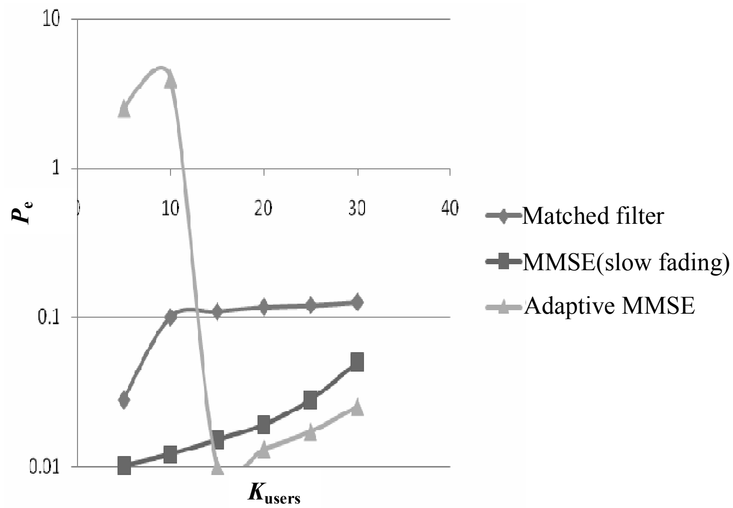

Fig. 5. Relation between Probability of Error and the Number of Users.

4. Conclusions

The interference rejection of signals by adaptive MMSE criterion over Rayleigh fading channels has been studied. It is observed that considering the effect of fading results in better estimation of the detected symbol. As the fading parameter increases, the required signal power is more for a constant value of MSE (mean square error) and vice versa.

When adaptive MMSE is put to fading environment, it adapts to dynamic environment. For the same number of simultaneous users, the present system has lower probability of error than the conventional/already existing receivers, i.e., matched filter, decorrelator. It has the advantage of being easily adapted and results in probability of error under most practical circumstances of wireless networks.

It is probable that wireless systems of near future will have elements, which adapt dynamically to changing patterns of interference. Adaptive techniques are becoming popular in various applications and the trend will grow.

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