Journal of Engineering Science and Technology
Vol. 17, No. 3 (2022) 1909 - 1920

MULTIDIMENSIONAL OPTIMIZATION FOR JOINT MAP CHANNEL AND PLL PHASE NOISE ESTIMATION IN OFDM

K. SHRIVASTAV[1], R. THAKUR[2], M. S. ANSARI[3],
A. PIMPALAPURE[3], S. K. VISHWAKARMA[4,*]

[1]Department of Electronics and Communication,
Bharat Institute of Engineering and Technology, Hyderabad, India
[2]Kalaniketan Polytechnic College, Jabalpur, India
[3]Department of Computer Science & Engineering Eklavya University, Damoh, India
[4]Manipal University Jaipur, India
[*]Corresponding Author: santosh.kumar@jaipur.manipal.edu

Abstract

A low complex algorithm of channel estimation in presence of phase locked loop phase noise (PLL PHN) for iterative orthogonal frequency division multiplexing (OFDM) receiver is addressed in this article. A maximum a posteriori (MAP) cost function for the joint estimation of channel transfer function (CTF) and PLL PHN is derived. The proposed joint estimation relaxes the restriction of small PLL PHN assumption and utilizes the prior statistical knowledge of PLL PHN spectral components to produce statistically optimal solution. The frequency domain estimation of unknown frequency selective fading makes the method simpler, compared with the estimation of channel impulse response (CIR) in time domain. Further a cyclic gradient descent optimization algorithm is proposed to minimize the joint MAP cost function. Simulation results are compared with Cramer-Rao lower bound (CRLB) to demonstrate that the proposed joint MAP estimation achieves near optimum performance.

Keywords: Channel transfer function, Maximum a posteriori, Orthogonal frequency division multiplexing, Phase locked loop phase noise

1. Introduction

OFDM system working for high spectral efficiency and data rates, necessitates the estimation of time varying channel in presence of receiver non idealities such as PHN, carrier frequency offset (CFO) and IQ (in and quadrature phase) imbalance effect. PHN is random fluctuation in the phase of the signal produced by the practical oscillator [1-3] whereas CFO and IQ imbalance are deterministic. Either the isolated or joint approach of the channel and PHN estimation has been used [4-12] for a reliable performance. While the isolated approach [4-6] results in poor estimation, the joint approach [7-14] produces the optimal estimates with increase in computation complexity.

A new frequency domain low complex approach for the estimation of CTF and PHN in joint is introduced in this paper. This paper focuses on more enhanced time varying PHN model as Ornstein-Uhlenbeck (O-U) process [15] without the assumption of small PHN. The derived joint MAP cost function is further minimized with proposed cyclic gradient descent optimization algorithm. The mean square error (MSE) of the proposed joint MAP algorithm is compared with the CRLB for an OFDM channel estimator without PHN.

The paper is organized in following manner. In Section 2 the PLL PHN modelling is presented which is further used in Section 3 to derive the PLL PHN corrupted OFDM signal. In Section 4 the proposed joint MAP cost function is derived which is further optimized globally with respect to CTF and PLL PHN by using proposed iterative cyclic gradient descent optimization algorithm in section 5. Section 6 is presenting the simulation results and Section 7 concludes the paper.

2. Phase Noise Modelling

In general, time varying PHN process ($\theta(t)$), can be written as following stochastic differential equation [15]:

$$d\theta(t) = \varphi(\mu - \theta(t))dt + \sigma dB(t) \quad (1)$$

where $\theta(t)$ and $B(t)$ are continues time Ornstein- Uhlenbeck (O-U) process and Brownian process respectively. μ is asymptotic mean, φ is the drift and σ^2 is the variance of the noise present in the system which is white noise in our case.

Solution for Eq. (1) with the initial condition $\theta(0)$is [15]:

$$\theta(t) = \theta(0)e^{-\varphi t} + \mu(1 - e^{-\varphi t}) + \sigma \int_0^t e^{-\varphi(t-s)} dB_s \quad (2)$$

If $\theta(t)$ is sampled with the sampling interval T_s/N, means $\theta_n = \theta(nTs/N)$ where $n = 0, 1, 2, \dots, N-1$ then:

$$\theta_{n+1} = \theta_n e^{-\varphi \frac{T_s}{N}} + \mu\left(1 - e^{-\varphi \frac{T_s}{N}}\right) + \phi_n \, . \quad (3)$$

Equation (3) represents the autoregressive process of order one (AR (1)) where ϕ_n is a sequence of identically and independently distributed (iid) random variables with mean zero and variance ${\sigma_{\phi_n}}^2$, such that:

$$\phi_n = \sigma_{\phi_n} \epsilon_n \quad (4)$$

where $\epsilon_n \sim iid\mathcal{N}(0, 1)$ and $\phi_n \sim iid\mathcal{N}(0,\ {\sigma_{\phi_n}}^2)$. In this case, Eq. (3) is known as discrete time regular O-U process with:

$${\sigma_{\phi_n}}^2 = \frac{\sigma^2}{2\varphi}\left(1 - e^{-2\varphi\frac{T_s}{N}}\right). \tag{5}$$

