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ENHANCMENT IR IMAGE USING WAVELET COEFFICIENTS AND HISTOGRAM EQUALIZATION

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Abstract

To obtain the best quality and the highest image feature resolution for target recognition system, many procedures have been used throughout the last years. In the current research, the procedure for contrast enhancement and resolution of infra-red (IR) images is based on lifting wavelet transform (LWT). The processing and combining with IR image of high sub bands (LH, HH and HL) by using LWT produced images with good recognition among different objects are captured by thermal camera for using LWT-HE technique. Where, Maxerr and L2RAT measures indicate that, the enhanced images preserved the general properties of IR image. An AG value indicates the increment of original IR images (38.1205, 35.9092 and 46.0523) to increase the enhanced images (41.4049 to 61.697). LWT-AHE measures indicate that the technique gives better enhancement than the usual AHE technique. Where, Mean, Entropy, Maxerr and L2RAT measures for enhanced images preserved the general properties of IR image. The values of AG are ranging from (39.4881-71.0950), it is a good indication of a significant contrast enhancement and clarifies of the image edges. The dark regions of the enhanced IR images become clearer for input IR image that having low temperature degrees. A comparison between the calculations of quality measures of the resulted images using the known AHE and proposed method of LWT-AHE. Good stability values of measures observed using the known AHE for all output images. It is found that AG of edges is enhanced entirely for output images by using the proposed method. For output enhanced images, the regions have more specific and detailed clarity, and then different objects can be distinguished in the same vision. The technique can be used in fields of development thermal camera, especially to clear night IR images and acquire good resolution.

Keywords: Contrast enhancement, Histogram equalization techniques, IR images, LWT.

1. Introduction

The infrared (IR) image used in detecting or tracking target, surveillance, object discrimination, remote sensing, and recently for medical applications [1]. Three spectral ranges are defined in thermography: Long-wavelength (LWIR 8-14 µm) region, Middle-wavelength (MWIR 3-5 µm) region and Short-wavelength (SWIR 1-3 µm) region [2]. In order to acquire an IR image of three wavelength-regions, a thermal observation device is required for each wavelength-region. Individual radiances corresponding to minimum/maximum temperatures of an arbitrary IR images calculated through the Planck's law. All objects above emit IR radiation as a result of their molecular motion. The wavelength of this radiation ranges from 0.7 to 1000 mm. The range from 0.7 to 14µm is best suited for thermal-IR imaging and is further subdivided into near-IR (0.7-1 µm), mid-IR (3-5 µm), and far-IR (8-14 µm). Most thermal cameras operate within the mid-IR portion of the spectrum [3]. Notice that the target and background are considered to be grey bodies so their emissivity is smaller than 1 (black body emissivity =1).

Some of IR images are hard to get feature from it because blurring, the quality of an IR image is more often linked to its contrast and brightness levels enhancing these parameters will certainly give us the best result [4]. Pixel brightness difference in IR image reflects the change of radiant energy intensity of target and scene [5]. Infrared images have low contrast and characteristics of high background because of the low temperature difference between the target and the background in the scene [6]. The goal of enhancement image technique is removing the distortion information, noise, intensity saturation, blurring pixels and determine the hidden information that are associated with image [7]. Enhancement methods classified as spatial domain and frequency domain transformation [8]. Enhancement the contrast in image techniques is an active task to be use added to it is difficult to take out a good contrast of image. There are many descriptions for an image to have lower contrast as a result to lower quality used in imaging device [9].

2. Related work

Many studies in this field are presented to get a comprehensive qualitative and quantitative image enhancement performance evaluation to verify the proposed theory and algorithm validity, efficiency, and reasonability. To compare the results selecting some of them in last years.

Yin and Yu [10] proposed a novel scheme for infrared image enhancement, which solved drawbacks of the histogram equalization algorithm that loss detail information of image and some areas are bleached, improved the visual effect of the infrared image. Bilateral filter performed on the original infrared image, obtaining a base layer and a detail layer. At base layer, most value normalizing and Gray linear mapping is applied to enhance the global contrast. At detail layer, S-curve algorithm is applied to enhance detail information. Weighted synthesis used to achieve the integration of base layer and detail layer, to improve the visual effect of the infrared image.

Sonal et al. [11] proposed a novel enhancement to Histogram Equalization technique by modifying the probability density function with a padding factor to improve background contrast. The proposed method also gives better enhancement results yielding a higher level of detail in the images.

Minjie et al. [12] proposed a particle swam optimization based local entropy weighted histogram equalization which involves the enhancement both local details and fore-and background contrast. Comparative experiments implemented on real infrared images prove that the algorithm outperforms other state methods in terms of both visual and quantized evaluations.

Ashiba et al. [13] presented a fabulous enhancement approach for infrared (IR) images. This approach mixes the benefits of the undecimated Additive Wavelet Transform (AWT) with the homomorphic transform and Contrast Limited Adaptive Histogram Equalization (CLAHE). The basic idea of this approach depends on applying the CLAHE on the IR image. Resultant image decomposed into sub-bands using the AWT. The homomorphic enhancement implemented on each sub-band, separately, up to the sixth sub-band. The homomorphic enhancement applied on the IR image in the log domain by decomposing the image into illumination and reflectance components. The illumination is attenuated, while the reflectance is magnified. Applying this method on each sub-band gives more details in the IR image. The performance quality metrics for the suggested approach are entropy, average gradient, contrast, and Sobel edge magnitude.

Yuanbin and Jian [14] showed that proposes an improved infrared image enhancement algorithm, using adaptive median filtering method for original image and platform histogram equalization with gamma transformation performed on the filtered image to improve the overall contrast of the image. At the same time, the filtered image performed by Laplacian sharpening to get the edge information of the image and weighted combination of gamma transformation and Laplacian sharpening

Ende et al. [15] showed that the infrared image has obvious stripe noise, which greatly affects its quality. The generation mechanism of stripe noise is analysed, and a new stripe correction algorithm based on wavelet analysis and gradient equalization is proposed, according to the single direction distribution of the fixed image noise of infrared focal plane array. The raw infrared image transformed by a wavelet transform, and the cumulative histogram of the vertical component is convolved by a Gaussian operator with a one-dimensional matrix, in order to achieve gradient equalization in the horizontal direction.

3. Histogram Equalization (HE)

Analytical histogram equalization is a technique for adjusting image intensities to enhance contrast. The contrast defined as a difference in colour or intensity between two objects in an image. If the contrast is too low, it is impossible to distinguish between two objects and they are seen as a single object. A good histogram means that almost all of the pixels are used because the whole available range of intensities contributes to the image [16]. The technique that produces image with regular distribution of pixels intensity is obtained as result of flattened and extended systematically the histogram image. The probability density function (PDF) and cumulative density function (CDF) for the histogram input of image and replacing the levels of input colour image with respect to the new image levels and generate the output image with histogram for the gray-scale level intensities are stretched and depressed systematically [17]. The basic concept of HE is to remap the value of the intensity of the original image to a new level of intensity through the transformation function. HE can be calculated using Eq. (1) [18].

$$S_k = \frac{(n_G-1)}{n} \sum_j^k n_{rj} \quad (1)$$

where S_k is the number of gray or for colored (R, G, B) levels in the image, $\boldsymbol{n}$ the number of all pixels in image$(\mathbf{0}, \mathbf{1}, \ldots\ldots (\boldsymbol{n_G} - \mathbf{1})$, and $\boldsymbol{n_{rj}}$ are the number of pixels that have $(\boldsymbol{r}, \boldsymbol{j})$ gray or colored level. Mapping the grayscale image based on the probability distribution of the input grayscale with a stretch dynamic range of histogram image, the results is a contrast enhancement. Histogram equalization significantly changes the brightness of the original which makes some areas of output image very bright or very dark [19].

4. Adaptive Histogram Equalization (AHE)

AHE is an improved technique for the HE, contrast enhances of images by the transforming the values in the intensity image I on small data regions (tiles) rather than the entire image. Each tile's contrast is enhanced, so that the histogram of the output region approximately matches the specified histogram. The contrast, especially in regular areas, can be limited to avoid amplifying the noise which might be present in the image AHE can be found according to Eq. (2) [20]:

$$S = T(r) = \int_0^r p_r(w) d(w) \quad (2)$$

The variable *(r)* represents the gray level of an image to be increased, *(T)* the transformation function and *(S)* the transformed value. If (p_r) and *(p_s)* represent the function of the probability density *(r)* and *(s)*. Then *ps* can be obtained by applying Eq. (3) as follows:

$$p_s(S) = p_r(r) \left| \frac{dr}{ds} \right| \quad (3)$$

From *T(r)* transformation function getting (p_s) so that $p_s(S)$ follows an almost uniform distribution yields an equalized image histogram. The HE and AHE are preform as known for the grayscale image, for the suggested technique are applied for each one of the components to get the enhanced IR image.