Though PHN from the PLL voltage controlled oscillator (VCO) follows the regular O-U process, but for wide sense stationary output from the PLL, asymptotic mean should be zero and with that Eq. (1) results in [15]:

$$d\theta(t) = -\varphi\theta(t)dt + \sigma dB(t) \tag{6}$$

with the solution [15]:

$$\theta(t) = \theta(0)e^{-\varphi t} + \sigma\int_0^t e^{-\varphi(t-s)}dB_s \tag{7}$$

which is celebrated O-U process[15]. The discrete time sampled version of Eq. (7) is:

$$\theta_{n+1} = \theta_n e^{-\varphi\frac{T_s}{N}} + \phi_{PLL_n} \tag{8}$$

where ϕ_{PLL_n} is a sequence of identically and independently distributed (iid) random variables with mean zero and variance:

$${\sigma_{\phi_{PLL_n}}}^2 = \frac{\sigma^2}{\varphi}\left(1 - e^{-\varphi\frac{T_s}{N}}\right) \tag{9}$$

Mehrotra [16] solved Eq. (7) for the PLL VCO with loop filter of order one (Fig. 1) resulting in zero mean and variance:

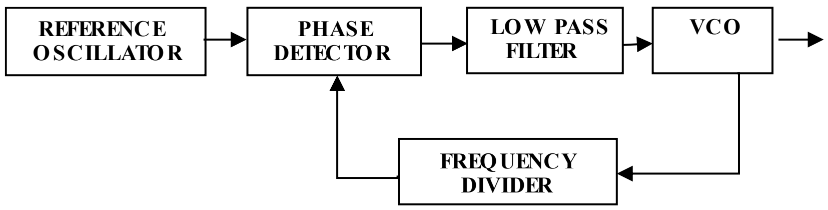

Fig. 1. Principal PLL block diagram.

$${\sigma_{\phi_{PLL_n}}}^2 = 4\pi^2{f_c}^2\left(C_{in}\frac{T_s}{N} + 2\sum_{i=1}^{2}(\xi_i + \zeta_i)\left(1 - e^{-\lambda_i\frac{T_s}{N}}\right)\right) \tag{10}$$

where:

$$\lambda_{1,2} = \frac{\omega_{lpf} \pm \sqrt{({\omega_{lpf}}^2 - 4\omega_{lpf}\sqrt{C_{PLL}})}}{2},$$

$$\xi_1 = \frac{C_{in}\lambda_2}{(\lambda_1 - \lambda_2)\lambda_1}, \qquad \xi_2 = \frac{-C_{in}\lambda_1}{(\lambda_1 - \lambda_2)\lambda_2},$$

$$\zeta_1 = \frac{C_{in} + C_{VCO}}{(\lambda_1 - \lambda_2)^2}\left(\frac{{\lambda_2}^2}{2\lambda_1} - \frac{\lambda_1\lambda_2}{2(\lambda_1 + \lambda_2)}\right),$$

and

$$\zeta_2 = \frac{C_{in} + C_{VCO}}{(\lambda_1 - \lambda_2)^2}\left(\frac{{\lambda_1}^2}{2\lambda_2} - \frac{\lambda_1\lambda_2}{2(\lambda_1 + \lambda_2)}\right)$$

where f_c is the centre frequency of VCO in Hz , ω_{lpf} is the angular corner frequency of the low pass filter in rad/sec and $\sqrt{C_{PLL}}$ is the PLL bandwidth in Hz. $C_{in}$and $C_{VCO}$are diffusion rates of the reference oscillator and VCO respectively.

If $\theta^m = [\theta_0^m, \theta_1^m, \ldots \theta_{N-1}^m]^T$ is the PHN vector for the m^{th} OFDM symbol, then:

$$P^m = [p^m_{-\frac{N}{2}}, p^m_{-\frac{N}{2}+1}, \ldots, p_0^m, \ldots, p^m_{\frac{N}{2}-2}, p^m_{\frac{N}{2}-1}]^T \tag{11}$$

defines a vector of the DFT (discrete Fourier transform) coefficients of one realization of $e^{j\theta_n}$ during m^{th} OFDM symbol where:

$$p_k^m = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\theta_n^m} e^{\frac{-j2\pi nk}{N}}, \quad -\frac{N}{2} \le k \le \frac{N}{2} - 1 \tag{12}$$

and the correlation matrix for PLL VCO is given as [5]:

$$\mathbf{R}_{\mathrm{P}^m}(a,b)_{PLL} = \frac{1}{N^2}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} e^{-4\pi^2 f_c{}^2\left(C_{in}\frac{|u-v|T_S}{N} + 2\sum_{i=1}^{2}(\xi_i+\zeta_i)(1-e^{-\lambda_i\frac{|u-v|T_S}{N}})\right)} - e^{\frac{-j2\pi(au-bv)}{N}},$$