5. Wavelet Transformations

The discrete wavelet transforms DWT defending to the function *ψ(x)* that known within a finite range and having the mean value of zero. The general concept of the wavelet technique is to illustrate in an arbitrary function of time an overlapping of a set of such wavelets or basis functions. Basis wavelets are gained from a single prototype expansion wavelet which named the mother wavelets by dilations and shifts [21, 22]. This function is called scaling function or mother function *φ(x),* and within certain conditions that is the properties of the wavelet function the wavelet function resulted as follow [23]:

$$\psi(x) = \sum_{k=-\infty}^{\infty} (-1)^k b_k \varphi(2x - k) \quad (4)$$

$$\psi(x) = \sum_{k=-\infty}^{\infty} (-1)^k a_{1-k} \varphi(2x - k) \quad (5)$$

DWT is resulted by sampling the parameters of wavelet on a net. Reconstruction the signal from its transform values naturally depending on the coarseness of the sampling net [24]. A fine grid mesh would permit easy reconstruction, but with evident redundancy, as a result oversampling. A too-coarse grid could result in loss of information [25]. For a signal x the DWT is computed bypass into x through a set of filters. First the samples or pixels are passing into through a low pass filter with impulse response g resulting as in equation follow

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]g[n-k] \quad (6)$$

The outputs of filter are down-sampled by 2 which is decimation operation in the filter bank; the following equations describe the decomposition process of $x[k]$ depending on filters g and h impulse response as follow:

$$y_{low}[n] = \sum_{k=-\infty}^{+\infty} x[k].g[2n-k] \quad (7)$$

$$y_{high}[n] = \sum_{k=-\infty}^{+\infty} x[k].h[2n-k]) \quad (8)$$

For image DWT decomposes into four sub band images, which are average (LL), vertical (HL), horizontal (LH) and diagonal (HH) information.

Lifting wavelet transform (LWT) is a new function of basis wavelet that added to build a new wavelet generation [26]. In the recent years there is an increased in the demand for better quality images in the various applications such as medical**,** astronomy, object recognition. Image enhancement process depends on two domains image domain and transform domain**.** Images are transformed in order to obtain high resolution [27]. The lifting scheme has numerous benefits compared with classical DWT [28]. LWT usually requires less mathematical operations compared with traditional approach convolution. LWT achievement does not require additional memory because of the in-place calculation features of the lifting. This is particularly suitable for the device's implementation of with a limited memory. LWT scheme submitted integer to integer transformation appropriate for lossless processing signal [29]. LWT coefficients are integers which reduced quantization error that arise from other wavelet schemes. The difference between the low-resolution input image and the LL sub band image are in their high-frequency components, in the intermediate stage this difference image used to correct the estimated high frequency components [30].

6. Image Quality Measurements

In image processing applications, objective measures are considered as the most common procedure for the measurement of difference between the input and output image signals. The objective measures that are used in image output folder that used for testing proposed algorithm to assess the performance. The objective measures are described briefly as follows:

1) Ratio of the signal squared (L2RAT) define as the ratio of energy between the input signal X and the output approximation signal (XAPP), which is a positive real number can be calculated as [29]:

$$L2RAT = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} B(i,j)}{\sum_{i=1}^{m}\sum_{j=1}^{n} A(i,j)} \quad (9)$$

when X and XAPP can be found as follow:

$$A = X^2(i,j) \quad (10)$$

$$B = XAPP^2(i,j) \quad (11)$$

2) Maximum absolute squared deviation (MAXERR) of image array *X* from the approximation image array XAPP which are of the same size, MAXERR returned as a positive real value and can be defined as follow:

$$MAXERR = \sum_{i=1}^{m} Max(|\, D(i,j)|) \quad (12)$$

where$|\, D(i,j)|$ is defined as:

$$|\, D(i,j)| = X(i,j) - XAPP(i,j) \quad (13)$$

3) Entropy Image is a measure which is used to characterize the randomness statistical measure of an image. Amount of information which should be coded for by a processing algorithm. Entropy can be used to characterize the quality of the input image defined as corresponding states of intensity level which individual pixels can adapted. It is possible to calculate as *H(x)* using following equation [31]:

$$H(x) = -\sum_{I=1}^{n} P(I)\, log_2 P(I) \quad (14)$$

In the above expression, *Pi* is the probability that the difference between two adjacent pixels is equal to *i*, and $\log_2$ is the base 2 logarithms.

The $P(I)$ value is the probability that the difference between two adjacent pixels is equal to *i*, and $log_2$is the base 2 logarithms. For single-channel 8-bit image (256 intensity levels), $P(I)$value computed as follows:

$$P(I) = \frac{\text{number of occurrences of the intensity level I}}{\text{number of intensity levels}} \quad (15)$$

4) Average gradient (*AG*) used to measure the change in contrast and edges in images. Gradients carry important visual image information that is essential to understanding a scene. Using this information, structural changes and contrast changes can be effective captured. Part from structural, contrast changes and image quality are also affected by intensity changes, which should also count towards a more robust and complete image quality [31]:

$$AG = \frac{1}{(m-1)\times(n-1)} \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \sqrt{\frac{f(i,j)-f(i+1,j)+f(i,j)-f(i,j+1)}{2}} \quad (16)$$

where *m* is numbers of rows; *n* is the number of columns in the image; *f* (*i*, *j*) is the pixel value of the image at point (*i*, *j*).

7. IR Image Enhancement

The enhancement resolution of IR images is a little different from classical image in treating with wide dark block and the small features. The main objective of proposed algorithm is the contrast enhancement resolution of IR image. In this study, one level LWT is implemented on IR image, which is decomposed into four sub bands coefficients [LL, LH, HL and HH]. Here, the LL sub band holds no edge information, unlike the sub bands [LH, HL and HH] inherently include the edge information within. Edge contrast and brightness of the three sub bands LH, HL and HH are improved by using HE or AHE within wavelet domain (LWT-HE or LWT- AHE). The two techniques LWT-HE and LWT- AHE are applied on images that resulted from the combination process with IR image. Figure 1 shows the flowchart of the proposed method. The proposed algorithm is achieved in MATLAB 2018a.

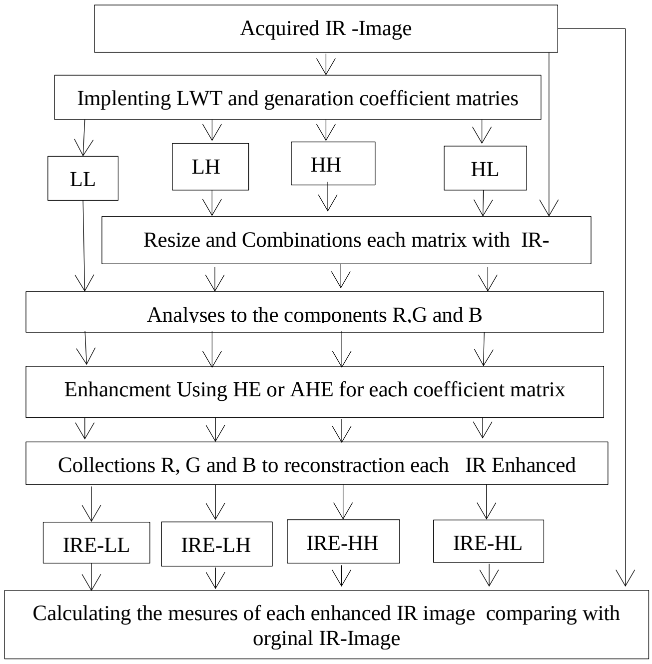


Fig. 1. Flow chart of the proposed technique.

The steps of enhancement IR image listed within algorithm in section 7.1. IR tested images are used to evaluate algorithm efficiency which is captured by thermal camera; Fig. 2shows the original IR images. The images are captured using thermal camera in different environments surrounding the objects presents in the scene image. Table 1 displays specifications mentioned in the details of the full information of the images used for testing the proposed technique in this study.

Table 1. IR input images information.