$$\frac{N}{2} \le a, b \le \frac{N}{2} - \tag{13}$$

=If θ_n is a celebrated O-U process then the cumulative PLL PHN increment in two received signals is an asymptotically Gaussian random variable. Thus the $\mathrm{P}^m$is complex Gaussian distributed, $\Pr(\mathrm{P}^m) = \mathcal{CN}(0, \boldsymbol{\Theta})$, with mean zero and covariance matrix, $\boldsymbol{\Theta} = \mathbf{R}_{\mathrm{P}^m}(a,b)_{PLL}$. Since the power spectral density (PSD) of PHN tappers off rapidly beyond the loop bandwidth, PLL PHN process can be sufficiently characterized by few lower order spectral components, containing most of the energy of a PHN sequence. These lower order spectral components of PLL PHN are given by $p_0{}^m, p_1{}^m, p_{-1}{}^m, p_2{}^m, p_{-2}{}^m$ etc. Here we define a variable Q, as an approximation order for which $2Q+1$ elements of the vector P^m, i.e., $p_{-Q}^m, \ldots, p_0^m, \ldots, p_Q^m$ can well approximate the PLL PHN process.

3. OFDM System Modelling

We model an OFDM system consisting of N subcarriers with sampling instant T_s/N. If$D_k^m, k = 0, 1, \ldots, N-1$, is the frequency domain quadrature amplitude modulation (QAM) modulated symbol on k^{th} subcarrier of m^{th} symbol then, $\mathrm{D}^m = [D_0^m, D_1^m, \ldots, D_{N-1}^m]^T$, defines a symbol vector. Let the discrete time composite CIR with order$L$is denoted by $h(l)$ and the CTF on the k^{th} subcarrier is denoted by H_k, then:

$$H_k = \sum_{l=0}^{L-1} h(l) e^{\frac{-j2\pi kl}{N}}. \tag{14}$$

The frequency domain received signal on thek^{th}subcarrier of the m^{th}symbol is:

$$r_k^m = \sum_{q=0}^{N-1} D_q^m H_q p^m_{\langle k-q\rangle} + Z_k^m, 0 \le k \le N-1 \tag{15}$$

where D_q^m is q^{th} element of symbol vector D^m, H_q is the q^{th} element of channel vector $\mathrm{H} = [H_0, H_1, H_2, \ldots, H_{N-1}]^T$, Z_k^m is additive white Gaussian noise (AWGN) in frequency domain and $p^m_{\langle k-q\rangle}$ is the $(k-q)^{th}$ spectral component of PLL PHN spectral component vector, P^m , which is modulo N indexed. For that the lower order spectral components of PLL PHN are given by $p_0, p_1, p_{N-1}, p_2, p_{N-2}$ etc. For further analysis we represent the matrix signal model as:

$$\mathbf{R}^m = \boldsymbol{\mathcal{H}}^m \mathbf{P}^m + \mathbf{Z}^m \tag{16}$$

where

$\mathbf{R}^m = [r_0^m, r_1^m, \ldots, r_{N-1}^m]^T$, $\mathbf{P}^m = [p_0^m, p_1^m, \ldots, p_{N-1}^m]^T$, $\mathbf{H} = [H_0, H_1, H_2, \ldots, H_{N-1}]^T$,

$\mathbf{D}^m = [D_0^m, D_1^m, \dots, D_{N-1}^m]^T$and $\boldsymbol{\mathcal{H}}^m$ is a column wise circulant matrix whose first column is vector $[H_0 D_0^m, H_1 D_1^m, \dots, H_{N-1} D_{N-1}^m]^T$.$\mathbf{Z}^m = [Z_0^m, Z_1^m, \dots, Z_{N-1}^m]^T$, is an uncorrelated white noise vector distributed as,$\Pr(\mathbf{Z}^m) = \mathcal{CN}(0, 2\sigma_\omega^2 \mathbf{I})$, with mean zero and covariance matrix $2\sigma_\omega^2\mathbf{I}$, which says:

$$\Pr(\mathbf{Z}^m) = \frac{1}{(2\pi)^N \sigma_\omega^{2N}} exp\left(\frac{-1}{2\sigma_\omega^2} \mathbf{Z}^{m^H} \mathbf{Z}^m\right) \quad (17)$$

4. Joint MAP Estimator

Here we will derive a joint MAP cost function for the CTF ($\mathbf{H}$) and the PLL PHN spectral components($\mathbf{P}^m$) estimation, when$\mathbf{R}^m$ is observed. By using Bayes' rule the "complete likelihood function", $\Pr(\mathbf{R}^m, \mathbf{H}, \mathbf{P}^m)$ which is proportional to a posterior distribution, $\Pr(\mathbf{H}, \mathbf{P}^m | \mathbf{R}^m)$ is given as:

$$\Pr(\mathbf{R}^m, \mathbf{H}, \mathbf{P}^m) = \Pr(\mathbf{R}^m | \mathbf{H}, \mathbf{P}^m) \Pr(\mathbf{H}) \Pr(\mathbf{P}^m) \quad (18)$$