	Beechcraft King Air twin	**Machine tool bit Turner**	**Secondary Power Station**
Type Emissivity	Image 10.6	Image 20.28	Image 30.95
Temperature	T_{min} = 299 K T_{max} = 413 K	T_{min} = 283.6 K T_{max} = 413.9 K	T_{min}=3 01.1 K T_{max}=3 61.5 K
Transmission	1	1	1
Image Range	-20°C to 350°C	20.6°C to 140.1°C	-9.4°C to 87.9°C
Camera Model	FLIR SC7000	Ti10 Fluke	Ti55FT Fluke
Lens description	25 mm	20 mm	20 mm/F0.8

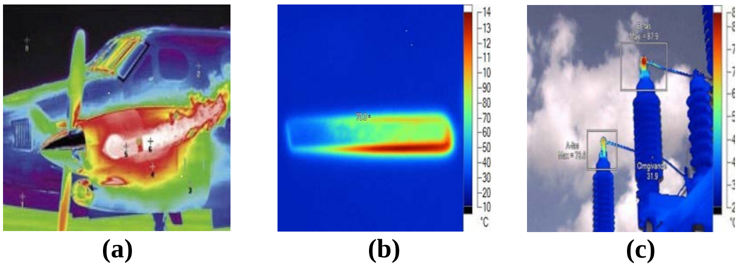


Fig. 2. IR images used to study :(a) Image 1 (b)Image 2 (c) Image3.

7.1. Proposed algorithm

Since the computational complexity is more effective and has high implementation speed, LWT is used to transform IR sequenced image to frequency domain. LWT unlike other frequency transformations techniques in enhancement that suffer from difficulty in implementation and utilization of more additional memory, due to the trouble of optimal parameters chosen. LWT coefficients are integers which remove quantization error that appear from the other wavelet transformations. Haar wavelet is adopted for the LWT scheme due to its faster computation and simplicity. LWT splits the transformed IR image into two bands of frequencies low (LL, LH) and high (HH, HL). In this study, the sub band of low frequency LL is used for enhancement without adding the IR image. The high sub bands (HH, HL and LH) are subjected to the re-sizing and adding with IR image. The algorithm details of image enhancement are given below:

Step1: Reade the sequenced original IR image and calculate its size.

Step2: Performs LWT to decompose the IR image into four sub bands (LL, LH, HH and HL) coefficients matrices which are processed later to produce the enhanced images.

Step3: Resize coefficients matrices to the dimensions similar to IR image size and process new matrices to integer values and produce four colour images.

Step4: The original IR image is added to each one of the colour images of step 3 output. Because of the noise sensitivity in the image which in most states is more stated within high frequency or image detail part, the sub bands LH, HH and HL are processed to get enhanced images. The sub band LL is left pure without adding to IR image.

Step5: Each one of the obtained images from step 4 is separated and analysed into three components R, G and B.

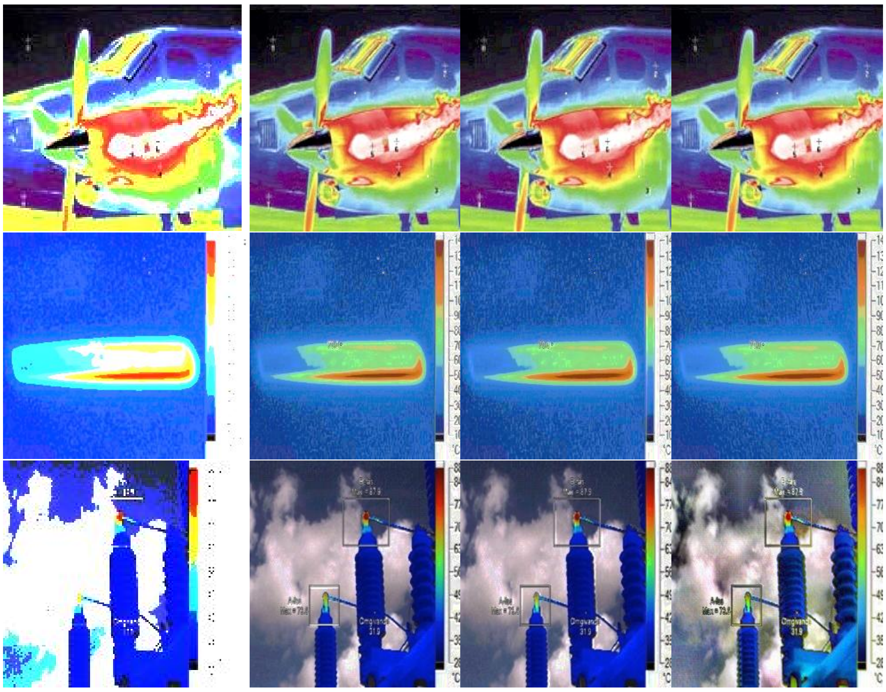
Step6: Performing HE or AHE technique for each coefficient matrix of the three components R, G and B output from step 5.