As $\mathbf{H}$ is not known in prior, $\Pr(\mathbf{H})$ is constant and with O-UPHN model, $\Pr(\mathbf{P}^m) = \mathcal{CN}(0, \mathbf{\Theta})$ which says:

$$\Pr(\mathbf{P}^m) = \frac{1}{\pi^N |\mathbf{\Theta}|} exp\left(-\mathbf{P}^{m^H} \mathbf{\Theta}^{-1} \mathbf{P}^m\right) \quad (19)$$

where $\mathbf{\Theta}$ is known. Minimizing the "complete negative log-likelihood function" and maximising the "complete likelihood function" in Eq. (18) is same so:

$$\widehat{\mathbf{H}}, \widehat{\mathbf{P}}^m = arg \min_{\mathbf{H}, \mathbf{P}^m} \{-log[\Pr(\mathbf{R}^m | \mathbf{H}, \mathbf{P}^m)] - log[\Pr(\mathbf{P}^m)]\} \quad (20)$$

Considering the signal model of Eq. (16) and the AWGN density of Eq. (17), the conditional density can be written as:

$$\Pr(\mathbf{R}^m | \mathbf{H}, \mathbf{P}^m) = \frac{1}{(2\pi)^N \sigma_\omega^{2N}} exp \left\{\frac{-1}{2\sigma_\omega^2} (\mathbf{R}^m - \boldsymbol{\mathcal{H}}^m \mathbf{P}^m)^H (\mathbf{R}^m - \boldsymbol{\mathcal{H}}^m \mathbf{P}^m)\right\}. \quad (21)$$

Using Eqs. (19), (20) and (21), the joint MAP estimate can be given as:

$$\widehat{\mathbf{H}}, \widehat{\mathbf{P}}^m = arg \min_{\mathbf{H}, \mathbf{P}^m} \{\mathcal{L}(\mathbf{H}, \mathbf{P}^m)\} \quad (22)$$

where:

$$\mathcal{L}(\mathbf{H}, \mathbf{P}^m) = \frac{1}{2\sigma_\omega^2} (\mathbf{R}^m - \boldsymbol{\mathcal{H}}^m \mathbf{P}^m)^H (\mathbf{R}^m - \boldsymbol{\mathcal{H}}^m \mathbf{P}^m) + \mathbf{P}^{m^H} \mathbf{\Theta}^{-1} \mathbf{P}^m \quad (23)$$

is the joint MAP cost function, which is simultaneously minimized for statistically optimal results of$\widehat{\mathbf{H}}$ and$\widehat{\mathbf{P}}^m$ with respect to $\mathbf{H}$ and$\mathbf{P}^m$.

5. Cost Function Optimization

A cyclic gradient descent optimization algorithm is proposed in this section to solve the critical problem of minimization of cost function. As Eq. (23) is not differentiable with respect to$\mathbf{H}$ in frequency selective fading, we minimise the Eq. (23) in respect of $\mathbf{H}$by searching the feasible finite set of CTF while $\mathbf{P}^m$is fixed. Further, Eq. (23) is a quadratic function in $\mathbf{P}^m$as $\mathbf{\Theta}$ is a non-singular covariance matrix. To find the solution of Eq. (22) in respect of $\mathbf{P}^m$, Eq. (23) should be analytic with respect to $\mathbf{P}^m$. With these two conditions if $\mathbf{H}$is fixed, we take the conjugate gradient of cost function with respect to$\mathbf{P}^m$ and equate it to zero to find the optimum minimization of Eq. (23) in respect of $\mathbf{P}^m$. We begin the optimization with $\widehat{\mathbf{P}}^{m^0}$, initial estimate of PHN spectral components with least square (LS) estimation of [6].At i^{th} iteration, the estimate of CTF can be calculated as:

$$\widehat{\mathbf{H}}^i = arg\min_{\mathbf{H}}\{\left(\mathbf{R}^m - \boldsymbol{\mathcal{H}}^m\widehat{\mathbf{P}}^{m^i}\right)^H\left(\mathbf{R}^m - \boldsymbol{\mathcal{H}}^m\widehat{\mathbf{P}}^{m^i}\right)\} \tag{24}$$

To find the solution of Eq. (24), a random search method is always preferred over expensive exhaustive grid search method in case of multidimensional optimization [17]. However it is important to begin with the best estimate of $\mathbf{P}^m$ possible to avoid local minima. To update the PHN spectral components estimate, we compute the conjugate gradient of the MAP cost function with respect to the vector $\mathbf{P}^m$. This gradient is given by:

$$\nabla_{\mathbf{P}^{m*}}\mathcal{L}(\mathbf{H},\mathbf{P}^m) = \frac{1}{\sigma_\omega^2}\left(\boldsymbol{\mathcal{H}}^{m^H}\boldsymbol{\mathcal{H}}^m\mathbf{P}^m - \boldsymbol{\mathcal{H}}^{m^H}\mathbf{R}^m\right) + 2\boldsymbol{\Theta}^{-1}\mathbf{P}^m \tag{25}$$

where:

$$\nabla_{\mathbf{P}^{m*}}\mathcal{L}(\mathbf{H},\mathbf{P}^m) = \begin{bmatrix} \frac{\partial\mathcal{L}(\mathbf{H},\mathbf{P}^m)}{\partial p_0^{m^*}} \\ \frac{\partial\mathcal{L}(\mathbf{H},\mathbf{P}^m)}{\partial p_1^{m^*}} \\ \cdot \\ \cdot \\ \frac{\partial\mathcal{L}(\mathbf{H},\mathbf{P}^m)}{\partial p_{N-1}^{m^*}} \end{bmatrix}. \tag{26}$$