Step7: Reconstruct the enhanced images by collecting the components R, G and B for each processed image, the output is four enhanced images (IRE-LL, IRE-LH, IRE-HH and IRE-HL).

Step8: Calculating the evaluation measures (Mean, Entropy, MAXERR, L2RAT, and AG) of each one of output images.

7.2. Experimental Results and Discussion

The proposed algorithm has been tested using three IR images each one in size (256 x 256). The LWT Haar filter is used to split the transformed IR image component into low frequencies (LL, LH) and high frequencies (HH, HL) having the size (128 x 128). The output sub bands coefficients (LH, HH and HL) are resized and combined with the original image; the result is four colour images were obtained. Figure 3 shows the enhanced images using LWT-HE where the details of dark region of output images become clearer comparing with input IR image. The results of evaluation measures between input image and output images are illustrated in Table 2, for every one of IR images measure the resulted numbers have been arranged according to the output images IRE-LL, IRE-LH, IRE-HH and IRE-HL respectively.



(a) (b) (c) (d)

Fig. 3. Enhanced IR image using LWT-HE technique for the three images used to study; (a) IRE-LL. (b) IRE-LH. (c) IRE-HH. (d) IRE-HL.

Table 2. Evaluation measures of enhanced images using LWT-HE technique.

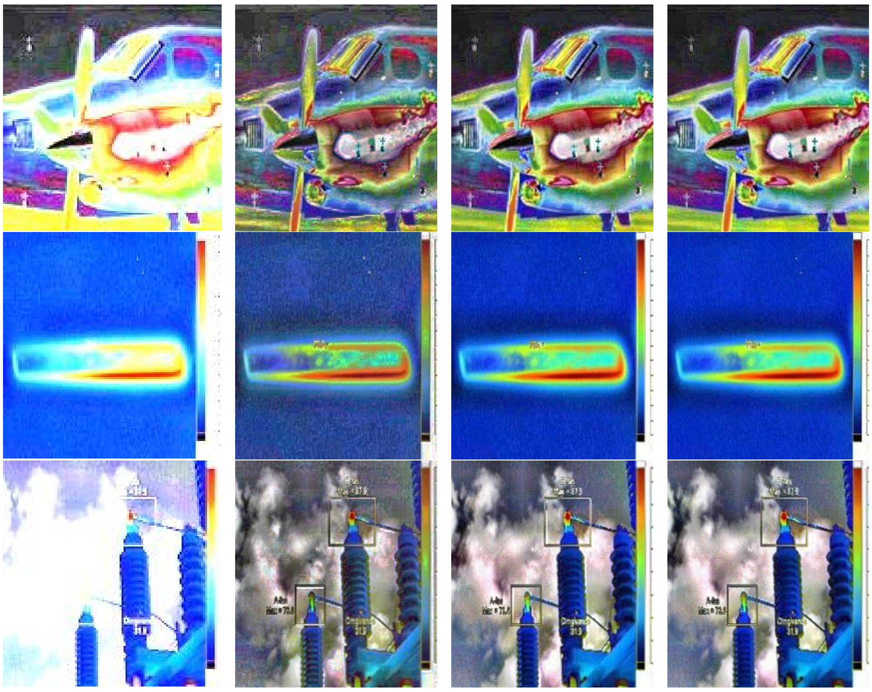
Image	mean	entropy	Maxerr	L2rat1	AG
Image 1	108.0127	7.6644			38.1205
	176.8373	4.0774	104	0.8087	57.6578
	127.4474	5.9868	33	0.9479	43.3108
	127.5109	5.9615	33	1.3081	42.5666
	127.6543	5.9871	36	1.3551	41.4049
Image 2	113.2647	6.3806			35.9093
	154.9529	3.4709	118	0.9610	39.0602
	128.7014	5.3048	72	0.9766	39.9939