At iteration i, $\mathbf{H} = \widehat{\mathbf{H}}^i$ so we put $\nabla_{\mathbf{P}^{m*}}\mathcal{L}(\mathbf{H},\mathbf{P}^m)\big|_{\mathbf{H}=\widehat{\mathbf{H}}^i}$ equal to zero and solve for $\mathbf{P}^m$. Thus we obtain the next PHN spectral components estimate as:

$$\widehat{\mathbf{P}}^{m^{i+1}} = [\widehat{\boldsymbol{\mathcal{H}}}^{m^{i^H}}\widehat{\boldsymbol{\mathcal{H}}}^{m^i} + 2\sigma_\omega^2\boldsymbol{\Theta}^{-1}]^{-1}\widehat{\boldsymbol{\mathcal{H}}}^{m^{i^H}}\mathbf{R}^m. \tag{27}$$

This updating procedure continues for $i = 0,1,2,3 \ldots$ till $\mathcal{L}\left(\widehat{\mathbf{H}},\widehat{\mathbf{P}}^m\right)$ stabilizes with $\left\|\widehat{\mathbf{H}}^{i+1} - \widehat{\mathbf{H}}^i\right\| / \left\|\widehat{\mathbf{H}}^i\right\| < \varepsilon$ (preset threshold) or a number of iterations is reached.

6. Simulation Results

Performance of the proposed joint MAP algorithm is simulated in this section, where each simulation point is conducted using 1,000 OFDM symbols in MATLAB. Simulation model is based on IEEE 802.11g like system with 64 subcarriers and 20 MHz of transmission bandwidth. OFDM symbols are generated using 16-QAM and 64-point inverse fast Fourier transform (IFFT), and then prepended by cyclic prefix (CP) of length 16 samples before transmitting over the channel. The discrete sampled CIR is modelled as 10 tapped delay lines having an exponentially decreasing power delay profile (PDP):

$$\alpha_l^2 = E\{|h(l)|^2\} = \frac{1}{\gamma}exp(-0.5l), l = 0,1,..,L-1 \tag{28}$$

where $\gamma = \sum_l exp(-0.5l)$ is choosen to normalise the PDP to unit energy. The 64-point FFT of the received signal is taken after receiver PLL PHN modelling followed by CP removal. The receiver PLL PHN is modelled as O-U process as shown in Fig. 2. PLL VCO parameters are, $f_c = 5$ GHz, loop corner frequency is 20 kHz, $C_{in} = 10^{-25} sec$ and $C_{VCO} = 10^{-19}$s. Assuming that the VCO is noisier than reference oscillator, $C_{PLL} = 4 * 10^8/\text{s}^2$.

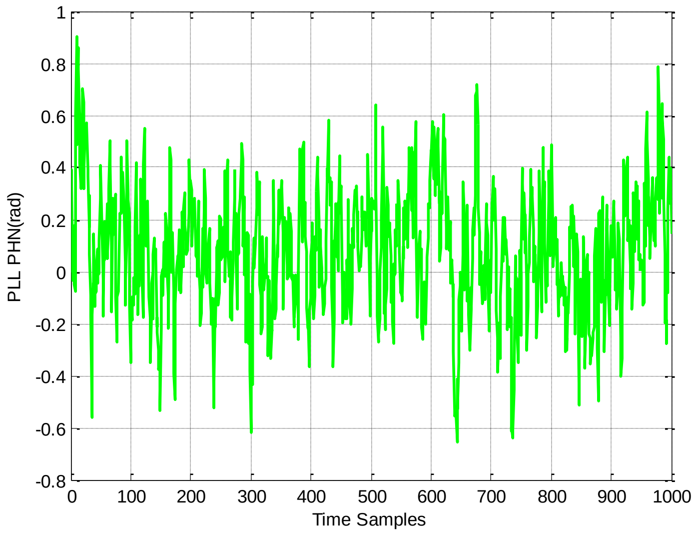

Fig. 2. Simulated PLL PHN Samples for PLL VCO (Ornstein-Uhlenbeck process).

Results for the matrix $\mathbf{\Theta}$ are evaluated and presented in Fig. 3.It is observed from Fig. 3 that, the cross correlation of some lower order PLL PHN spectral components cannot be neglected when compared with auto correlation terms. The $\widehat{\mathbf{P}}^{m^0}$ is obtained with LS estimation [6] with common phase error (CPE) correction [18]. For that channel has been LS estimated [19], without PHN. Equation (24) is minimised with random search algorithm [17].

In Fig. 4 the MSE of the proposed channel estimation for estimating normalised CTF against system signal to noise ratio (SNR) for$i = 6$ and $Q = 4$, is compared with the posterior CRLB for an OFDM channel estimator without PHN distortion. MSE performance curves for the channel estimation with CPE correction only and non-iterative maximum likelihood (ML) joint estimation of [9] are also simulated and presented in Fig. 4.