	128.3370	5.3267	71	0.9649	38.4864
	128.2472	5.3165	72	0.9857	39.3467
Image 3	159.2213	7.5634			46.0523
	186.1609	2.3005	175	0.8534	61.6979
	128.1668	5.8007	54	0.6959	54.6775
	127.9034	5.7500	54	0.6917	54.5119
	127.8961	5.8210	52	0.7029	53.6591

7.3. Results of LWT-AHE technique

Figure 4 shows the enhanced IR images using enhancement LWT-AHE. Table 3 shows the evaluation measures of this technique, the measures illustrate the enhanced image of high contrast edges. Figure 5 displays the edges detection images of Sobel effect for IR image 3. It is noticed that there are details of components and objects which are exposed within enhanced images. Figure 6 displays the output image from processing IR image in wavelet domain of AHE algorithm known in the field of digital image processing where there are no clear details in images.

From Figs 3 and 4, it is found that the sub bands have contained high frequency especially HH and HL sub bands which are more suitable to get enhanced IR image. By performing LWT-AHE, it is noticed that the extension of histogram for output images regions, this is an evidence of getting more specifics and details with distinguishable different objects in same image vision. The histogram figures of the three original IR images and enhanced image output from processing the LWT-HE and LWT-AHE techniques for LH sub band are illustrated in Fig. 6.

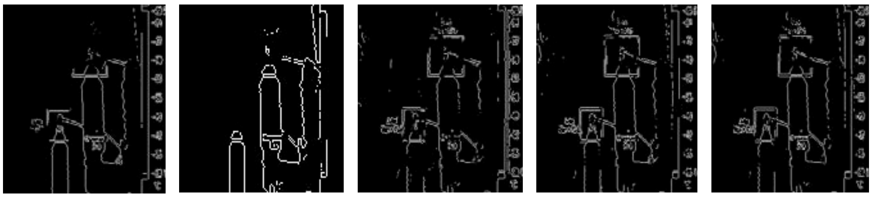


(a) **(b)** **(c)** **(d)**

Fig. 4. Enhanced IR images using LWT-AHE technique for the three images used to study: (a) IRE-LL(b) IRE-LH (c) IRE-HH(d) IRE-HL.

Table 3. Evaluation measures of enhanced images for LWT-AHE technique.

Image	mean	entropy	Maxerr	L2rat1	AG
Image 1	108.0127	7.6644			38.1205
	180.7557	5.5724	120	1.0166	51.9895
	114.9463	7.6577	149	1.2941	71.0950
	112.9444	7.8389	122	1.0478	64.6236
	113.3209	7.8338	120	1.0743	62.9206
Image 2	113.2647	6.3806			35.9093
	157.3095	4.3999	96	1.0346	36.0520
	115.2393	6.7629	132	0.7838	56.2507
	112.8573	6.8444	113	0.9329	44.3225
	113.8396	6.8558	116	0.9383	46.4273
Image 3	159.2213	7.5634			46.0523
	219.7758	3.3380	79	1.0072	39.4881
	132.8633	7.4156	171	0.6711	70.4147
	142.9644	7.5672	129	0.8308	67.7146
	143.9624	7.6665	132	0.8283	68.0569



(a) (b) (c) (d) (e)

Fig. 5. Sobel edge detection technique of IR Image 2 and enhanced images(a) Image 2, (b) IRE-LL, (c) IRE-lH, (d) IRE-HH, (e)IRE-HL.

Table 4 represents a comparison between the calculations of quality measures of the resulted images from using the known AHE and proposed method of LWT-AHE. The instability and unfamiliar values of measures (Maxerr, L2rat1) are observed using the known AHE for all output images (EIR-LH, EIR-HH and EIR-HL). It is found that AG of edges is enhanced entirely for output images by using the proposed method.

Table 4. Quality measures of AHE with LWT-AHE technique of IRE-LH, IRE-HH and IRE-HL enhanced images.