As calculated in Appendix A, the posterior CRLB for estimating the CTF, ($CRLB_{\mathrm{H}}$) is calculated as:

$$CRLB_{\mathrm{H}} = L/SNR \tag{29}$$

and the MSE is obtained as:

$$MSE = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=0}^{N-1}\left(H_n^m - \widehat{H}_n^m\right)^2 \tag{30}$$

where$M$represents the number of simulated OFDM symbols.

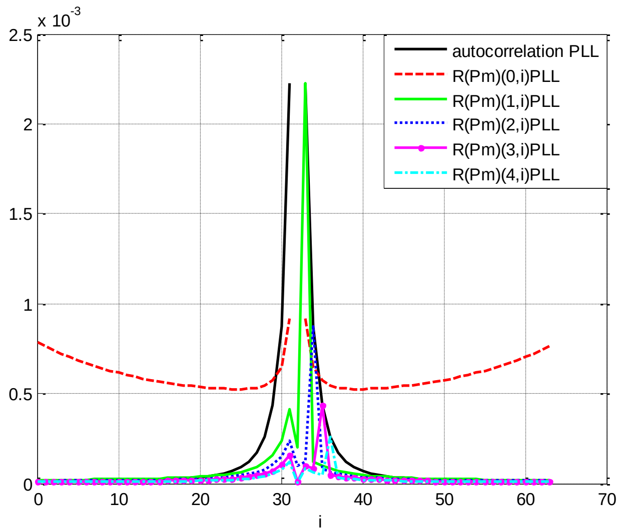

Fig. 3. Correlation property of PHN spectral components for PLL VCO.

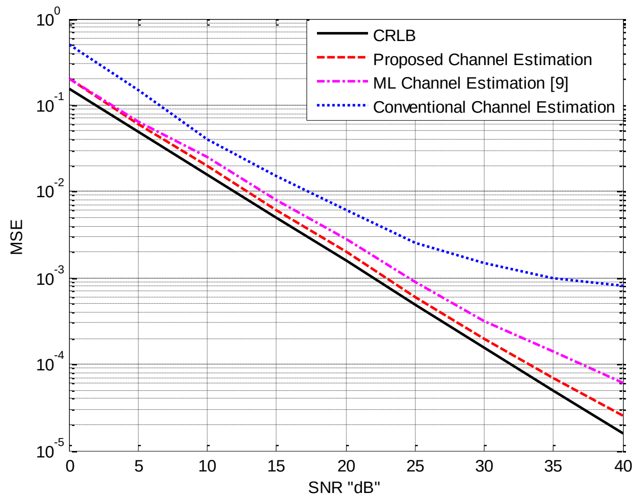

Fig. 4. MSE performance of CTF estimation as a function of SNR.

In Fig. 4 the proposed joint MAP algorithm outperforms the conventional channel estimator with CPE correction only for a wide range of SNR. As shown in Fig. 4, to achieve the MSE=10^{-2}, proposed joint MAP algorithm shows 4dB improvement in SNR over conventional method. This performance gap increases with SNR as with high PHN levels random inter carrier interference (ICI) dominates over CPE for large values of SNR [4]. As shown in Fig. 4, the proposed joint MAP algorithm achieves MSE=10^{-4} at SNR= 33 dB whereas the conventional method causes a significant SNR degradation in the estimation and produces an error floor.

It can also be observed from Fig. 4 that the proposed joint MAP algorithm shows better improvements than the non-iterative ML joint estimator [9].As shown in Fig. 4, to achieve the MSE=10^{-2}, proposed joint MAP algorithm shows 1dB improvement in SNR over ML joint estimator [9]. It is because the proposed algorithm performs optimization before each iteration to combat the sever sensitivity towards high PHN level. As we apply the statistical knowledge of the PLL PHN spectral components without the assumption of small PHN this adds in the performance improvement even for the large SNR values. It can be observed from Fig. 4 that the proposed joint MAP algorithm achieves MSE=10^{-4} at SNR= 33dB whereas the ML joint estimator [9] needs SNR= 37 dB which shows the improvement of 4dB for higher SNR values.

7. Conclusions

A statistically near optimum joint MAP channel estimation algorithm for OFDM system in presence of PLL PHN is presented in this paper. A more advanced and enhanced model for time varying PHN, produced by PLL VCO is presented and analysed. Simulation results prove that with the proposed iterative cyclic gradient descent optimization algorithm the estimator achieves near CRLB MSE performance. Relaxing the assumption of small PHN improves over the cost function minimization and joint MAP estimation. The potential improvement in the performance proves the competence of proposed joint MAP algorithm to mitigate the high level of PHN even in fading environment. As the performance improves with increasing the order of approximation and number of iterations a trade-off should be maintained between performance improvement and computational complexity. Incorporation of fading statistical study, done with different models of channel, can further enhance the performance.