	Measures of traditional AHE in wavelet domain			**Measures of proposed algorithm (LWT-AHE) in wavelet domain**		
Image	**Maxerr**	**L2rat1**	**AG**	**Maxerr**	**L2rat1**	**AG**
Image1	111	1.0788	65.8727	149	1.2941	71.0950
	91	8.7741	39.3887	122	1.0478	64.6236
	60	18.2536	12.8289	120	1.0743	62.9206
Image2	95	13.5448	29.8660	132	0.7838	56.2507
	84	2.5278	31.4421	113	0.9329	44.3225
	55	3.9828	8.8681	116	0.9383	46.4273
Image3	91	4.3747	43.9615	171	0.6711	70.4147
	56	9.0488	12.4003	129	0.8308	67.7146
	56	9.0488	12.4003	132	0.8283	68.0569

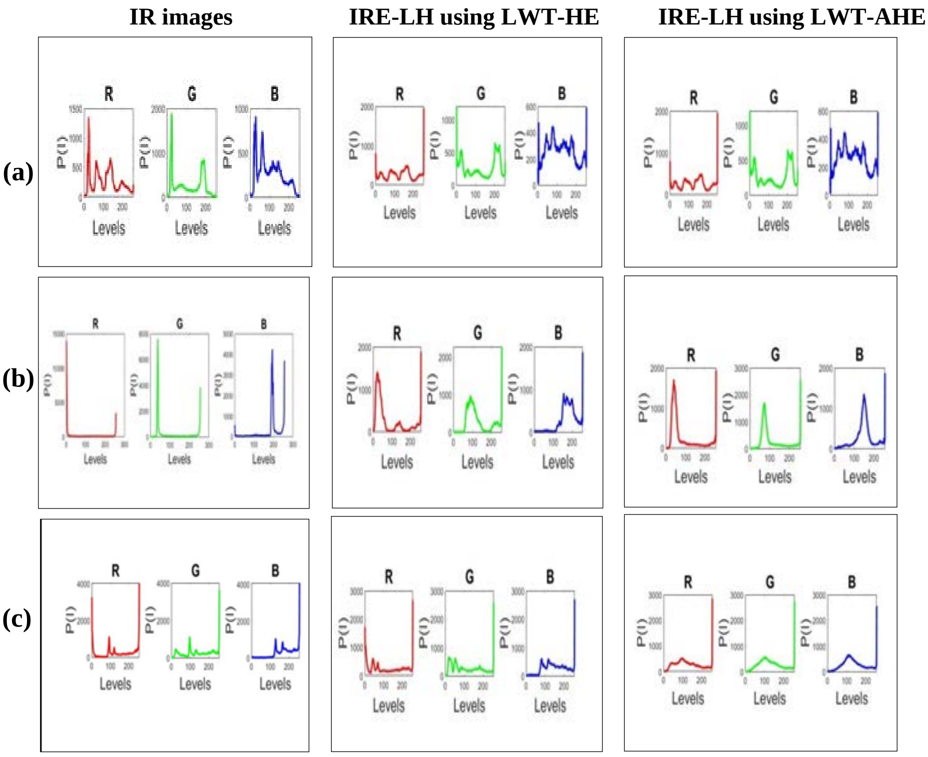


Fig. 6. Histogram for original IR images and enhanced images (IRE-LH:(a) Image 1(b) Image 2 (c) Image 3.

8. Conclusion

The proposed algorithm shows good results for output enhanced images which appear relatively clearer, and their details became more accurate than input IR images. A summary of the conclusions of this research is as follows:

- The enhancement techniques in wavelet domain by using LWT-AHE can be used to get a good enhancement resolution of thermal camera.
- The dark regions for LL sub band become clearer than IR image before, especially for the thermal image that are captured during the night times at low temperatures differences degrees, the objects within image can be correctly recognize because of the enhanced contrast and lighting edges.
- The detail coefficients LH, HH and HL of high sub bands of LWT, produced a good, enhanced IR images that are captured during the day times at high temperatures differences degrees.

Nomenclatures

n	the number of all pixels in image
Sk	the number of gray or for coloured (R, G, B) levels in the image
T	Transformation function

Greek Symbols

Ψ	Wavelet function
φ	Mather function

Abbreviations

AHE	Adaptive Histogram Equalization
CDF	Cumulative Density Function
DWT	Discreet Wavelet Transform
HE	Histogram Equalization
HL	High- Low Band
HH	High- High Band
IR	Infrared
LL	Low- Low Band
LH	Low- High Band
LWT	Lifting Wavelet Transform
PDF	Probability Density Function

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