Nomenclatures

$B(t)$	Brownian process
$\sqrt{C_{PLL}}$	PLL bandwidth in /sec
C_{in}	Diffusion rate of the reference oscillator in sec
C_{VCO}	Diffusion rate of the VCO in sec
D^m	m^{th}symbol vector
f_c	Centre frequency of VCO in Hz
$h(l)$	Discrete time composite CIR
H_k	CTF on the k^{th} subcarrier
i	Iteration number
M	Number of simulated OFDM symbols
N	No. of subcarriers
P^m	PHN vector for the m^{th} OFDM symbol
Q	Approximation order
r_k^m	frequency domain received signal on thek^{th}subcarrier of the m^{th}symbol
T_s	Symbol time in sec
Z_k^m	AWGN in frequency domain

Greek Symbols

$2\rho^2$	E_s (Symbol energy per subcarrier)
α_l^2	Exponentially decreasing PDP
γ	Normalising coefficient for PDP
ε	Preset threshold
μ	Asymptotic mean of noise process
φ	Drift of the noise process
Θ	Covariance matrix of P^m
$\theta(t)$	Time varying PHN process
ϕ_{PLL_n}	PLL PHN process sequence
ϕ_n	PHN process sequence
σ^2	Variance of the noise process
${\sigma_{\phi_n}}^2$	Variance of the PHN
${\sigma_{\phi_{PLLn}}}^2$	Variance of the PLL PHN
ω_{lpf}	Angular corner frequency of the low pass filter in rad/s

Abbreviations

AR(1)	Autoregressive process of order one
AWGN	Additive white Gaussian noise
CFO	Carrier frequency offset
CIR	Channel impulse response
CP	Cyclic prefix
CPE	Common phase error
CRLB	Cramer-Rao lower bound
CTF	Channel transfer function
DFT	Discrete Fourier transform
ICI	Inter carrier interference
IFFT	Inverse fast Fourier transform
IQ	In and quadrature phase
iid	Identically and independently distributed
LS	Least square
MAP	Maximum a posteriori
ML	Maximum likelihood
MSE	Mean square error
OFDM	Orthogonal frequency division multiplexing
O-U	Ornstein-Uhlenbeck
PDP	Power delay profile
PHN	Phase noise
PLL	Phase locked loop
PSD	Power spectral density
QAM	Quadrature amplitude modulation
SNR	Signal to noise ratio
VCO	Voltage controlled oscillator

References

1. Robertson, P.; and Kaiser, S. (1995). Analysis of the effects of phase noise in orthogonal frequency division multiplex (OFDM) systems. *Proceedings IEEE International Conference on Communications ICC '95*, Seattle, WA, USA.
2. Armada, A.G. (2001). Understanding the effects of phase noise in orthogonal frequency division multiplexing (OFDM). *IEEE Transactions on Broadcasting*, 47(2), 153-159.
3. Wu, S.; and Bar-Ness, Y. (2002). Performance analysis on the effect of phase noise in OFDM systems. *Proceedings IEEE Seventh International Symposium on Spread Spectrum Techniques and Applications*, Prague, Czech Republic.
4. Wu, S.; and Bar-Ness, Y. (2004). OFDM systems in the presence of phase noise: consequences and solutions. *IEEE Transactions on Communications*, 52(11), 1988-1997.
5. Petrovic, D.; Rave, W.; and Fettweis, G. (2007). Effects of phase noise on OFDM systems with and without PLL: Characterization and compensation. *IEEE Transactions on Communications*, 55(8), 1607-1616.
6. Syrjälä, V.; Valkama, M.; Tchamov, N.; and Rinne, J. (2009). Phase noise modelling and mitigation techniques in OFDM communications systems. *Proceedings Wireless Telecommunications Symposium*, Prague, Czech Republic, 1-7.
7. Lin, D.D.; Pacheco, R.A.; Lim, T.J.; and Hatzinakos, D. (2006). Joint estimation of channel response, frequency offset, and phase noise in OFDM. *IEEE Transactions on Signal Processing*, 54(9), 3542 - 3554.
8. Tao, J.; Wu, J.; and Xiao, C. (2009). Estimation of channel transfer function and carrier frequency offset for OFDM systems with phase noise. *IEEE Transactions on Vehicular Technology*, 58(8), 4380-4387.
9. Rabiei, P.; Namgoong, W.; and Al-Dhahir, N. (2010). A non-iterative technique for phase noise ICI mitigation in packet-based OFDM systems. *IEEE Transactions on Signal Processing*, 58(11), 5945-5950.
10. Munier, F.; Eriksson, T.; and Svensson, A. (2008). An ICI reduction scheme for OFDM system with phase noise over fading channels. *IEEE Transactions on Communications*, 56(7), 1119-1126.
11. Salim, O.H.; Nasir, A.A.; Mehrpouyan, H.; Xiang, W.; Durrani, S.; and Kennedy, R.A. (2014). Channel, phase noise, and frequency offset in OFDM systems: Joint estimation, data detection, and hybrid Cramer-Rao lower bound. *IEEE Transactions on Communications*, 62(9), 3311–3325.
12. Septier, F.; Delignon, Y.; Menhaj-Rivenq, A.; and Garnier, C. (2007). OFDM channel estimation in the presence of phase noise and frequency offset by particle filtering. 2007 *IEEE International Conference on Acoustics, Speech and Signal Processing - ICASSP '07*, Honolulu, HI, USA.
13. Nguyen-Duy-Nhat, V.; Bui-Thi-Minh, T.; Tang-Tan, C.; Bao, V.N.Q.; and Nguyen-Le, H. (2016). Joint phase noise and doubly selective channel estimation in full-duplex MIMO-OFDM systems. 2016 *International Conference on Advanced Technologies for Communications (ATC), Hanoi, Vietnam*, 412-417.
14. Lee, T.-J.; and Ko, Y.-C. (2017). Channel estimation and data detection in the presence of phase noise in MIMO-OFDM systems with independent oscillators. *IEEE Access*, 5, 9647-9662.

15. Ghosh, A.P.; Qin, W.; and Roitershtein, A. (2016). Discrete-time Ornstein-Uhlenbeck process in a stationary dynamic environment. *Journal of Interdisciplinary Mathematics*, 19(1), 1-35.
16. Mehrotra, A. (2002). Noise analysis of phase-locked loops. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 49(9), 1309-1316.
17. Nelder, J.A.; and Mead, R. (1965). A simplex method for function minimization. *The Computer Journal*, 7(4), 308-313.
18. Wu, S.; and Bar-Ness, Y. (2002). A phase noise suppression algorithm for OFDM based WLANs. *IEEE Communications Letters*, 6(12), 535-537.
19. Coleri, S.; Ergen, M.; Puri, A.; and Bahai, A. (2002). Channel estimation techniques based on pilot arrangement in OFDM systems. *IEEE Transactions on Broadcasting*, 48(3), 223-229.
20. Shrivastav, K.; Yadav, R.P.; and Jain, K.C. (2017). Cyclic gradient descent optimization for joint MAP estimation of channel and phase noise in OFDM. *IET Communications*, 12(12), 1485-1490.

Appendix A

Derivation for $CRLB_{\mathrm{H}}$

For an OFDM system with frequency selective channel, the$CRLB_{\mathrm{H}}$ is given as [20]:

$CRLB_{\mathrm{H}} = N \times CRLB_{\mathrm{h}}$

where$CRLB_{\mathrm{h}}$ is MSE for the estimation of CIR. The received signal without PHN distortion is (from Eq. (15) and (16)):

$\mathbf{R}^m = \mathbf{X}^m \mathbf{F}\mathbf{h} + \mathbf{Z}^m$

where$\mathbf{X}^m$ is a diagonal matrix with entries $[D_0^m, D_1^m, \ldots, D_{N-1}^m]$, $\mathbf{F}$ is $N \times L$ DFT matrix with $F(n,l) = exp\left(-\frac{j2\pi nl}{N}\right)$ and $\mathbf{h} = [h(0), h(1), \ldots, h(L-1)]^T$ is the time domain channel vector. Thus:

$\log\left[\Pr(\mathbf{R}^m|\mathbf{h})\right] = \frac{-1}{2\sigma_\omega^2}(\mathbf{X}^m\mathbf{F}\mathbf{h} - \mathbf{R}^m)^H(\mathbf{X}^m\mathbf{F}\mathbf{h} - \ \mathbf{R}^m).$

$\frac{\partial}{\partial \mathbf{h}^*}\{\log\left[\Pr(\mathbf{R}^m|\mathbf{h})\right]\} = = \frac{-1}{2\sigma_\omega^2}\mathbf{F}^H\mathbf{X}^{m^H}\mathbf{Z}^m$

The Fisher information matrix [7-8] is given as:

$FIM(\mathbf{h}) = \left\{\left[\frac{\partial}{\partial \mathbf{h}^*}(\log[\Pr(\mathbf{R}^m|\mathbf{h})])\right]\left[\frac{\partial}{\partial \mathbf{h}^*}(\log[Pr(\mathbf{R}^m|\mathbf{h})])\right]^H\right\} = \frac{1}{2\sigma_\omega^2}\mathbf{F}^H\mathbf{X}^{m^H}\mathbf{X}^m\mathbf{F}$.

where $\mathbf{X}^{m^H}\mathbf{X}^m = 2\rho^2\mathbf{I}$,where $2\rho^2 = E_s$(symbol energy per subcarrier). Thus:

$FIM(\mathbf{h}) = \frac{2\rho^2}{2\sigma_\omega^2}\mathbf{F}^H\mathbf{F} = \frac{\rho^2}{\sigma_\omega^2}N\mathbf{I}$.

Therefore the$CRLB_{\mathrm{h}}$ is given by [7-8] $CRLB_{\mathrm{h}} = Tr[FIM^{-1}(\mathbf{h})] = \frac{L\sigma_\omega^2}{N\rho^2} = \frac{L}{N \times SNR}$

where $SNR = {E_s}/{N_0} = \frac{2\rho^2}{2\sigma_\omega^2}$. Thus, $CRLB_{\mathrm{H}} = \frac{L}{SNR}$.